Spin Foam Models: Lessons from 2+1 dims

Kirill Krasnov
Albert Einstein Institute
Golm/Potsdam
Germany
**Spin foam model:**

Split space (space-time) $M$ into simple blocks, assign a quantum amplitude to every block, multiply amplitudes, sum over labels.

Usually—**triangulated** $M$, building blocks are simplices.

$$Z_M \text{(triangulation)} = \sum_{\text{labels}} \prod_{\Delta} \Psi_{\Delta}(\text{labels})$$

Also known as **state sum models**
Examples:

- Ponzano Regge model \rightarrow Euclidean 2+1 gravity
- Paraeu-Viro model
- Freidel - Lorentzian $\Lambda = 0$ 2+1
- Ooguri - $\Lambda = 0$ BF in any II
- Crane-Yetter BF+$\Lambda$BB in 4 dims

All examples: TQFT

$Z^m$ is independent of triangulation chosen.

Invariant of $M$
Spin foam models of gravity.

Obtained by quantifying a geometric simplicial in 3 dimensions Barbieri.

In 4d leads to the whole class of models: Barrett-Crane, Di Pietri, Rovelli, Perez, ...

Similar models exist in any D.

EM depends on triangulation chosen. Good! Not PQFP!
What to do with triangulation dependence?

4) Sum over triangulations
   Group field theory
   Rovelli, Perez, ...

2) Take limit as triangulation becomes more refined.
   Weckl, Markopoulou, ...

Problems:

1) \[ \sum_{\text{triangulations}} = \sum_{\text{topologies}} \sum_{\text{triangulations at fixed topology}} \]

Even in 2+1 the second sum diverges.
Original problem of the Bouloutaou model

Need to gauge fix: Freidel, Louapre

2) Not clear if any limit exists
Alternative:
Start with the classical space $M$ and find the canonical decomposition.

Let $M$ be a solution of Einstein eqs.
Find the value of the action on $M$.

$$S[M] = \sum S(\text{piece})$$

pieces

2+1 Lesson #1: There is the simplest possible decomposition (for given $M$) that does the job. It is unique (almost)!

2+1 Lesson #2: The canonical pieces are not in general tetrahedra.
"Canonical" spin foam:

Assign amplitudes $\sim e^{iS[\text{piece}]}$ to every canonical piece. Multiply, sum over labels $Z_M$ depends on decomposition. But now different decompositions correspond to different manifolds $M$.

The sum over decompositions becomes meaningful.

Similar to Freidel's gauge fixing. But propose a concrete way to go about it: understand the geometry of $M$ and its canonical decomposition.

Definitely not refinement - rather its opposite.
Examples:

- Euclidean $2+1$, $\Lambda = 0$. It is always possible to triangulate $M$ by a triangulation (singular) with a single vertex.

  The way one computes $\mathbb{Z} M$ in practice

- Euclidean $2+1$, $\Lambda < 0$. Finite volume manifolds $S^3 / K$

Decomposition of $M$ exists into ideal tetrahedra (unique)

Corresponding spin foam model exists (Freidel, KK) in prep.

 Practically impossible to get it decomposing $M$ into the usual tetrahedra!

- Euclidean $\Lambda < 0$, Infinite volume case: Handlebodies

Exists decomposition into the core and the rest. Pieces are not tetrahedra.

Spin foam not yet understood. Work in progress
Summary:

2+1 gravity lesson: Look for a canonical decomposition of $M$ that is unique once the geometry of $M$ is fixed.

Spin foam based on such a decomposition is a gauge fixed model.

Can sum over manifolds

3+1 How to decompose a general spacetime into simple pieces?