Black hole horizons and holography

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Tommy Levi & SFR, hep-th/0304150

Kraus, Ooguri & Shenker, hep-th/0212277, Maldacena, hep-th/0106112,

Louko, Marolf, & SFR, hep-th/0002111, Horowitz & SFR, hep-th/9803085

- Poincaré horizon
- BTZ black hole
- Correlation functions in CFT
- Correlation functions in bulk
- Another analytic continuation
- General lesson & open issues
Poincaré horizon

Start with a horizon we do not understand:
Poincaré coordinates

\[
 ds^2 = \frac{1}{z^2} (-dt^2 + dz^2 + d\vec{x}_n^2)
\]

Boundary at \( z = 0 \): \( \mathbb{R}^{n,1} \). Horizon at \( z \to \infty \).

Global coordinates:

\[
 ds^2 = -\left(\frac{1 + R^2}{1 - R^2}\right)^2 d\tau^2 + \frac{4}{(1 - R^2)^2} (dR^2 + R^2 d\Omega_n)
\]

for \( 0 \leq R < 1 \): boundary \( \mathbb{R} \times S^n \)

CFT on \( \mathbb{R}^{n,1} \) dual to Poincaré region. Extension to ESU describes global coord region.  

Lorentzian AdS/CFT: theory ↔ boundary conditions, state ↔ initial data.

Horowitz & Ooguri

Balasubramanian, Kraus & Lawrence
**Black holes**

Euclidean black hole $\leftrightarrow$ thermal state in field theory

CFT on $S^1 \times \mathbb{R}^n$ $\leftrightarrow$ BH geometry

**Analytically continue:**

- What does CFT describe?
- Analytic continuation gives CFT on 2 copies of $\mathbb{R}^{n,1}$
- Completely specifies geometry

**Concrete question:**
bulk calc of correlation functions integrates over what region?
Rotating BTZ black hole

\[ ds^2 = - \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} dt^2 + \frac{r^2 dr^2}{(r^2 - r_+^2)(r^2 - r_-^2)} + r^2 \left[ \frac{r_-}{r^2 + r_2^2} (r^2 - r_+^2) dt + d\phi \right]^2 , \]

Event hor \( r = r_+ \), Cauchy hor \( r = r_- \), “Sing” \( r = 0 \).

▷ Locally AdS$_3$:

\[ T_1 = \sqrt{\alpha} \cosh (r_+ \phi), T_2 = \sqrt{\alpha - 1} \sinh \left( \frac{r_+^2 - r_-^2}{r_+} t - r_- \phi \right) , \]

\[ X_1 = \sqrt{\alpha} \sinh (r_+ \phi), X_2 = \sqrt{\alpha - 1} \cosh \left( \frac{r_+^2 - r_-^2}{r_+} t - r_- \phi \right) . \]

\[ \alpha = \frac{r_+^2 - r_-^2}{r_+^2 - r_-^2} . \] Identifications along \( \xi = \partial_{\phi} \).

★ Discard region where \( \xi \) is timelike.

Euclidean section: \( t \rightarrow -i\tau, r_- \rightarrow -i\tilde{r}_- \)

\[ ds^2 = \frac{(r^2 - r_+^2)(r^2 + \tilde{r}_-^2)}{r^2} d\tau^2 + \frac{r^2 dr^2}{(r^2 - r_+^2)(r^2 + \tilde{r}_-^2)} + r^2 \left[ - \frac{\tilde{r}_-}{r^2 + r_2^2} (r^2 - r_+^2) d\tau + d\phi \right]^2 . \]
Correlation functions in CFT

Euclidean CFT on boundary $T^2$,

$$ds^2_{\Sigma} = d\tau^2 + (d\phi - \frac{\tilde{r}_-}{r_+} d\tau)^2$$

Consider, e.g., 3-point function:

$$G(b_1, b_2, b_3) \sim \lambda \int d\tau d\phi G(b_1, b') G(b_2, b') G(b_3, b')$$

Define Lorentzian correlation function by analytic continuation in $\tau, \tilde{r}_-$:

Two copies of $\mathbb{R} \times S^1$: vertical segments represent entanglement.

Niemi & Semenoff
Correlation functions in bulk

Use AdS/CFT to relate correlation function to integral over Euclidean geometry:

$$G(b_1, b_2, b_3) \sim \chi' \int_{r_+} dr \int d\tau d\phi K(b_1, x') K(b_2, x') K(b_3, x')$$

Perform same analytic continuation $\tau \to it$:

Integral over the two regions $r \geq r_+$.

▷ Doesn’t include whole spacetime—but does include whole boundary.
▷ Region beyond horizon determined by this region.
◆ Look for another analytic cont to make this explicit.
Another analytic continuation

New coordinates on Euclidean spacetime:

\[
\begin{align*}
\frac{ds^2}{(1 - X^2 - Y^2)^2} &= \frac{4}{dX^2 + dY^2 + \tilde{r}_-(X dY - Y dX) d\phi + \tilde{r}_+^2 (X^2 + Y^2) d\phi^2 + \frac{r_+^2}{4} (1 + X^2 + Y^2)^2 d\phi^2} \\
0 &\leq X^2 + Y^2 < 1. \text{ Closely related to global coords.} \\
\text{Coordinate trans:} \\
X &= \left(\frac{\sqrt{\alpha} - 1}{\sqrt{\alpha} + 1}\right)^{1/2} \cos \left(\frac{r_+^2 + \tilde{r}_-^2}{r_+} \tau\right) \\
Y &= \left(\frac{\sqrt{\alpha} - 1}{\sqrt{\alpha} + 1}\right)^{1/2} \sin \left(\frac{r_+^2 + \tilde{r}_-^2}{r_+} \tau\right) \\
\text{Analytic continuation } Y \to iT, \tilde{r}_- \to i r_-:
\end{align*}
\]

\[
\begin{align*}
\frac{ds^2}{(1 - X^2 + T^2)^2} &= \frac{4}{dT^2 + dX^2 - r_- (X dT - T dX) d\phi + \tilde{r}_-^2 (X^2 - T^2) d\phi^2 + \frac{r_+^2}{4} (1 + X^2 - T^2)^2 d\phi^2}.
\end{align*}
\]

▷ NB: real Lorentzian metric.
\[ X^2 - T^2 = 1 \text{ is } r = \infty, \quad X^2 - T^2 = 0 \text{ is } r = r_+, \]
\[ X^2 - T^2 = -1 \text{ is } r = r_- \]

- Note similarity to Minkowski vs Rindler coordinates

Euclidean integral

\[ G(b_1, b_2, b_3) \sim \lambda' \int_{X^2 + Y^2 < 1} dX dY \int d\phi \]
\[ K(b_1, x') K(b_2, x') K(b_3, x') \]

★ How to analytically continue?

Metric invariant under inversion:

\[ X, Y \rightarrow \frac{X, Y}{X^2 + Y^2}; \quad X, T \rightarrow \frac{X, T}{X^2 - T^2} \]
If $K(b_i, x')$ invariant under inversion, can write

$$G(b_1, b_2, b_3) \sim \frac{1}{2} \lambda' \int dXdY d\phi K(b_1, x') K(b_2, x') K(b_3, x')$$

Analytic continuation in $Y, \tilde{r}_-$:

$$G(b_1, b_2, b_3) \sim \frac{1}{2} \lambda' \int dXdTd\phi K(b_1, x') K(b_2, x') K(b_3, x')$$

$$\sim \lambda' \int_1 dXdTd\phi K(b_1, x') K(b_2, x') K(b_3, x')$$

(with a complicated presc in the region $r_- < r < r_+$)
Lesson & open issues

▷ Region beyond event horizon is included in the holographic description.

▷ CFT is more than a local observer; it encodes information which will never be accessible asymptotically.

▷ Region beyond Cauchy horizon is not included - no boundary conditions at timelike singularity.

▷ AdS/CFT respects strong cosmic censorship.

▷ Higher dimensions: technically hard, but in principle should be similar.

▷ See blueshifting singularity at Cauchy horizon in CFT?