New Insights into Black Hole Physics

Olaf Dreyer

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O.D., Quasinormal Modes, the Area Spectrum, and Black Hole Entropy

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Outline

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Introduction

1st and 2nd law of black hole mechanics + Hawking temperature give

\[ S = \frac{A}{4} \]

What does \( S \) count?

Answers so far from: String Theory, Loop Quantum Gravity, Horizon Symmetries, ...

New insights from Quasinormal Modes, the ringing frequencies of a black hole.
Quasinormal Modes

Spin $\sigma$ perturbations $\phi$ of a black hole are governed by

$$\left(-\frac{d^2}{dr_*^2} + V(r_*) - \omega^2\right)\phi = 0$$

With the Regge - Wheeler potential

$$V(r_*(r)) = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{1-\sigma^2}{r^3}\right)$$

With outgoing boundary conditions

$$\phi(r_*) \sim e^{\pm i\omega r_*} \quad \text{as} \quad r_* \to \mp \infty$$
Quasinormal Modes: The Spectrum

Low damping Quasinormal mode spectrum for Schwarzschild and Kerr
The Large Damping Limit

\[ \text{Re}(\omega) \rightarrow \frac{\ln 3}{8\pi M} \]

Hod '98: Observation based on numerical data by Nollert

Motl '02: Analytic proof
Question

There is thus one particular frequency associated with a black hole:

\[ \omega_R = \frac{\ln 3}{8\pi M} \]

In a **Quantum Theory** of a black hole this particular frequency should play a role.

Does it?
Loop Quantum Gravity: The Variables

Instead of $q_{ab}$ and $P^{ab}$ we use Sen - Ashtekar - Immirzi - Barbero variables.

Introduce a triad $e^i_a$ such that

$$q_{ab} = e^i_a e^i_b$$

New SU(2) gauge freedom.

Let $\Gamma^i_a$ be the corresponding spin - connection compatible with $e^i_a$.

$$A^i_a = \Gamma^i_a + \beta K_{ab} e^b_i$$

$$E^a_i = \sqrt{q} \frac{e^a_i}{\beta}$$

And $\beta \in \mathbb{R}_{>0}$ is called the Immirzi parameter.
Loop Quantum Gravity: Hilbert Space

Spin Network States

\[ \bigotimes \pi_{j_i}(U(A, \gamma_i)) \cdot \bigotimes I_j \]
Area Operator

Area corresponding to a surface $\sigma$ can be quantized. Spin networks are eigenstates.

$$A = 8\pi\beta l^2_{\text{Planck}}\sqrt{j(j + 1)}$$
LQG and Black Hole Entropy
(Smolin, Rovelli, Krasnov, Ashtekar, Baez, ...)

Horizon area created by punctures from spin network edges.
LQG and Black Hole Entropy II

On the horizon lives a Chern-Simons theory. Each puncture of spin $j$ increases the dimensionality of boundary Hilbert space by

$$2j + 1$$

For large number of intersections the dimension is

$$\prod_i (2j_i + 1)$$

Lowest spin $j_{\text{min}}$ is most important.

$$S = N \ln(2j_{\text{min}} + 1)$$

Thus for $j_{\text{min}} = 1/2$

$$S = \frac{A \ln 2}{4\pi \sqrt{3} \beta l_{\text{Planck}}^2}$$

Use this to fix $\beta$ to $\ln 2 / \pi \sqrt{3}$. 
A Connection

The classical frequency $\omega_R$ corresponds to a quantum transition.

Possible candidate in LQG: Appearance or disappearance of an edge.
A Connection II

The mass changes by

$$\Delta M = \hbar \omega_R$$

Since \( A = 16\pi M^2 \) we have

$$\Delta A = 32\pi M \Delta M = 4 \ln 3 \ l_{\text{Planck}}^2$$

This gives for the entropy:

$$S = \frac{A}{\Delta A} \ln(2j_{\text{min}} + 1)$$

$$= \frac{A}{4l_{\text{Planck}}^2} \frac{\ln(2j_{\text{min}} + 1)}{\ln 3}$$

Switching from SU(2) to SO(3) gives \( j_{\text{min}} = 1 \) and thus the Bekenstein-Hawking value.
Remarks

- Provides independent way of fixing $\beta$. Entropy becomes a prediction.

- Is it true? To answer this question we need to
  - understand what is going on at the horizon.
  - get rid of the surface we put in by hand.
  - understand how black holes arise; learn about the classical limit.
  - solve the Hamiltonian constraint and tackle the problem of time.

This seems to be hard ...
Near Horizon Symmetries

Generalized symmetry vector fields in near horizon geometry

$$\xi_n, n \in \mathbb{Z}$$

Central charge in classical Poisson bracket

$$\{H_{\xi_1}, H_{\xi_2}\} = H_{[\xi_1, \xi_2]} + \text{c.c.}$$

Cardy formula gives Entropy.
Discussion

Hints from Classical Theory → Quantum Theory of Gravity → Direct Approach