Recent progress in QG Phenomenology

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Quantum gravity in the Americas III
Penn State
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What is QG phenomenology?

The search for effects at low energies (<< Planck scale) induced by quantum gravity.

Phenomenology: The translation of theory to observations

Ideally we could calculate from our favorite QG model (strings, LQG, causal sets)

What is the state corresponding to our universe?

How did we get to it?

What are the testable unique predictions?
What is QG phenomenology?

Unfortunately, such a clean approach is a ways off...

We still pick a QG theory, but relax our standards

What qualitative characteristics might the quantum state for our universe have?

Do these characteristics deviate from classical general relativity/QFT?

Look hard for these deviations even though they are not firm predictions but educated guesses.
Where should we look?

Easiest place: A number of QG ideas hint that the local symmetries of the solution to classical gravity that describes our universe (Minkowski space on small scales) may not be exact.

- **String theory**
  - Tensor VEV's violating Lorentz invariance (Kostelecky and Samuel 1989)

- **Quantum gravity**
  - Violations of Lorentz invariance in LQG?
  - DSR as flat space limit of QG? (Freidel et al. 2003)

- **Causal sets**
  - Statistical violation of translation invariance (Sorkin, Dowker, and Henson 2004)

- **Canonical N-C geometry**
  - Non-commutativity tensor is Lorentz violating

Other possibilities: early universe cosmology, brane worlds, decoherence, etc.
An analogy with particle physics

Gauge anomaly with chiral fermions

Classical Action

Classical gauge field theory
General relativity

Classical symmetry of the theory

Gauge symmetry
Spacetime symmetry

Quantization

Gauge invariance broken
Spacetime symmetries broken/modified

Information about quantum theory

Gauge invariance required, restricts hypercharges in the Standard Model
Should classical symmetries be respected?
Three symmetries of spacetime

- **Lorentz invariance**
  - Relatively easy experiments for QG sized violations
  - Involves multiple QG models (strings & LQG)
  - Extensive testing over last decade, violations tightly constrained

- **Translation invariance**
  - Hypothesized to be not respected only in causal sets
  - Fewer good tests

- **Scale invariance**
  - Much harder to test
  - Likely violated by QG
First symmetry: Lorentz invariance

Two primary frameworks for Lorentz violation/modification

**Effective Field Theory**
- Lorentz invariance explicitly broken by adding new background tensors to the action. New particle dispersion relations.
- Translation invariance and conservation usually left unmodified (energy and momentum add in usual way)

**Doubly special relativity**
- Lorentz invariance preserved, but transformation laws differ for energy and momentum
- Addition of energy and momentum modified
Lorentz violation: EFT framework

Dozens upon dozens of operators that can be added to Lorentz invariant standard model that preserve gauge invariance

Renormalizable operators – the *Standard Model Extension* of Colladay and Kostelecky

Non-renormalizable operators up to mass dimension six have also been studied (c.f. Myers-Pospelov 2003 and others)

Simplify for the purposes of this talk to rotationally invariant Lorentz violation – *preferred frame* theories.
A relatively simple Lagrangian for a preferred frame

One fermion species, one gauge field (think QED), matter sector only

<table>
<thead>
<tr>
<th>Operators</th>
<th>CPT Even</th>
<th>CPT Odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Usual</td>
<td>(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i \bar{\psi} (\gamma \cdot D) \psi)</td>
<td></td>
</tr>
<tr>
<td>Dim 3 LV</td>
<td>(-\frac{1}{4} (k_F) u_\nu \eta_{\lambda \mu} u_\nu F^{\kappa \lambda} F_{\mu\nu} + \frac{i}{2} \bar{\psi} (u \cdot \gamma) (u \cdot D) \psi + \frac{1}{2} i d \bar{\psi} \gamma_5 (u \cdot \gamma) (u \cdot D) \psi)</td>
<td>(-b E_{Pl} \bar{\psi} \gamma_5 (u \cdot \gamma) \psi)</td>
</tr>
<tr>
<td>Dim 4 LV</td>
<td>(-\frac{1}{4} (k_F) u_\nu \eta_{\lambda \mu} u_\nu F^{\kappa \lambda} F_{\mu\nu} + \frac{i}{2} \bar{\psi} (u \cdot \gamma) (u \cdot D) \psi + \frac{1}{2} i d \bar{\psi} \gamma_5 (u \cdot \gamma) (u \cdot D) \psi)</td>
<td></td>
</tr>
<tr>
<td>Dim 5 LV</td>
<td>(-\frac{1}{E_{Pl}} \bar{\psi} (\alpha_L^{(5)} P_L + \alpha_R^{(5)} P_R) (u \cdot D)^2 \psi)</td>
<td>(\frac{\xi}{E_{Pl}} u^\mu F_{\mu \alpha} (u \cdot \partial) (u_\nu \tilde{F}^{\nu \alpha}))</td>
</tr>
<tr>
<td>Dim 6 LV</td>
<td>(-\frac{1}{2 E_{Pl}} \beta^{(6)} F^{\mu\nu} u_\mu u_\sigma (u \cdot \partial)^2 F_{\sigma \nu} - \frac{i}{E_{Pl}} \bar{\psi} (u \cdot D)^3 (u \cdot \gamma) (\alpha_L^{(6)} P_L + \alpha_R^{(6)} P_R) \psi)</td>
<td>(\frac{1}{E_{Pl}} \bar{\psi} (u \cdot \gamma) (\eta_L P_L + \eta_R P_R) (u \cdot D)^2 \psi)</td>
</tr>
</tbody>
</table>

\(P_{L,R}\) are projection operators, \(u^\alpha\) is the vector defining the preferred frame, \(E_{Pl}\) is the Planck energy, and \(D\) is the gauge covariant derivative.

Combination of SME, Myers-Pospelov action, dim 5 and 6 CPT even terms

Still 14 LV operators in the Lagrangian!
New couplings, new free field dispersion

Modified particle dispersion:
$$E^2 = p^2 + m^2 + \eta \, p^n$$

New interaction vertices (suppressed by Planck scale)
New couplings, new free field dispersion

Dispersion drastically modifies kinematics when $m^2 \approx \eta p^n$

<table>
<thead>
<tr>
<th>$n$</th>
<th>Operators responsible</th>
<th>Energy needed (protons)</th>
<th>Energy needed (electrons)</th>
<th>Energy needed (neutrinos)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>dim 3</td>
<td>All</td>
<td>All</td>
<td>All</td>
</tr>
<tr>
<td>2</td>
<td>dim 4</td>
<td>All</td>
<td>All</td>
<td>All</td>
</tr>
<tr>
<td>3</td>
<td>CPT Odd dim 5</td>
<td>$10^6$ GeV</td>
<td>$10^4$ GeV</td>
<td>1 GeV</td>
</tr>
<tr>
<td>4</td>
<td>CPT even dim 5,6</td>
<td>$10^{9.5}$ GeV</td>
<td>$10^8$ GeV</td>
<td>$10^5$ GeV</td>
</tr>
</tbody>
</table>

All energies well below Planck scale!!
Doubly special relativity

Lorentz invariance is deformed (not broken) to admit an invariant length

(Recent review see J. Kowalski-Glikman 2006)

<table>
<thead>
<tr>
<th>Important phenomenological assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified universal dispersion n=3</td>
</tr>
<tr>
<td>Coefficient η=1</td>
</tr>
<tr>
<td>Also modified energy conservation equations for interactions</td>
</tr>
<tr>
<td>Speed of particles given by usual group velocity?</td>
</tr>
</tbody>
</table>
Lorentz invariance: Effects for each framework

<table>
<thead>
<tr>
<th>Effect</th>
<th>EFT</th>
<th>DSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified time of flight for astrophysical particles</td>
<td>Yes</td>
<td>Maybe</td>
</tr>
<tr>
<td>Birefringence</td>
<td>Sometimes</td>
<td>No</td>
</tr>
<tr>
<td>New particle reactions allowed by kinematics</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Modified thresholds for existing reactions</td>
<td>Yes</td>
<td>Yes, but tiny</td>
</tr>
<tr>
<td>Susceptible to preferred frame experiments</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>New Goldstone bosons from broken symmetry</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>New neutrino oscillation signatures</td>
<td>Yes</td>
<td>Likely?</td>
</tr>
</tbody>
</table>

EFT has more effects than DSR, is easier to test and is more tightly constrained.
A selective summary of some current direct bounds

A cheat for simplicity: mush all fermion bounds together

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Best two sided bound (log 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b (dim 3 fermion)</td>
<td>-43</td>
</tr>
<tr>
<td>$k_F$ (dim 4 photon)</td>
<td>-32</td>
</tr>
<tr>
<td>c (dim 4 fermion)</td>
<td>-27</td>
</tr>
<tr>
<td>$\xi$ (dim 5 photon)</td>
<td>-4</td>
</tr>
<tr>
<td>$\eta$ (CPT odd dim 5 fermion)</td>
<td>-2</td>
</tr>
<tr>
<td>Dim 6</td>
<td>Large parts of phase space forbidden</td>
</tr>
</tbody>
</table>

With some additional mild assumptions about how coefficients behave for SM generations can put log($\eta$) at -13 and log (dim 6) at -2.

Moral: If you believe in just SM physics and QG induced Lorentz violation, you’ve got to be a good tap dancer
Even worse, radiative corrections cause major problems

In general if there is no new physics between accessible energies and $E_{\text{Pl}}$ the size of the dimensionless coefficients will be of roughly the same order due to radiative corrections. (c.f. Collins, Perez, Sudarsky 2006)

Therefore constraints on dimension four operators give strong constraints on dim six operators (they are irrelevant).

However there are symmetries that can protect dangerous dimension 4 and 5 operators, so we really should be testing each operator without regard to hierarchy.
What are these symmetries?

How does one prevent lower dimension LV operators without assuming Lorentz invariance?

**Answer: Field transformation symmetries**

If there is a symmetry that takes one field into another then the speed of propagation must be the same for both fields. Works in both condensed matter analogs and the real world w/ SUSY.

Weinfurter et. al. 2006, Groot-Nibbelink and Pospelov 2005

The problem with SUSY is that the higher dimension operators don’t modify dispersion significantly either...fairly impervious to high energy tests.
Yes, but do we live in a SUSY world?

SUSY obviously not a feature of current world. Therefore we will have dimension 4 LV operators, but their amplitude will be suppressed by a ratio of scales.

\[ \text{LV dim 4 operators of size } \left( \frac{E_s}{E_p} \right)^2 \]

Likely need SUSY scale to be near 1-100 TeV to be compatible w/ experiment. Requirement for low energy SUSY becomes stronger with improved bounds.
Experiments for CPT even dimension 5 and 6 operators

Ultra high energy cosmic rays and neutrinos are the only things that have enough energy to probe these operators

Well known and well discussed Griesen-Zatsepin-Kuzmin cutoff (or lack thereof)

GZK reaction

$$p + \gamma_{CMB} \rightarrow \pi^0 + p \text{ or } \pi^+ + n$$

Lorentz violation

Modified dispersion

New GZK cutoff energy

Modified reaction kinematics
Waiting for Auger and others...

Contradicting results from AGASA and HiRes on existence of cutoff, Pierre Auger Observatory should resolve differences

First data results (April 2006)

Definitive results Summer 2007?
If we see it, is it conclusive?

Bound operators at 0.001 level...but won’t remove all of phase space

Example, modified proton and pion dispersion (in units where $M_{pl}=1$)

\[
E^2 = m_p^2 + p^2 + \eta_p p^4
\]

\[
E^2 = m_{\pi}^2 + p^2 + \eta_{\pi} p^4
\]

How far is GZK cutoff from $5 \times 10^{10}$ GeV?

One sided bounds, typical of threshold reactions...

Requires coefficients to be almost equal and negative or no proton modification. Both constitute a tuning!
For better bounds, another particle required

First guess, try something similar to GZK reaction

Only other high energy particle that might reach us are GZK neutrinos generated by subsequent decay of n, $\pi^+$ or initial acceleration

<table>
<thead>
<tr>
<th>Particle</th>
<th>Energy kinematics begin to change (dim 5 &amp; 6 ops)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>$10^{10}$ GeV</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$10^5$ GeV</td>
</tr>
</tbody>
</table>

Example: LV could induce new reactions for neutrinos above 100 TeV like

$$\nu \rightarrow \nu + \nu + \bar{\nu}$$  Leads to rapid energy loss!

Reaction rates very low, need near GZK energies for observable rates. ANITA should detect these neutrinos, launches in December 2006
How about DSR for particle reactions?

- EFT
  - Modified dispersion
  - Unmodified conservation
  - Shifted thresholds, new reactions

- DSR
  - Modified dispersion
  - Modified conservation
  - No observable effects
For DSR use a different method

Modified dispersion yields different speeds for particles

Time of flight delays for high energy particles

Arrival of GRB photons (Ellis et. al. 2006) bounds coefficients for n=3 at O(100)

Can’t touch n=4

Hard to improve bound significantly due to scattering

Neutrinos as secondary particles (Piran 2006)

Predicted to be emitted from GRB’s

Don’t scatter

Can achieve higher energies and probe n=3 much better
A different (and better?) approach with neutrinos

Neutrino oscillations

Modified dispersion \[ E^2 = p^2 + m_i^2 + \eta_i p^n = p^2 + N_i^2 \]

\( P_{ij} \) is probability initial neutrino flavor eigenstate I is measured in state J after travelling distance L

\[ P_{ij} = \delta_{ij} - \sum_{i,j>i} 4 F_{ij} \sin^2 \left( \frac{\delta N_{ij}^2 L}{4E} \right) + 2 G_{ij} \sin \left( \frac{\delta N_{ij}^2 L}{2E} \right) \]

No LV, fixed baseline, high energies, observed flux \( \sim P_{ij} \sim L/E \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>Deviation energy</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1 GeV</td>
<td>( E^2 )</td>
</tr>
<tr>
<td>4</td>
<td>( 10^5 ) GeV</td>
<td>( E^3 )</td>
</tr>
</tbody>
</table>
First MINOS data

Normalized flux

Reconstructed Neutrino Energy

GeV

Not looking very much like $E^2$! Hard for DSR to explain!? Very unpleasant for CPT violating dim 5 EFT operators...
A wrap up of Lorentz violation

LV in an EFT framework still possible, but need some other unknown physics to prevent large renormalizable operators.

SUSY + CPT can perhaps do it but really need low energy SUSY.

Even then dimension 6 operators should soon be bounded at $10^{-2-3}$ level (or an explosion of research if GZK cutoff is missing).

DSR evades all the best EFT constraints, but neutrino oscillations could cause serious problems...
And now we swerve to something completely different...

**Causal sets!**

The swerve effect:
A particle propagating in a causal set will not stay on its mass shell exactly.

Postulated in Sorkin, Dowker, and Henson (2004)

Leads to a statistical diffusion in momentum space.

Conservation laws don’t hold exactly due to Poincare breakdown at fundamental level.
The diffusion equation

Since causal sets are pretty much Lorentz invariant diffusion leads to a LI diffusion equation

\[
\frac{\partial \rho}{\partial \tau} = k \nabla^2_p \rho - \frac{1}{m c^2} p^\mu \frac{\partial}{\partial x^\mu} \rho
\]

\(\tau=\)proper time, \(k\) diffusion constant, \(\rho\) distribution in momenta
Laplacian is on the momentum space mass shell...

Reduces for non-relativistic particles to

\[
\frac{\partial \rho}{\partial t} = k \nabla^2 \rho
\]
Heating of gases

Net effect: a gas in thermal equilibrium heats up over time according to

\[
\frac{dT}{dt} = \frac{2k}{m}
\]

Relic neutrino background would heat up due to swerve effect, lead to more hot dark matter.

Amount of hot dark matter is bounded by cosmology, leads to limits on \( k \) of \( 10^{-61} \) GeV\(^3\).

Length scale of \( k \), \( 10^{-20} \) GeV is far smaller than any length scale in the problem so very unclear from first principles why \( k \) must be so small...
A problem for causal sets?