Singularities, Effective Actions and String Theory

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References: a biased collection
Singularities \(\rightarrow\) important role in the evolution of our still incomplete understanding of both nongravitational and gravitational theories

w/o gravity quantum origin \(\rightarrow\) IR singularities: definition of physical states IR-safe observables

UV singularities: high energy modes renormalized away – not always possible

What theories have a UV completion?

When is it needed? Is it unique?

— An effective theory does not know when and why it breaks down

w/ gravity classical level \(\rightarrow\) classically cannot be avoided (Hawking, Penrose) geodesics terminate in regions with large curvature

Should they be removed? How? Is it unique?
Singularities → important role in the evolution of our still incomplete understanding of both nongravitational and gravitational theories

- An effective theory does not know when and why it breaks down w/ gravity
  classical level → classically cannot be avoided (Hawking, Penrose)
  geodesics terminate in regions with large curvature

  Should they be removed? How? Is it unique?

Sometimes, but not always (Horowitz, Myers)

- There exist classical “geometries” which we classify as unphysical
e.g. negative-mass Schwarzschild

- If resolved and new physics is localized near the singularity
  → negative energy, unstable flat space

◊ need singularity to rule solution as unphysical
Singularity is important in the evolution of our understanding of both nongravitational and gravitational theories.

**w/o gravity**

- **IR singularities**: definition of physical states
  - **IR-safe observables**

- **UV singularities**: high energy modes renormalized away—it’s not always possible

What theories have a UV completion?

When is it needed? Is it unique?

- An effective theory does not know when and why it breaks down

**w/ gravity**

- **classically cannot be avoided** (Hawking, Penrose)

  - Geodesics terminate in regions with large curvature

**Possible cures:** quantization

- **LQG & string theory**

  - High deriv. terms (classical)

- In string theory, some GR-singular spaces are smooth...
some singularities are morally-similar to QFT IR singularities – related to changes in the field content:

– away from singularities: GR is fine

– near resolved singularity: GR extended with additional light states
  • corresponding fields: massive away from singularities or stuck there

Precise details depend on the type of singularity –

• orbifolds $\mathbb{C}^n/\Gamma$ (twisted fields) (Dixon, Harvey, Vafa, Witten 1985/86)

• some singular plane waves (higher derivatives?) (Horowitz, Steif 1990)

• some $N = 2$ effective act. (wrapped branes) (Strominger 1995; +Vafa 1996)
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Not all cases treatable geometrically
- (extremal) black holes=ensemble of nonsingular microstates (Mathur)
- string-scale $AdS_5 \times S^5/\Gamma$; instability cured “holographically” (Dymarsky, Klebanov, RR;+Franco)

Not all singularities are smoothed out
- some plane waves remain singular (strings get ripped apart as they pass through the front wave) (Horowitz, Steif; David; RR)
- unclear how to treat most space-like singularities; sometimes some fields see a smooth geometry (Liu, Moore, Seiberg; Horowitz, Polchinski) (Buchel, Langfelder, Walcher/Dijkgraaf, Verlinde, Verlinde)
orbifolds of flat space $\mathcal{M} = \mathbb{R}^{(9-2n,1)} \times \mathbb{C}^n/\Gamma$: exact CFTs

- asymptotic states propagate on $\mathcal{M}$; $\text{GR} + B_{\mu\nu} + \phi + F_{2n}(+1)$
- modular invariance $\rightarrow$ more states: asymptotic on $\mathbb{R}^{(9-2n,1)}$
  localized on $\mathbb{C}^n/\Gamma$
- If no tachyons, graviton scattering amplitudes on $\mathcal{M}$ are finite.

Partial resolution: analytical continuation of 2d bh $\text{SL}(2)/\text{U}(1)$
(Buchel, Langfelder, Walcher)

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<th>Momentum</th>
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- due to higher derivative terms, suppressed by $1/k$: resummation
\( \mathcal{N} = 2 \) effective actions from CY compactifications \( \text{ (Strominger)} \)

- for fixed topology, the moduli space of CY manifolds contain singularities; generic type – conifold

\[
u^2 + v^2 + w^2 + t^2 = 0 \quad \rightarrow \quad u^2 + v^2 + w^2 + t^2 = \epsilon^2 \quad \longrightarrow \quad S^3
\]

- conifold singularity appears as log singularity moduli space metric

- Moduli appear as fields in effective action; at generic point on moduli space kinetic term is given by moduli space metric

The cure:

- 3-branes wrap \( S^3 \) and are massless at singularity; effective action derived at generic point does not apply near this point

- extra field in effective action; quanta=extremal black holes!

- recover log singularity by integrating out this field
Less-than-trustworthy geometry

- $\mathbb{C}^3/\Gamma$ is a real cone over $S^5/\Gamma \rightarrow$ if localized tachyons on $\mathbb{C}^3/\Gamma$ and $\Gamma$ acts w/o fixed points on $S^5$, then for sufficiently small $S^5$, $S^5/\Gamma$ will still have tachyon(s)
  - How comes? – twisted states get lighter as size of $S^5$ is decreased
    - Supergravity not valid (and not trustworthy anyway $R^2_{AdS} \sim \alpha'$)
    - Hard to guess how to correctly include all relevant terms
    - Hard to describe the condensation of the tachyon(s)

- A solution: use holographic description
  - Dual theory is *perturbative* $\Gamma$-orbifold of $\mathcal{N} = 4$ SYM
  - Tachyon(s)=quantum-induced (2-trace) coupling *naively* running to negative infinity in finite RG time
  - Tachyon condensation=$\langle \mathcal{O} \rangle \neq 0$ and $\Gamma(\mathcal{O}) \neq 0$
Less naive: coupled RG flow: 2-trace + YM coupling + masses

\[ M \propto \langle \mathcal{O} \rangle \]

\[ M \geq \Lambda \] the corresponding field decouples and RG flow changes

Spectrum of \( \mathcal{N} = 4 \) SYM appears for sufficiently large \( \langle \mathcal{O} \rangle \)

- rank of gauge group is reduced

- if we are to attempt a geometric interpretation –

the condensation of the marginally-tachyonic fields on string-size \( AdS_5 \times S^5 / \Gamma \) leads to \( AdS_5 \times S^5 \) of size reduced by \( |\Gamma| \)
holography (AdS/CFT) has interesting suggestions about the fate of black hole singularities → Mathur’s conjecture

- non-extremal black holes ↔ gauge theory thermal states
  \[ \rho_{bh} = \sum_{E} e^{-\beta E} |E\rangle\langle E| \]

- entropy of extremal, asymptotically $AdS_3$ black holes can be computed (in controllable situations) as
  \[ S = k_B \ln N_{op} \]

  $N_{op} = \text{nr. of operators with the same charges as the black hole}$

- the gauge theory deformed by a gauge-invariant operator is dual to a nonsingular asymptotically $AdS_3$ geometry; differences are “localized” near the origin
Is it possible that a(n extremal) black hole is a statistical ensemble of nonsingular geometrical microstates, each of them smooth and without a horizon?

Mathur’s conjecture: Yes.

- horizon = where microstates “differ significantly” from each other

**The Plan:**
- construct the microstates and understand entropy
  - works great for 1-charge bh (Mathur, Lunin, Saxena, Maldacena, Maoz, etc)
  - entropy vanishes classically; string scale horizon; $S \propto A$
  - complicated for 2-charge bh; only few examples are known
  - arguments at linearized level

- classically, solutions depend on continuous parameters; need to understand quantization of the space of solutions

**Main point:** physics “explaining” bh singularity is not localized at the classical singularity, but extends all the way to horizon
Summary

- Singularities are useful for ruling out unphysical effects

- Singularities can be artifacts of effective actions breaking down
  - signaling the appearance of additional light states
  - signaling that low energy approximation breaks down

- Geometrically, signal that classical geometry is not trustworthy
  - holographic description may be better
  - “statistical” description in terms of microstates