Numerical simulations of black holes: where avoiding the key issue is crucial

- **Singularities**: The inescapable consequence of simulating black holes
- **The Excision Concept**: Black holes without black holes
- **Numerical Implementation**: Bearing out the physical concept
- **Highlights**: The last and only 10 years of excision
- **Premonitions of the Future

K. Smith, Penn State graduate student
Singularities: The inescapable consequence of simulating black holes

A huge great enormous thing, like — like nothing. A huge big — well, like a — I don’t know — like an enormous big nothing ...

Piglet describes the Heffalump, in *Winnie the Pooh* by A.A. Milne

Singularity: Space curvature becomes infinite.
Horizon: Hides the singularity. Nothing, not even light escapes

1. Assume the cosmic censorship conjecture holds
2. Not interested in black-hole’s interior
3. Must avoid the singularity to compute exterior spacetime
The Excision Concept: Black holes without black holes

**Question:** Is it possible to model the effects of black hole singularities without including them in the calculation?

**Answer:** YES

Take advantage of cosmic censorship and remove or **EXCISE** the singularity **inside the horizon**.

**Boundary conditions at excision boundary?**

None! Excision boundary is an **OUTFLOW** boundary.

Caution: unphysical modes
Numerical Implementation: Bearing out the physical concept

Rewritten \( \mathcal{G}_{ab} = 8\pi \mathcal{T}_{ab} \) as an initial value problem. Solving set of partial differential equations in space at each instant of time.

1. Need a tool to identify location of horizon on a spatial slice: Apparent Horizon (See Jonathan’s talk)

2. Excise inside of the horizon to be conservative; therefore need horizon penetrating coordinates.

3. Freedom to choose form of equations, gauge and numerical method.

4. Physical modes propagate into the black hole, but unphysical modes can propagate out of the excision boundary.
Consequences of excision

**Excision**

- Computational Mesh
  - Boundary conforming
  - Non-conforming

- Discretization of Derivatives
  - Spectral methods
  - Finite differencing
Excision details

Excision in finite differencing

Simple excision (Alcubierre & Bruegmann)

Formal excision (LSU)

Example: Simple Excision

Excision in spectral methods: Boundary point is the same as an interior point
Highlights: 10 years of excision

- 1984
  - Excision with apparent horizon introduced by Unruh
- 1992
  - Excision implemented (causal differencing) in 1D by Seidel and Suen
- 1995
  - Collapse to black hole in Brans-Dicke theory Scheel, Shapiro and Teukolsky
- 1996
  - Black hole-scalar field interactions in spherical symmetry Choptuik and Marsa
- 1998
  - Stable black hole, BBH Grand Challenge Characteristic code (t > 60,000M)
  - Wobbling black hole, Pitt Characteristic code
  - Boosted black hole, BBH Grand Challenge ADM code (t ~60M)
- 2000
  - Grazing collision, PSU / Pitt / Texas Agave code (t ~30M)
Highlights in the last 3 years

- 2000
  - Single black hole using parameterized hyperbolic eqns, Cornell’s pseudospectral code (t ~600, 1300M)
  - Scalar field driven black hole collapse, Pretorius & Choptuik axisymmetric code with constrained evolutions
- 2001
  - Simple excision for single black hole, Alcubierre & Bruegmann (t ~500M)
  - Simple excision for dynamic black holes, AEI
  - Scalar field in moving black hole background, improved excision Yo, Baumgarte & Shapiro
- 2002
  - Fine tuned parameters, single black hole, Cornell/Caltech (t ~8000M)
- 2003
  - Scalar field in black hole background, LSU
  - Moving black holes, Penn State (t ~100M, now t >6000M)
BBH Grand Challenge

Surface area vs time
wobbling black hole

Normalized Hamiltonian constraint
boosted black hole

Gomez et al PRL 80 (1998)

Cook et al PRL 80 (1998)
Grazing collision (PSU / Pitt / Texas)

Brandt et al, PRL 85 (2000)
Recent Results

Single black hole

\[ ||C||^{1/2} \]


Distorted black hole

Alcubierre et al PRD 64 (2001)
Latest results

Moving black hole: PSU
2-d Slice of the trace of extrinsic curvature

Distorted black hole: (DS & Laguna)
2-d Slice of the trace of extrinsic curvature
Excision and Future Black-Hole Simulations

• The idea of excision is well founded although alternatives exist
  – Singularity avoiding gauge conditions (Alcubierre CQG 20 (2003)).
  – Stuffing the black hole
• Future binary black hole codes will need mesh refinement
  – Pseudospectral
  – Finite differencing with adaptive mesh refinement (Goddard)
  – Others
• They will also need a robust form of the equations of motion
  – Symmetric hyperbolic
  – Second order or
  – Constrained evolutions (Choptuik)
  None show a clear superiority yet.

• Implementing elliptic equations with excision (Cook, Diener, Grandclement, Jansen, Marronetti, Pfeiffer, …)

• Watch for in the future: Mathematics vs. Experience

• Excision will play a role in the future simulations; however, the complexity of its implementation will depending on the choices we make.