Generalizing the Kodama State

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Introduction

• Kodama State is one of the only known solutions to all constraints with well defined semi-classical interpretation

• Represents quantum (anti)de-Sitter space

• Cosmological data suggest we are in increasingly lambda dominated universe
  • World appears to be asymptotically de-Sitter

• Exact form in connection and spin network basis:
  \[
  \langle A | \Psi \rangle = N e^{\frac{3}{4k\Lambda}} \int Y_{CS}[A] \quad \langle \Gamma | \Psi \rangle = K_{\Gamma}
  \]
But...

- Not normalizable under kinematical inner product
  - Not known to be under physical inner product
  - Linearized solutions not normalizable under linearized inner product
- Not invariant under CPT
  - Violates Lorentz invariance?
  - CPT inverted states have negative energy?
- Not invariant under large gauge transformation
  
  (This could be a good thing)
- Loop transform in complex variables not as rigorous as with real variables
- Reality constraint must be implemented
Resolution

- Problems can be tracked down to complexification
  - Immirzi is pure imaginary: $\beta = \pm i$

- Need to extend state to real values of Immirzi parameter

- This can be done

- Answer is surprising:
  - Opens up a large Hilbert space of states
  - de-Sitter/Chern-Simons is one element of this class
The Generalized States

• States are WKB states corresponding to first order perturbations about de-Sitter spacetime

\[ S_0 \simeq \frac{3}{2k\Lambda} \int_M \star R \wedge R - \frac{2}{\beta} R \wedge R \]

• Write action as functional of Ashtekar-Barbero connection, \( A = \Gamma - \beta K \)

\[ S_0[A] \simeq \frac{3}{4k\Lambda\beta^3} \int_{\partial M = \Sigma} Y_{CS}[A] - (1 + \beta^2)(Y_{CS}[\Gamma] - 2\beta K \wedge R_{\Gamma}) \]

• States are pure phase WKB states when Immirzi is real

\[ \Psi_R[A] = Ne^{iS_0[A]} \]

• States appear to depend on position and momentum
  • This requires careful interpretation
Interpretation

• Generalized Kodama state is like NR momentum state

\[ \langle x|p \rangle = \Psi_p[x] = \mathcal{N} \exp[i \cdot x \cdot p - i E \cdot t] \]

\[ \langle A|R_\Gamma \rangle = \Psi_{R_\Gamma}[A] = \mathcal{P} \text{exp} \left[ i \alpha \int A \wedge R_\Gamma - i \varepsilon Y_{CS}[A] \right] \]

• Infinite set of states parameterized by curvature

• Expect different states to be orthogonal

\[ \langle p'|p \rangle \sim \int dx \ e^{-ix \cdot (p' - p)} = \delta(p' - p) \]

\[ \langle R'|R \rangle \sim \int \mathcal{D}\phi \mathcal{D}A \ e^{-i\alpha \int A \wedge (\phi R' - R)} = \delta(R' - R) \]
Levi-Civita Curvature Operator

• Momentum operator in momentum basis:

\[ \hat{p} = \int dp \ p |\Psi_p\rangle\langle\Psi_p| \rightarrow \hat{p} |\Psi_p\rangle = p |\Psi_p\rangle \]

• Similarly can construct gauge *covariant* curvature operator

\[ \int_\Sigma \lambda \wedge \hat{R}_\Gamma = \int \mathcal{D}\phi \mathcal{D}\Gamma' \left[ \left( \int_\Sigma \lambda \wedge \phi R'_\Gamma \right) |\Psi_{\phi R'}\rangle\langle\Psi_{\phi R'}| \right] \]

\[ \int_\Sigma \lambda \wedge \hat{R}_\Gamma |\Psi_R\rangle = \int_\Sigma \lambda \wedge R_\Gamma |\Psi_R\rangle \]

• Using this definition of curvature operator:

\[ \hat{H} |\Psi_R\rangle = 0 \quad !!! \]
## Properties

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<tr>
<td>![checkmark]</td>
<td>Delta-function normalizable</td>
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<td>Solve Hamiltonian constraint</td>
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<td>Semi-classical interpretation (WKB)</td>
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<td>CPT invariance</td>
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<td>Negative Energies</td>
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References

• gr-qc/0504010, “A Generalization of the Kodama State for Arbitrary Values of the Immirzi Parameter”

• Upcoming papers:
  • Generalizing the Kodama State I: Construction
  • Generalizing the Kodama State II: Properties and physical interpretation