RENORMALIZATION FOR SPIN FOAM MODELS OF QUANTUM GRAVITY

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CONVENTIONAL RENORMALIZATION

- prototype: lattice models

\[ \begin{array}{c}
\text{rescale} \\
\hline
a \quad \sim \quad \frac{\text{discretization}}{\text{determined by}} \\
\hline
a' \end{array} \]

- changes of discretization are elements \( g \) of the group \( G \) of scale transformations
- observables \( \Theta(a, \tau) \) depend on length \( a \) and coupling constants \( \tau \)

RENORMALIZATION PROBLEM:

- tune couplings \( \tau \) such that they compensate the change of observables induced by a change \( g \) of discretization

\[ \Theta(g \cdot a, g \cdot \tau) = \Theta(a, \tau) \]

- solutions exhibit renormalization group flow (orbits of \( G \) in \( \Lambda \))

\[ \begin{array}{c}
\text{IR fixed point} \\
\hline
\text{UV fixed} \end{array} \]

arrows point in direction of increasing a (or decreasing \( \tau \))
consider model defined on arbitrary
discretization \( \lambda \) (no background, no length scales)

\[
\begin{align*}
\text{discretizations} & \quad \text{can change locally} \\
\end{align*}
\]

changes \( g \) of discretizations are
pairs \((a, a')\) of discretizations
\(\rightarrow\) these are elements of a groupoid \( G \)

need model with \underline{local couplings} \( \lambda \) to tune,
i.e. \( \lambda \) represents an assignment of couplings
to elements of a discretization \( a \)

observables \( \Theta(a, \lambda) \) depend on discretization \( a \)
and couplings \( \lambda \)

\[
\text{RENORMALIZATION PROBLEM:}
\]

\( \rightarrow \) find action of groupoid \( G \) on spaces
\( \Lambda(a) \) of couplings such that:

\[
\Theta(g \cdot \lambda, g \cdot \lambda') = \Theta(a, \lambda')
\]

[\text{note: if } g = (a, a') \text{ then } g : \Lambda(a) \rightarrow \Lambda(a')]

\( \uparrow \) action often only makes sense if \( g \) is
\underline{coarsening} as local couplings cannot
be "created" out of nothing
The Cellular Moves

Consider compact topological manifolds and their cellular decompositions, i.e. decompositions as CW-complexes.

In dimension 1 define a set of n local changes of cellular decomposition, called the cellular moves.

In dimension 3:

• 3-move

\[
\begin{array}{c}
\text{3-cell } \sigma^3 := \sigma_0 \cup \sigma_1 \cup \sigma_2 \cup \sigma_3
\end{array}
\]

• 2-move

\[
\begin{array}{c}
\text{3-cell } \sigma^2 := \sigma_0 \cup \sigma_1 \cup \sigma_2
\end{array}
\]

• 1-move

\[
\begin{array}{c}
\text{0-cell } \nu
\end{array}
\]

Completeness conjecture:

Any two cellular decompositions of a compact orientable manifold are related (up to cellular homeomorphism) by a sequence of cellular moves and their inverses.
AN "INTERPOLATING" MODEL

- start with BC model:
  - is not discretization independent
  - has no (local) parameters
  - dominated by trivial representations for "natural" choice of weights
  - wrong state space on boundary?

- view BC model as BF theory with projection operators inserted per edge & face

- main idea: interpolate between no projection (BF) and total projection (BC)
  + take heat kernel operator from LGT

pictorially:

identity  \[ \lambda \rightarrow 0 \]

full projection  \[ \lambda \rightarrow \infty \]

BF  \[ \sim e^{-\lambda} \]

BC  \[ \text{quadratic Casimir of SU(2)} \]

\[ \text{local couplings } \lambda \text{ (positive real number)} \]

\[ \text{recover LQG type state spaces} \]

BF theory is UV-fixed point

BC model is "like" IR-fixed point