Baryogenesis from Inflationary Gravity Waves

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Outline

- Introduction: Sakharov’s dream
- Overview of the Mechanism (The Physics)
- Gravity, Chiral Fermions and CTP
- Gravitational Chiral Anomaly and CP Violation
- Gravity Wave Quantum Mech. and Inflation
- Evaluation of Fermion Number
- Heterotic String Embedding: Constraining string
- Consistency Check for CMB
- Conclusion and Outlook
The Baryon Asymmetry Problem

\( \frac{n_b}{s} \simeq (6.5 \pm 0.4) \times 10^{-10} \)

- WMAP and BBN :

- Why this number? Where does it come from?

- Our Standard Model Makes Wrong Prediction in the context of SBB!

- What is missing?
Sakharov’s Dream

- Sakharov realized the 3 necessary conditions for Baryogenesis

<table>
<thead>
<tr>
<th>Baryon number Violation</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP and C VIOLATION</td>
</tr>
<tr>
<td>Departure from Equilibrium</td>
</tr>
</tbody>
</table>
The Past Mechanisms

- The Sakharov conditions were not unified by a simple physical process. (This of course is our (APS) prejudice)

- CP & B usually come from non-perturbative and finite temperature effects in the Standard Model and its extensions.

- Baryogenesis usually thought to occur after inflation. We show that it can happen during inflation naturally and economically.
Baryogenesis is observed in CMB

The Cosmic Microwave Background Radiation
The diagram illustrates the relationship between vacuum energy density (cosmological constant) and mass density in cosmology. The axes represent different density parameters, with the top axis indicating vacuum energy density and the bottom axis representing mass density. The regions labeled 'No Big Bang,' 'Supernovae,' 'Clusters,' 'WMAP,' and 'CMB' are indicative of various cosmological models and observations.

Key observations:
- The area marked 'expands forever' suggests a universe that continues to expand indefinitely.
- The area labeled 'recollapses eventually' indicates a universe that will eventually collapse back together.
- The 'closed' region corresponds to a universe with a finite amount of mass.
- The 'flat' region represents a universe with a critical density.

172 supernovae for $0.01 < z < 1.7$ are plotted, indicating their impact on understanding the universe's expansion.

The pie chart below the diagram shows the contribution of different components:
- Dark energy: $\Omega_{\Lambda} \approx 0.65$
- Baryon: $\Omega_{\text{baryon}} \approx 0.05$
- Matter: $\Omega_{\text{matter}} \approx 0.30$
- Dark matter

These values highlight the dominance of dark energy in the universe, with dark energy contributing the majority of the total density parameter.
Structure needs matter asymmetry!

- We will provide a mechanism which unifies the Sakharov conditions during cosmic inflation. (The advantage being a natural out of equilibrium phase)

- Gravity waves are produced during inflation and sources chiral anomaly $\rightarrow$ Baryon number violation naturally out of equilibrium. (Intuition: Gravity waves are not strong enough-Quantum mechanics proves us wrong as we will see)

- Baryon asymmetry will be related to inflationary observables (ie. Power spectrum of tensor perturbation) Direct connection with structure formation and CMB experiments (PLANCK)
The Theme

- Inflationary Gravity Waves unify all three Sakharov conditions!

- Scalar perturbations in inflation seed Galaxies

- NEW: Gravity waves in inflation seed matter asymmetry.

- We claim that there is a connection between baryon to entropy ratio and the observed CMB power spectrum.

- IMPORTANT: We provide an observational and physical role for Inflationary Gravity Waves—they mirror scalar perturbations in LSS formation.
The Mechanism

- A non-vanishing complex phase of the inflaton field sources CP asymmetric (birefringent) gravity waves.

- Since these Gravity waves encode CP violation in their dispersions, they interact with chiral fermions and violate CP and fermion number simultaneously.

- Inflation inherently provides us with the out of equilibrium condition satisfying 3 Sakharov conditions.
First: Some Technical Background

- Currents and Fermions in General Relativity
- Physics of Gravity Waves
- Concrete Model: Natural Inflation (Adams, Freese et al.)
Fermions in the SM interact covariantly with Gravity via the spin connection

\[ L_l = (\text{det } e) (\bar{\Psi} i \gamma^5 \gamma^\mu D_\mu \Psi) \]

\[ D_\mu = \partial_\mu + \frac{1}{4} \omega^{ab}_\mu \gamma_{ab} \]

Which can be solved in terms of the vierbein.

\[ D_\mu e^a_{\nu} = \partial_\mu e^a_{\nu} - \Gamma^\rho_{\mu\nu} e^a_{\rho} + \omega^{ab}_{\mu} e_{\nu b} \equiv 0 \]
CP and CPT

Using the following identity

\[
\gamma^a \gamma^{bc} = \frac{i}{2} (\eta^{ab} \gamma^c - \eta^{ac} \gamma^b) - \epsilon^{abcd} \gamma^5 \gamma^d
\]

\[
L_l = (\det e)(\bar{\Psi}_L i \gamma^\mu \partial_\mu \Psi_L + L_{int})
\]

We arrive at the interaction Lagrangian

\[
L_{int} = -\frac{1}{4} \bar{\Psi}_L (\omega_a \gamma^a + i \tilde{\omega}_a \gamma^5 \gamma^a) \Psi_L + c.c.
\]

Where

\[
\tilde{\omega}_d = \epsilon^{abcd} \epsilon_\mu^a \omega^b_\mu .
\]
Gravity gives us a universal interaction

\[ \mathcal{L}_{int} = \frac{1}{4} \left[ \bar{\Psi}_L (\omega_a + i\tilde{\omega}_a) \gamma^a \Psi_L \right] + c.c. \]

This is an interaction between

The lepton number and the gravitational spin connection

We need to understand the CP, CPT properties and quantum corrections of this interaction to explore leptogenesis.
We are concerned with the local C, P and T transformations.

Local PT is the $\mathbb{Z}_2$ part of $O(3,1)$ which is $SO(3,1)$ the local Lorentz symmetry whose gauge field is the spin connection.

Under local PT:

$$\partial_\mu \longrightarrow \partial_\mu , \quad e^a_\mu \longrightarrow -e^a_\mu$$

They are both invariant under C.

Therefore, from the definition of the spin connection of the derivative of vierbein and Christoffel symbols.

$$\omega^{ab}_\mu^{C,PT} \omega^{ab}_\mu$$
Therefore Under Local CPT

\[ \omega_a, \tilde{\omega}_a \longrightarrow -\omega_a, -\tilde{\omega}_a \]

Important: time dependent spin connection can violate CP.

Under Local PT

\[ \bar{\Psi} \gamma^a \gamma^5 \Psi \longrightarrow - \bar{\Psi} \gamma^a \gamma^5 \Psi \]

Under C

\[ \bar{\Psi} \gamma^a \gamma^5 \Psi \longrightarrow + \bar{\Psi} \gamma^a \gamma^5 \Psi \]

Hence, interaction Lagrangian is CPT even

\[ \mathcal{L}_{int} = -\frac{1}{4} \bar{\Psi}_L (\omega_a \gamma^a + i\tilde{\omega}_a \gamma^5 \gamma^a) \Psi_L + c.c. \]
The Lagrangian describes the lepton number current interacting with the spin connection.

Is there a connection between the discrete symmetries and the global symmetry (lepton number)?

**YES, BUT QUANTUM MECHANICALLY**

Let us study this connection.
Gravitational Chiral Anomaly

In the interaction the global lepton current is classically conserved

\[ \partial_\mu J_{\mu 5} = \partial_\mu \bar{\Psi} \gamma_\mu \gamma_5 \Psi = 0 \]

However, this is not the case quantum mechanically

The expectation value up to 2nd order in coupling

\[ \langle J_{\mu 5} \rangle = \langle J_{\mu 5} \mathcal{L}_{\text{int}} \mathcal{L}_{\text{int}} \rangle \]
Chiral-Current Anomaly

Duff, Deser, Isham 88; Alvarez-Gaume, Witten. 90
This leads to the well known ABJ triangle anomaly

$$\partial_{\mu} J_{l}^{\mu} = \frac{1}{16\pi^2} R \tilde{R}$$

$$J_{l}^{\mu} = \bar{l}_{i} \gamma^{\mu} l_{i} + \bar{\nu}_{i} \gamma^{\mu} \nu_{i}$$

$$\tilde{R} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R_{\gamma\delta\rho\sigma}$$

$$< \partial_{0} J_{05} >= \frac{dn}{dt}$$

- This current vanishes in FRW backgrounds so at first sight it is not useful for cosmology.

- However for Gravity Waves the lepton number can be non vanishing. Let us explore this possibility.
Realistic FRW backgrounds include metric perturbations

\[ ds^2 = a^2(\eta) \left[ ((1 + 2\psi)\delta_{ij} + h_{ij}) \, dx^i \, dx^j \right] - (1 + 2\phi) \, d\eta^2 + w_i \, d\eta \, dx^i \right] \]

We are interested in the tensor modes with both helicities excited. For waves moving the $z$-direction, the metric simplifies to...

\[ ds^2 = a^2(\eta) \left[ -d\eta^2 + (1 - h_+) \, dx^2 + (1 + h_+) \, dy^2 + 2h_x \, dx \, dy + dz^2 \right] \]
To see the CP violation explicitly it’s convenient to use the helicity basis

\[
 h_L = \frac{1}{\sqrt{2}}(h_+ - i h_\times), \quad h_R = \frac{1}{\sqrt{2}}(h_+ + i h_\times)
\]

In the presence of gravity waves, the Chiral Anomaly is non vanishing

\[
 R\tilde{R} = \frac{4i}{a^3} \left[ \left( \frac{\partial^2}{\partial z^2} h_R \frac{\partial^2}{\partial t \partial z} h_L - \frac{\partial^2}{\partial z^2} h_L \frac{\partial^2}{\partial t \partial z} h_R \right) 
+ a^2 \left( \frac{\partial^2}{\partial t^2} h_R \frac{\partial^2}{\partial t \partial z} h_L - \frac{\partial^2}{\partial t^2} h_L \frac{\partial^2}{\partial t \partial z} h_R \right) 
+ \frac{1}{2} \frac{\partial}{\partial t} a^2 \left( \frac{\partial}{\partial t} h_R \frac{\partial^2}{\partial t \partial z} h_L - \frac{\partial}{\partial t} h_L \frac{\partial^2}{\partial t \partial z} h_R \right) \right]
\]

Lepton number can be generated from GWs
What configuration of Gravity Waves Generate a non vanishing lepton number?

This was first realized by R. Jackiw in flat space PRD: 68

The General Solution:

\[
\begin{align*}
  h(z, t)_+ &= \frac{1}{2\pi^3} \int d^3 k h_+(k) e^{A(t)k t} e^{i k(t-z)} \\
  h(z, t)_x &= \frac{1}{2\pi^3} \int d^3 k h_x(k) e^{B(t)k t} e^{i k(t-z)}
\end{align*}
\]

Since the different helicities have different dispersions we say that they are “birefringent”
The Lesson: It takes a birefringent configuration of Gravity Waves to produce lepton number.

But what about the CP connection we sought?

What sources this birefringence?
Inflation to the rescue

In the past, small patch of spacetime underwent superluminal expansion, sourced by a field with negative pressure (inflaton field). This process also gives the out of equilibrium condition.

Forward LC
The Answer: **Inflation**

(Guth)

Inflatons can explicitly violate CP from a non-vanishing complex phase (axion). This field will couple to gravity through the interaction.

\[ \mathcal{L}_{\text{int}} = f(\theta) R^{\alpha}_{\sigma \mu \nu} \tilde{R}^{\sigma \mu \nu} \]

Before inflation the complex modulus field varies from point to point in modulus and phase. Inflation blows up a small region in this field to size larger than visible universe.

This value then rolls slowly toward the minimum of the potential.

This process is out of equilibrium as well as CP asymmetric. This gets transmitted to gravitational waves which are amplified quantum mechanically during inflation.
We need to calculate the quantum expectation value of the lepton number during the history of inflation.
This field will have the following gravitational coupling

\[ \mathcal{L}_{\text{int}} = f(\theta) R_{\sigma \mu \nu} \tilde{R}^{\sigma \mu \nu} \]

From Green-Schwartz Mechanism

\[ f(\phi) = \frac{1}{16\pi^2 M_{Pl}} \mathcal{N} \phi \]

Later, we will derive this model independent function from Heterotic String theory. The above is identical to the divergence of Chiral current discussed earlier, from ABJ anomaly.

But how are gravity waves affected by this term? Let's see.
Gravity Wave Production

After linearizing the Einstein Equations with the new term
In harmonic gauge

\[ \partial^\nu h_{\mu\nu} = \frac{1}{2} \partial_\mu h_\nu^\nu \]

\[ M_p^2 \partial_\mu \partial^\mu h_{\alpha\beta} = 2f'' \dot{\Phi}^2 \epsilon^{ijk} \eta_{i\alpha} (\partial_\beta \partial_j h_{0k} + \partial_0 \partial_k h_{\beta k} + 2f' \dot{\Phi} \epsilon^{ijk} \eta_{i\alpha} \partial_j \partial_\mu \partial^\mu h_{\beta k} + (\alpha \leftrightarrow \beta) \]

We need to now find the quantum expectation value of GW which will allow us to get the lepton number
In the Inflationary Epoch the Gravity Wave E.O.M simplifies

\[
\partial_\mu \partial^\mu h_L = -2i a^{\Theta} \dot{h}_L'
\]

\[
\partial_\mu \partial^\mu h_R = 2i a^{\Theta} \dot{h}_R'
\]

\[
M_{pl}^2 \Theta = 4(F'' \dot{\phi}^2 + 2HF' \dot{\phi})
\]

Convenient to use conformal time

\[
\eta = \frac{1}{Ha} = \frac{1}{H} e^{-Ht}
\]

\[
\frac{d^2}{d\eta^2} h_L - 2 \frac{1}{\eta} \frac{d}{d\eta} h_L + k^2 h_L = +2k\Theta \frac{d}{d\eta} h_L
\]
If we ignore r.h.s we get a spherical Bessel function (note with r.h.s we have a Coloumb Wavefunction eq)

\[
\frac{d^2}{d\eta^2} h_L - 2 \frac{1}{\eta} \frac{d}{d\eta} h_L + k^2 h_L = 0
\]

Whose solution is:

\[
h^+_L(k, \eta) = e^{+ik(\eta+z)}(1 - ik\eta)
\]

Lets take the ansatz

\[
h_L = e^{ikz} \cdot (-ik\eta)e^{k\Theta_\eta}e^{ik\eta} g(\eta)
\]
This is the eq of a Schrodinger particle with l=1 in a Coulomb potential which changes sign under helicity change. This leads to attenuation/amplification of opposite helicity. This is cosmological Birefringence discussed by Kamionkowski et al

In the regime (subhorizon) \( k\eta \gg 1 \)

we can ignore the potential terms and arrive at the solution

\[
g(\eta) = \exp[ik(1 - \Theta^2)^{1/2}\eta(1 + \alpha(\eta))]\]
Computing Green’s Function

Our lepton number is dominated by the quantum part of the gravity wave evolution. In this regime we can calculate the expectation value by contracting $h_{l,r}$ in $R\tilde{R}$ using the following Green’s fn.

$$G(x, x') = \langle h_L(x)h_R(x') \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot (x-x')} G_k(\eta, \eta')$$

$$G_k = \begin{cases} 
    e^{-k\Theta \eta} G_{k0} e^{+k\Theta \eta'} & \eta < \eta' \\
    e^{-k\Theta \eta} G_{k0} e^{+k\Theta \eta'} & \eta' < \eta
\end{cases}$$
Hence the quantum expectation value:

\[
\langle \tilde{R} \tilde{R} \rangle = \frac{16}{a} \int \frac{d^3 k}{2\pi^3} \frac{H^2}{2k^3 M^2_{Pl}} (k\eta)^2 k^4 \Theta
\]

We pick up only the leading behavior for \( k\eta >> 1 \)
Which corresponds to UV Sub-Horizon modes

A reminder: The above expression is non-zero because of the effect of inflation in producing CP asymmetry out of equilibrium.

WE ARE FINALLY READY TO COMPUTE LEPTON NUMBER :)
Recalling from the 18th transparency $< \partial_0 J_{05} > = \frac{dn}{dt}$

From $\partial_{\mu} J_{l}^{\mu} = \frac{1}{16\pi^2} R \tilde{R}$

we compute the lepton number during the whole course of inflation

$$n = \int_{0}^{H^{-1}} d\eta \int \frac{d^3k}{(2\pi)^3} \frac{1}{16\pi^2} \frac{8H^2 k^5 \eta^2 \Theta}{M_P^2}$$

The integral over $k$ runs over all momentum space, up to the scale at which our eff. Lagrangian breaks down (mu). The dominant effect comes not from super horizon modes $k/H < 1$

but from short distances compared to horizon scale.
Integrating momentum up the cutoff scale $\mu$ we obtain the number:

$$ n = \int_0^{H^{-1}} d\eta \int \frac{d^3k}{(2\pi)^3} \frac{1}{16\pi^2} \frac{8H^2k^5\eta^2\Theta}{M_{Pl}^2} $$

This integral represents a compromise between two effects of inflation. First, to blow up distances and thus carry us to smaller physical momenta and second to dilute the generated lepton number through expansion.
This result has nice physical interpretation:
The factor \(\left(\frac{H}{M_{\text{Pl}}}\right)^2\) is the usual magnitude of the GW power spectrum

The factor \(\Theta\) gives the magnitude of effective CP violation

The surprise, the factor \(\left(\frac{\mu}{H}\right)^6\) gives the enhancement over one’s first guess due to strongly quantum short distance fluctuations to generate \(R\bar{R}\) rather than the super-horizon modes which effectively behave classically

\[
n = \frac{1}{72\pi^4} \left(\frac{H}{M_{\text{Pl}}}\right)^2 \Theta H^3 \left(\frac{\mu}{H}\right)^6
\]
Entropy Density

- Our entropy density comes from spontaneous reheating.

- This leads to baryon to entropy ratio:

  \[ \frac{n}{s} \sim 1 \times 10^{-6} \cdot \sqrt{\epsilon N} \left( \frac{H}{M_{\text{Pl}}} \right)^{11/2} \left( \frac{\mu}{H} \right)^6 \]

  \( N \) can be a large dimensionless number, we guess 100. We shall return to this in the Stringy Embedding of the mechanism.
Structure formation requires: \[
\frac{\delta \rho}{\rho} \sim \frac{H}{M_{Pl}} / \sqrt{\epsilon} \sim 10^{-5}
\]

WMAP constrains \[
\frac{H}{M_{Pl}} \sim 10^{-4}
\]

Which implies that \[
\epsilon \sim 10^{-2}
\]

Hence \[
\frac{n}{s} \sim 1 \times 10^{-5} \cdot (H/M_{Pl})^{-1/2} (\mu/M_{Pl})^6
\]

The mass scale from the kinetic term for the field is set at a lower mass $F$, then Theta can be larger

\[
\Theta \sim \sqrt{2\epsilon N (H^2/M_{Pl} F \pi^2)} \sim (H/M_{Pl})^2 (M_{Pl}/F)
\]
We arrive at the final result for the baryon to entropy ratio. We can find \( \mu \) for a range of Hubble that is acceptable by CMB constraints.

\[
\frac{n_s}{s} \sim 1 \times 10^{-5} \cdot \left( \frac{H}{M_{\text{Pl}}} \right)^{-1/2} \left( \frac{\mu}{M_{\text{Pl}}} \right)^{5}
\]

\[10^{-30} \lesssim H/M_{\text{Pl}} < 10^{-4}\]

The range is \( 3 \times 10^{14} < \mu \lesssim 10^{17} \)

Which is the scale of the right handed neutrino! From this we can get

\[
\frac{n}{s} \sim 10^{-10}
\]
Stringy Embedding

- What is the cutoff scale in terms of a UV completed theory?
- Where does the Chern-Simons term originate from?
- We still need an enhancement controlled by the cutoff.
- We will now show that these questions are resolved in a model independent way in String Theory.
Heterotic Origin of 4D Chern-Simons Coupling

We postulated a Chern-Simons form interaction with a linear coupling to the axion. Let us derive this from Heterotic String Theory.

\[ S = M_{10}^8 \int d^{10}x \sqrt{g_{10}} \left( R - \frac{1}{2} \partial_{A} \phi \partial^{A} \phi - \frac{1}{12} e^{-\phi} H_{ABC}^2 - \frac{1}{4} e^{-\phi/2} \text{tr}(F_{AB}F^{AB}) \right) \]

\[ H_3 = dB_2 - \frac{1}{4} (\Omega_3(A) - \alpha' \Omega_3(\omega)) \]

- We see that the antisymmetric 3-form gets modified to cancel chiral anomaly-(Green-Schwarz)
Compactifying to 4D we obtain the N=1 SUGRA

\[ S_{4D} = \frac{M_{pl}}{2} \int d^4 x \sqrt{-g} \left[ \mathcal{R} - \frac{2 \partial_\mu S^* \partial^\mu S}{(S + S^*)^2} - \frac{1}{2} G^{ij} G_{kl} \partial_\mu T_{il} \partial^\mu T_{jk} \right] \]

It was shown by Gukov et al. that all Moduli (internal dimensions) are stabilized in this limit.

But there is also a Chern-Simons term inherited from 10D. This is important for us.

Also, inflation can be driven by superpotential Linde et al.

\[ W = W_0 + A e^{-2\pi T/N} + B e^{-2\pi T/M} \]
DERIVATION OF CHERN-SIMONS

\[ M_{10}^8 \int d^{10}x e^{-\phi} H_{ABC}^2 \]

\[ H_3 = dB_2 - \frac{1}{4}(\Omega_3(A) - \alpha'\Omega_3(\omega)) \]

\[ 2M_{10}^8 \int d^4x e^{-\phi} \frac{\alpha'}{4} \star dB \wedge \Omega_3(\omega) \int d^6y \]

\[ \star dB_{[2]} = da \]

We arrive at

\[ S_{CS} = -\frac{M_{10}^2}{2} g_s^{-1}(Vol \ M_{10}^6) \int d^4xa R \wedge R \]
But this is exactly the form we postulated (APS) for $F(a)$

$$\mathcal{L}_{\text{int}} = F(\phi) \mathcal{R}^\alpha_\sigma \mathcal{R}_{\alpha}^{\sigma \mu \nu}$$

But now we can determine the form of $F$ from String Theory in a Model Independent manner.

$$F(a) = \mathcal{V} a$$

$$\mathcal{V} = \frac{1}{2} (\alpha' M_{10}^2) g_s^{-1} Vol \ M_{10}^6$$

Where $\mathcal{V}$ is the volume enhancement factor.
Enhancement of Leptogenesis from strings

- We immediately see that the volume enhancement from string theory can be much larger than our original guess.

- We can constrain(determine) the string scale from baryon asymmetry index.
Constraining String Scale
Model Independently

hep-th/040914: S.A, S.J Gates

Some simple algebra gives:

\[ \Theta = \left( \frac{H}{M_{pl}} \right)^2 \mathcal{V} \sqrt{\epsilon}. \]

\[ \mathcal{V} \propto (\text{Vol } M_{10}^6) \]

The 4D Planck scale is related to string scale via.

\[ M_{pl}^2 = M_{10}^8 \text{Vol} \]

\[ M_{10}^8 = \alpha'^{-4} g_s^2 \]

We finally get

\[ \mathcal{V} = \frac{1}{2} g_s^{-1/2} \left( \frac{M_{pl}}{M_{10}} \right)^2 \]
Using our original result for baryon to entropy index

\[ \frac{n}{s} = 10^{-6} \Theta \left( \frac{H}{M_{pl}} \right)^{7/2} \left( \frac{\mu}{H} \right)^6 \]

\[ \frac{n}{s} = 10^{-7} g_s^{-1/2} \left( \frac{H}{M_{pl}} \right)^{-1/2} \left( \frac{M_{10}}{M_{pl}} \right)^4 \]

WMAP and BBN tells us

\[ \frac{n}{s} \sim 10^{-10} \]

This determines string scale to be

\[ g_s^{-1/2} \frac{M_{10}}{M_{pl}} \sim 10^{\frac{-5}{4}} \]

for \( g = 1 \), \( l_s \) is \( 10^{\{-3\}} \) cm
The Consistency Check and Birefringent GWs

- One can ask: What about associated superhorizon GWs, are they ruled out by current WMAP observation?

- Since we have a model independent realization, are they any predictions for string theory?

- We will now rederive the consistency relation including the Tensor to Scalar ratio of our model and explore this question.
On Superhorizon scales our GWs obey

\[
\left(1 - \lambda^s k \frac{f'}{a^2}\right) (h^s_k)'' + \left(2\dot{\mathcal{H}} - \lambda^s k \frac{f''}{a^2}\right) (h^s_k)' + \left(1 - \lambda^s k \frac{f'}{a^2}\right) k^2 h^s_k = 0, \quad s = R, L
\]

\[
\mu^s_k \equiv z^s_s h^s_k
\]

\[
f = \frac{N}{16\pi^2 M_{\text{Pl}}^2} \frac{\phi}{M_{\text{Pl}}}
\]

\[
z^s_s (\eta, k) \equiv a(\eta) \sqrt{1 - \lambda^s k \frac{f'}{a^2}}
\]

\[
(\mu^s_k)'' + \left(k^2 - \frac{z^s_s''}{z^s_s}\right) \mu^s_k = 0
\]

potential differs from typical scenarios since it depends on polarization, wavenumber as well as conformal time.
Effective Potential

\[ x = \Theta \kappa \eta / 8 \]

\[ f_R(x) = \frac{2 + 3\epsilon}{x^2} + \frac{1}{x(1 - x)} - \frac{1}{4} \frac{1}{(1 - x)^2} \]

\[ f_L(x) = \frac{2 + 3\epsilon}{x^2} - \frac{1}{x(1 + x)} - \frac{1}{4} \frac{1}{(1 + x)^2} \]
Power Spectrum Calculation

After much algebra we find the solution in the linear regime.

\[
\mu_k^L = -\frac{4\sqrt{\pi \ell_{P1}^3}}{\sqrt{2k}} e^{ik\eta_i} e^{-\pi\Theta/32} W_{i\Theta/16,0} \left[ \frac{16i(1+x)}{\Theta} \right]
\]

Where \( W \) is the Whittaker function which can be expressed in terms of the CHGF

\[
\langle h_{ij}(\eta, x) h^{ij}(\eta, x) \rangle = \frac{1}{\pi^2} \sum_{s=L,R} \int_0^{+\infty} \frac{dk}{k} k^3 |h^s_k|^2
\]
We now expand the result at first order in the slow roll parameter

\[ k^3 P_h^s(k) = \frac{k^3}{\pi^2} \left| \frac{\mu_k^s}{a(\eta) \sqrt{1 - \lambda^s k f'/a^2}} \right|^2 \]

On Large Scales it becomes

\[ k^3 P_h^s = \frac{16 \ell_{\text{Pl}}^2}{\pi} \frac{k^{-2\epsilon}}{\ell_0^2} \frac{\Gamma^2(2\xi)}{2^{2\xi}} \left| \frac{\Gamma(1/2 + \xi - i\lambda^s \Theta/16)}{\Gamma(1/2 + \xi)} \right|^2 e^{-\lambda^s \pi \Theta/16} \]

Where \[ \xi \equiv \frac{3}{2} + \epsilon \]

We now expand the result at first order in the slow roll parameter

\[ a(\eta) = \ell_0(-\eta)^{-1-\epsilon}, \quad \phi' \approx -M_{\text{Pl}} \mathcal{H} \sqrt{2\epsilon} \]
Straightforward calculations yield

\[ k^3 P_h^s(k) = \frac{16 H^2_{\text{inf}}}{\pi m^2_{\text{Pl}}} \frac{1}{2} \mathcal{A}^s(\Theta) \left[ 1 - 2(C+1)\epsilon - 2\epsilon \ln \frac{k}{k_*} - \epsilon \mathcal{B}(\Theta) \right] \]

\[ \mathcal{A}^R = 1 - \frac{\pi}{16} \Theta + \left( \frac{\pi^2}{384} - \frac{1}{256} \right) \Theta^2 + \mathcal{O}(\Theta^3) \]

At leading order in the slow roll parameter, the spectral index remains unmodified

\[ n_T^s = \frac{d \ln (k^3 P_h^s)}{d \ln k} = -2\epsilon \]

Our mechanism is consistent with WMAP
The scalar power spectrum is unmodified however, the tensor to scalar ratio is modified!

\[
\frac{(T/S)_{\Theta \neq 0}}{(T/S)_{\Theta = 0}} \simeq 1 + 0.022 \times \Theta^2
\]

In String Theory

The value of Theta is proportional to N

\[
\mathcal{N} = \pi^2 \sqrt{\frac{g_s}{2}} \left( \frac{M_{P_1}}{M_{10}} \right)^2
\]

And we arrive at the final result

\[
\frac{(T/S)_{\Theta \neq 0}}{(T/S)_{\Theta = 0}} \simeq 1 + \frac{0.022}{4} \left( \frac{H_{\text{inf}}}{M_{10}} \right)^4 g_s \epsilon
\]

The correction is model independent but unfortunately too small. For larger values of Theta a non-linear treatment is necessary—work in progress

S.A. Daniel Greene, Jerome Martin
We have presented a novel leptogenesis mechanism in the context of inflation which unifies all three Sakharov conditions in one shot!

The **Gravitational Chiral anomaly** is the source of lepton number violation and the quantum expectation value is proportional to the power spectrum of **birefringent gravity waves** produced during the inflationary.

We found a string theoretic embedding of the mechanism and realized a model independent connection via Green-Schwarz mech to the string scale and the observed baryon number.

We derived CMB signatures of this effect from the consistency relation and tensor to scalar ratio. We still need to understand the non-linear regime of the power spectrum. (To appear: S.A. and Jerome Martin)

To show that these gravity waves produce isocurvature density perturbations which underly the relationship between

\[ \frac{\delta \rho}{\rho} \sim \frac{n_B}{s} \]

(Work in progress S.A and M. Kamionkowski)
Level Crossing

\[ ds^2 = a^2(\eta) \left[ -d\eta^2 + (\delta_{ij} + h_{ij})dx^idx^j \right] \]

\[ \omega_{\mu}^{ab} = (\omega_{\mu}^{ab})_0 + (\omega_{\mu}^{ab})_h \]

\[ \gamma^\mu \nabla_\mu \psi = \gamma^\mu (\partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma^{ab}) \psi = 0 \]

\[ \hat{H}_0 |l, n, j_\pm, j_3> = \mu_\pm |l, n, j_\pm, j_3>, \quad j_\pm = \frac{l}{2} \pm \frac{1}{2} \]

\[ \psi_{1}^{\pm} = a^{-3/2} \chi_1 \exp \pm i \int^{\eta} E_1(\eta) d\eta \]

\[ \psi_{2}^{\pm} = a^{-3/2} \chi_2 \exp \pm i \int^{\eta} E_2(\eta) d\eta \]