The Asymptotic Structure of Homogeneous Plane Waves (and related topics in 30 min. or less...)

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What, in general, is the boundary of a spacetime?
Plane waves: spacetimes $\gamma$ cov. cons.
null $KVF$
$\sim$ planar symmetry.

motivation: String Theory, but also simple exact sol's to GR, or any other theory of the form

$$S = \int V^2 (R + \text{other curvature scalars})$$

null $KVF$ reduces other curvature terms to $R_{\mu\nu}$.
Curvature scalars vanish.

$$ds^2 = -2 dx^+ dx^- - \sum_{ij} m^2 \cdot x^i x^j (dx^+ dx^-)$$

$$R_{++} = \text{Tr} m^2 \geq 0 \text{ for } m^2 \geq 0 \text{ for } m \in \mathbb{C}$$
Lorentz sig., any # of dim.

$$m^2_{ij} = m^2_{ji} (x^+)^n$$ \text{ not necessarily pos. def.}

Hom. $\Rightarrow \frac{2}{dx^+}$ is sym, $m^2_{ij}$ = constant.
Simple case #1: $\mathbf{m}_{ij} = \delta_{ij}$

(Berenstein \& Nastase)

$$ds^2 = -2dx^i dx^i - (\delta_{ij} x^i x^j) dt^2 + \delta_{ij} dx^i dx^j$$

Conformally flat!
Conformally-flat

Other simple case: $\mu_{ij} = -\delta_{ij}$

(fails null conv. cond.)

$\Rightarrow \eta_{ij} E$

$ds^2 = -2dx^i dx^j + (\delta_{ij} x^i x^j) (dx^i)^2$

$+ \delta_{ij} dx^i dx^j$

Conformal to slice of Minkowski between two parallel null planes
Infinity for other Hom. Plane waves?

NOT Con formally Flat!

\[ \Rightarrow \text{Con formal embedding fails} \]

If \( d^2 = \Omega^2 d^5 \)

G.H.

\[ \text{Weyl (c) \rightarrow 0} \]

\[ \Rightarrow c \sim r^2 \rightarrow \infty \]

What to do?
GKP strategy

(y Timelike $\Rightarrow$ Construct
IP: $\mathbb{I}^{- \left[ I^\ast \right]} = \mathbb{I}^\ast$
(Past)

$\mathbb{I}^\ast$ (original)

$\mathbb{P}^I$ and $\mathbb{T}^I$ (original)

$\mathbb{P}_1$, $\mathbb{P}_2$, $\mathbb{P}_3$, $\mathbb{P}_4$, $\mathbb{P}_5$ (original)
GKP: 1) Identify $\mathcal{P}$ and $\mathcal{P}^*$
if past/future of $p_0$.

2) Further identify $\mathcal{T}$ to make a certain topology Hausdorff.
Also need IF'S
$I^+[\mathcal{G}]:=P^*$
\( \cap I^-(x_n) = \bigcup I^a(y_n) \)

\( \cap I^+(y_n) = \bigcup I^+(x_n) \)
For each \( P \), \( P^* \)

Construct "post" \( p(P^*) = \text{Inf} \{ x : \text{Inf} \subseteq P^* \} \)

Now, find all mutually maximal pairs; i.e. \( P, P^* \)

\( P \) is largest \( I \subseteq p(P^*) \)

\( P^* \) is largest \( I \subseteq f(P) \)

S2 abadado: identify \( P \cup P^* \), but we...
Various Improvements...

Our rule:

For each IP \( P \), construct "Future":

\[ f(P) = \text{Int}(x : \mathcal{I}(x) \supseteq P) \]
\( \overline{M} = \left\{ (p, p^*) : \ p \& p^* \text{ are mutually maximal} \right\} \)

OR \( f(p) = \emptyset = p^* \)

OR \( p(p^*) = \emptyset = p \)

**Results**

1) For \( p \in M \), \( I^+(p) \) appear in exactly one pair \( (I^-(p), I^+(p)) \).

2) Chronology (timelike relation) extends to \( \overline{M} \), weakly distinguishing.

3) Correct point sets in simple examples (e.g., cont. flat plane wave).

4) All plane waves but \( \Delta \xi_j = -\delta_{ij} \) have causal boundary = Single null curve (same for Stabaders)
Trivial 1+1 Example

Minkowski - half line

 maximal pairs:
 (C, A*)
 (C, B*)

⇒ our causal
 boundary contains
 both points

vs. identification rule: A* ∼ C ∼ B**
more important 2+1 Example
Show Space

before $x = 0$

$A, B, C I^- (0)$

max. pairs

$(A, C^*)$

$(B, C^*)$

$(A, D^*)$

$(B, E^*)$

after $x = 0$

$C^*, D^*, E^* C I^+$

Note: Identifications would lead to $q \not\in \mathcal{E} I^+ (p)$ !!!!
Summary

1) Conformal boundary method not applicable to plane wave \( \Rightarrow \) need new approach.

2) Causal Boundary for simple flat case:

3) More generally, causal boundary is single curve (like above) but diagram = ??

4) Causal boundary construction is still under construction!?
   (we suggested improvements but room for more)
Beyond the point-set...

A topology?

Historically, a problem, more help needed!!!

We introduce two.

Both satisfy

1) $M$ homeo to $MC_\overline{M}$

2) All timelike curves in $M$ have (at least) one endpoint in $\overline{M}$.

3) Each $x \in \overline{M}$ is an endpoint of some timelike curve.

4) $M$ is dense in $\overline{M}$

But which gives 'intuitive' results depends on example...