Binary Neutron Stars: Simulations in Full General Relativity

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Code Developments

3+1 BSSN Code to handle:
- vacuum spacetimes
- relativistic hydrodynamics
- relativistic MHD

Binary Neutron Stars

“Thin-sandwich” calculations
Quasi-equilibrium approaches
Full 3+1 dynamical simulations

Binary Black Holes

Radial & non-radial instabilities
Full dynamical simulations to assess final state

Rotating Neutron Stars

Formation of
Supermassive
Black Holes

Formation and collapse of rotating:
supermassive stars
collisionless clusters, etc.

General Relativistic MHD

Magnetic braking of differential rotation
Magnetic SN core collapse, etc.
Initial Value & Time Evolution

Challenges

\[ G_{\mu\nu} = 8\pi T_{\mu\nu} \]

- Initial Value Problem
  - The mathematical solution of the Hamiltonian & momentum constraints

- Time-Independent Elliptic Equations
  - Challenge
    - To find astrophysically realistic solutions

- Time Evolution
  - The mathematical solution of the Cauchy problem for the given initial data

- Time Dependent Hyperbolic Equations
  - Challenge
    - To achieve stable evolution that conserves the constraints
Initial Value Data: Some History
(always remember your roots…)

Gravitational Fields

Hamiltonian Constraint
momentum Constraints
+ some evolution eqs.

Wilson-Mathews Conformal “Thin-Sandwich”

• Initial Data Stationary in the Rotating Frame

  Killing vector $\xi_\mu$ (approx.)

  \[ \xi^{\text{Inertial}}_\mu = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \varphi} \quad \xi^{\text{Rotating}}_\mu = \frac{\partial}{\partial t} \]

• Spatial Metric is Conformally Flat

  \[ \gamma_{ij} = \psi^4 \delta_{ij} \]

• Five elliptic eqs. for the metric fields $\alpha, \beta^i$, and $\psi$

Neutron Stars
  Wilson et al. '95, '96
  Baumgarte et al. '97, '98
  Bonazzola et al. '99
  Marronetti et al. '98, '99
  Uryû et al. '00
  Usui et al. '00, '02

Black Holes
  Grandclement et al. '02
  Pfeiffer et al. '02
  Yo et al. (in prep.)
  Friedman et al. (in prep.)
Initial Value Data: Some History
(always remember your roots…)

Hydrodynamical Fields

Continuity Equation + Euler Equations

1) Fluid-Relaxation Binary: Fluid “relaxes” in background
   - Very complex to implement
   - No control over the fluid motion

2) Corotating Binary: Stars are “tidally locked”
   - Very simple to implement
   - Astrophysically unrealistic for NSs

3) Irrotational Binary: Stars have “zero spin”
   - Simpler than 1) but more complex than 2)
   - More astrophysically realistic

Wilson et al. ‘95, ‘96
Baumgarte et al. ‘97, ‘98
Marronetti et al. ‘98
Usui et al. ‘00, ‘02
Bonazzola et al. ‘97, ‘98
Teukolsky, ‘98
Shibata, ‘98
Marronetti et al. ‘99
Uryū et al. ‘00
New Method: Constant Circulation
(Marronetti & Shapiro, \textit{in preparation})

Irrotational Formalism

Continuity Eq. \( (\rho_0u^\mu)_{;\mu} \)

Null vorticity \( \omega_{\mu\nu} = 0 \)

\[ \nabla^2 \Psi = \rho_\Psi \]

\[ u^\mu = \frac{1}{h} \nabla^\mu \Psi \]

Bonazzola \textit{et al.} '97 / Asada '98
Teukolsky '98
Shibata '98
Gourgoulhon '98

Constant Circulation Formalism

Continuity Eq. \( (\rho_0u^\mu)_{;\mu} \)

\[ V^i \equiv \frac{u^i}{u^0} \]

\[ \vec{V} = (\vec{\Omega} \times \vec{r}_c) + \vec{\Omega}_s \times (\vec{r} - \vec{r}_c) + \nabla \sigma \]

\[ \nabla^2 \sigma = \rho_\sigma \]

where

\( r_c \) : stellar center
\( \vec{\Omega} \) : Orbital angular velocity
\( \vec{\Omega}_s \) : Spin angular velocity

\[ \vec{\Omega}_s \equiv a\vec{\Omega} \]

with \( a \) an arbitrary \( (spin) \) parameter
New Method: Constant Circulation
$\Gamma=2$ Polytrope in the Rotating Frame

Full Fluid Velocity $\vec{V}_{rotating}$

Irrotational Component only $\vec{V}_\sigma$

If we do NOT solve for $\vec{V}_\sigma$

<table>
<thead>
<tr>
<th>Full Solution</th>
<th>Solution with $\sigma=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 0.13587$</td>
<td>$M = 0.13580$</td>
</tr>
<tr>
<td>$J = 0.08494$</td>
<td>$J = 0.08268$</td>
</tr>
</tbody>
</table>
New Method: Constant Circulation
Constructing Quasi-Equilibrium Sequences

How do we put together a quasi-equilibrium sequence?

1) Keep the baryonic mass $M_0$ constant

2a) For Corotating Sequences:
   Keep the spin parameter $a = 1$

2b) For arbitrary spin Sequences:
   Keep the relativistic equatorial circulation $C$ constant

\[
C = \oint_C hu_{\mu} d\lambda^\mu = \text{const}
\]

For every separation, find the spin parameter $a$ that keeps the equatorial circulation $C$ at a pre-determined value

Naively, we would expect the $C=0$ sequence to approximate the irrotational sequence
New Method: Constant Circulation

Code tests & comparisons ($\Gamma=2$ polytrope)

- $a = 1$ seq. vs. Baumgarte et al. ‘97
- $C = 0$ seq. vs. Uryu & Eriguchi ‘00

$M_0 = 2m_0$

Turning Point (ISCO?)
New Method: Constant Circulation

Quasi-Equilibrium Sequences

\[
\left( \frac{M}{R} \right)_\infty = 0.14
\]

\[
c \equiv \frac{\text{Circulation}}{\text{Circulation}_{\text{Mass-Shedding}}}
\]

\[
BE \equiv \frac{(M - 2m_\infty)}{M_0}
\]
New Method: Constant Circulation
Spin-Orbital Momentum Coupling

Null Circulation ($C=0$) Sequences

\[ \frac{\Omega_3}{\Omega} = \begin{cases} \infty & R_M \end{cases} \]

\[ \frac{M}{R} = 0.14 \]

\[ \frac{M}{R} = 0.19 \]

PPN (analytic)
Intermezzo

Coming up next:

Time Evolution of Neutron Star Binaries
Evolution: Neutron Star Binaries

(Duez, Marronetti, Shapiro & Baumgarte PRD 2002)
Some Details of the Runs

Gravitational Fields

- **BSSN Formalism** (Shibata & Nakamura ’95, Baumgarte & Shapiro ’99)
- **K-Driver** for the Lapse function (Balakrishna et al. ’96)
- **Gamma-Driver** for the Shift Vector (Alcubierre & Brügmann ’01)
- Sommerfeld boundary conditions

Hydrodynamics

- **van Leer Hydro with Artificial Viscosity** (van Leer ’77)
- No-atmosphere scheme (Duez et al. ’03)
- $\Gamma = 2$ polytrope, Compaction Ratio $(M/R)_\infty = 0.14$

**We Work in the Rotating Frame!**

Greatly improves conservation of Angular Momentum throughout the run
ISCO freq. \((\Omega M_0)\) for Corotating Dynamical Conf. thin-sandwich \(0.0147 \pm 0.0006 \quad 0.0187\) Null Circulation/Irrotational Dynamical Conf. thin-sandwich \(0.0143 \pm 0.0010 \quad 0.0192\)

ISCO frequency insensitive to stellar spin!

\[
\left( \frac{M}{R} \right) = 0.14
\]

(Marronetti, Duez, Shapiro & Baumgarte, in preparation)
Are you sure?
(Corotating Runs)

Rest Mass
Gravitational Mass
Angular momentum

\[ \Delta M_0 \approx 0.02\% \quad \Delta M_0 \approx 0.04\% \]
\[ \Delta M_G \approx 0.05\% \quad \Delta M_G \approx 0.23\% \]
\[ \Delta J \approx 0.50\% \quad \Delta J \approx 4.70\% \]

Normalization to Initial Values
Are you sure?

Segunda Parte

||Ham. constr.||_2

||mom. constr. X ||_2

||mom. constr. Y ||_2

Your typical normalization for constraints
Conclusions

**Initial Data**

- The *Constant Circulation* scheme permits the construction of initial data sets with arbitrary spin (as long as it is proportional to $J_{orb}$).

- The spin angular frequency evolves during the inspiral:
  - For $C=0$ sequences, the spin increases, reaching frequencies > 10% of the orbital frequency for the innermost orbits.

- This effect is potentially observable (GW + EM) if one of the NS is a pulsar.

**Dynamical Evolution**

- The ISCO can be found with (relatively) short simulations.

- The ISCO orbital frequency appears to be (roughly) insensitive to the stellar spins.
Has this been done before?
Kind of ...

Analysis of a sequence of Conformally Flat Initial Data sets

This one is for stars with $(M/R)_\infty = 0.15$

Baumgarte et al. ‘97
ISCO? What ISCO?

**Innermost Stable Circular Orbit**

For massless test particle in circular orbit around a Schwarzschild Black Hole the ISCO radius is $6M$ ($5M$) in Schwarzschild (Isotropic) Coordinates.

How about massive point particles?
- or massive finite-size bodies (black holes)?
- or massive non-vacuum finite-size bodies (neutron stars)?

How far in can we go and still see circular orbits?
More Details of the Runs

Cartesian Coordinates

$256^3$ grid points, which reduces to $256 \times 128^2$ when using $\pi$ and Eq. Symm.

20 (Low Res.) and 40 (High Res.) grid points across the star

16 processors for typical runs (IBM Regatta p690 at NCSA)
Separation $\sim 5 R_s$

$\Delta$ Orbital Frequency $\sim 8\%$