Gravitational Aspects of String Theory The past 10 years
Outline

1) Preliminaries
2) New ideas
   a) Branes
   b) Duality
   c) Holography
3) New results
   a) BH microstates
   b) Resolving singularities
String Theory

A promising candidate for
- a quantum theory of gravity
- a unified theory of all forces and particles
Brane-worlds:

Extra spatial dimensions may be large (~10 microns) or infinite, all standard model particles confined to 3+1 dimensional brane
Flat space description

Surface where open strings can end

D( Dirichlet)-brane

(Polchinski)

D-branes are charged
Branes

String theory includes other extended objects

Nonperturbative:

\[ T = \frac{1}{g}, \quad M = \frac{V}{g} \]

So \( GM = gV \)

Gravitational field is small at weak coupling
Supersymmetry

Symmetry relating fermions and bosons

In SUSY theories, $M = cQ$ states with $M = cQ$ are special (BPS states):
Mass does not have quantum corrections
String coupling: \( g \)

\[ G \sim g^2 l_s^2 \]

String theory predicts space has more than 3 dimensions.

(Kaluza-Klein)
Elementary particles are excitations of a one dimensional "string".

Strings interact by splitting and joining,

\[ \text{0} \rightarrow \text{8} \rightarrow \text{8} \]
Duality

Theory 1
with coupling $g_1$

Theory 2
with coupling $g_2 = \frac{1}{g_1}$

For $g_2 \gg 1$, D-strings are much lighter than fundamental strings. They interact via $\tilde{g} = \frac{1}{g}$. Recover weakly coupled theory.
Check of this duality:

A string _instanton_ is a 2D minimal surface with (Euclidean) worldsheet wrapped around.

Re-expressing a perturbative calculation in dual theory, one sees a sum of instanton contribution.

Can extract # of instantons.
Looking Ahead

1) Better understanding of dictionary relating string theory and gauge theory

2) Applications to cosmology

3) Discovery of SUSY (and more?) at LHC
More Results from String Theory

Singularities:
Definition is different from GR - use test strings. Some spacetimes which are singular in GR are nonsingular in string theory e.g. conical singularities, certain curvature singularities in extra dimensions.
How many BPS states are there with charge $Q$?

Number of states is precisely $e^{S_{BH}}$!

(Strominger & Vafa) 1996
Connection to Black Holes

\[ g = 0 \]

\[ \text{String states with } M = cQ \]

\[ g \gg 0 \]

\[ \text{black hole with } M = cQ \]

G.R. provides only a crude description of the state at strong coupling.
Plane waves

Exact analogs of linearized solutions

Penrose: Every spacetime has a plane wave limit

Taking this limit on $\text{AdS}_5 \times S^5$ allows one to extend AdS/CFT to plane wave spacetimes
Evaporation of a small black hole is described by ordinary evolution in gauge theory.

No violation of quantum mechanics.
1) Add mass terms to gauge theory and study RG flow.

2) Find asymptotically AdS$_5 \times S^5$ soln to E. e.g. with nonconstant scalars. Compare small radius behavior of solution with endpoint of RG flow and find detailed agreement!
Einstein's eq. knows a lot about QFT:

In a QFT, one can integrate out high energy degrees of freedom and obtain low energy effective theory. Called renormalization group (RG) flow.
A large Schwarzschild BH in AdS$_5$ has

$$ T \propto r $$

So

$$ S_{BH} = \frac{A}{4} \propto T^3 $$

just like 3+1 field theory
Evidence

1) Symmetries agree

2) Low energy spectra agree

3) Some interactions have been shown to agree

4) Can reproduce entropy of black holes in AdS from gauge theory
Radial direction in AdS$_5$ corresponds to energy scale in gauge theory:
large radii $\rightarrow$ high energy
Reconstructing 10D from 4D (near AdS$_5 \times S^5$)

Dependence on $S^5$ = coef. of spherical harmonics

Let $T_{i...j}$ be symmetric & traceless tensor in $\mathbb{R}^6$. Then $T_{i...j} x^i ... x^j$ is spherical harmonics.

Gauge theory has 6 scalars $\phi^i$. Obtain $S^5$ dependence from $\langle T_{i...j} \phi^i ... \phi^j \phi \rangle$ other operator
Example:

String theory with $AdS_5 \times S^5$ bdy. cond. is described by ordinary four dim. gauge theory.

This is a nonperturbative + (mostly) background independent formulation of string theory.
AdS/CFT Conjecture
(Maldacena): String theory on spacetimes that approach Anti de Sitter $x$ (compact) at $\infty$ is completely described by a conformal field theory (CFT) "living on boundary at $\infty"
Holography

Physics in a region of space can be described by fundamental degrees of freedom living on the boundary.

(‘t Hooft, Susskind)
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Table 5.2: The numbers of rational curves of degree $k$ for $1 \leq k \leq 10$. 

(Candelas et al.)

The number of minimal surfaces of various topologies in a compact 6D manifold
Summary

1) String theory includes other extended objects
2) Strong coupling = weak coupling
3) For some bdy conditions string theory = gauge theory
4) Have microscopic description of BH thermodynamics
5) Some spacetime singularities have been resolved