Emergent Inflation

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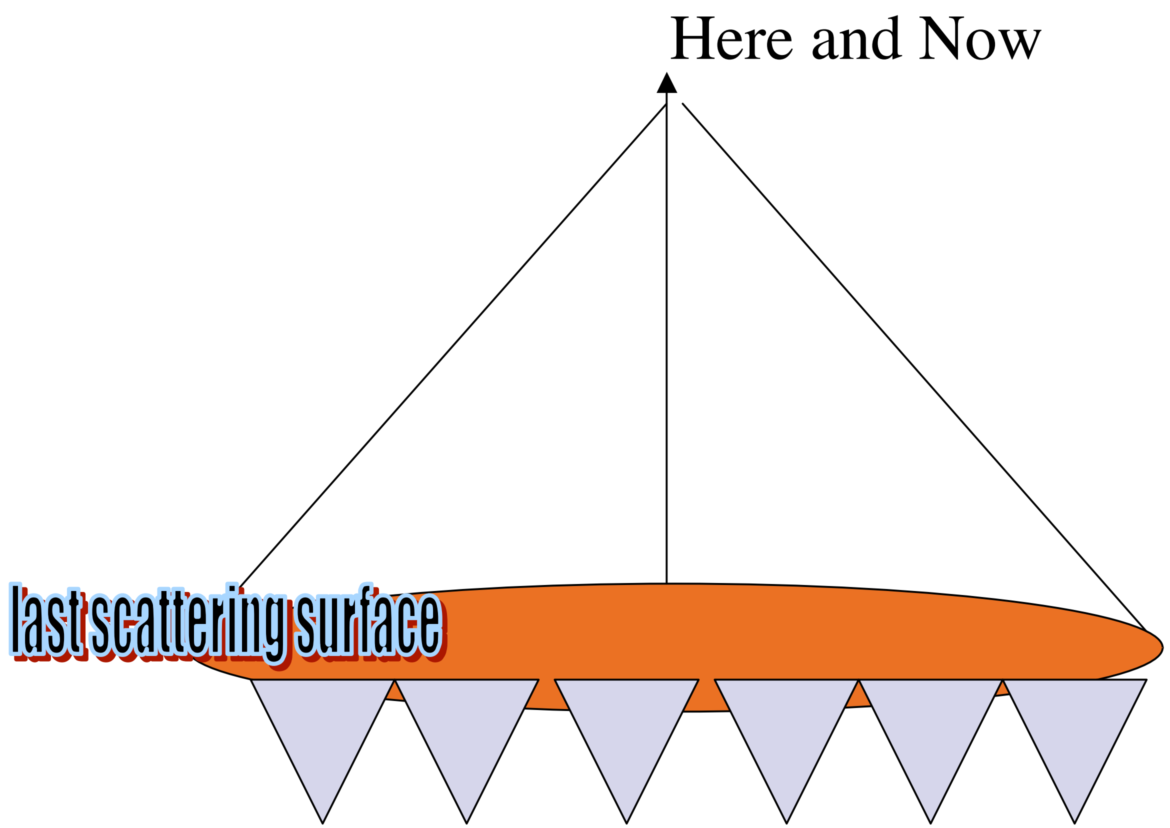
Outline

- Overview of Inflation
- Main Idea: The Instability of the Perturtbative Vacuum
- Review of the microscopic BCS theory
- Calculation and Realization
- Conclusion Outlook

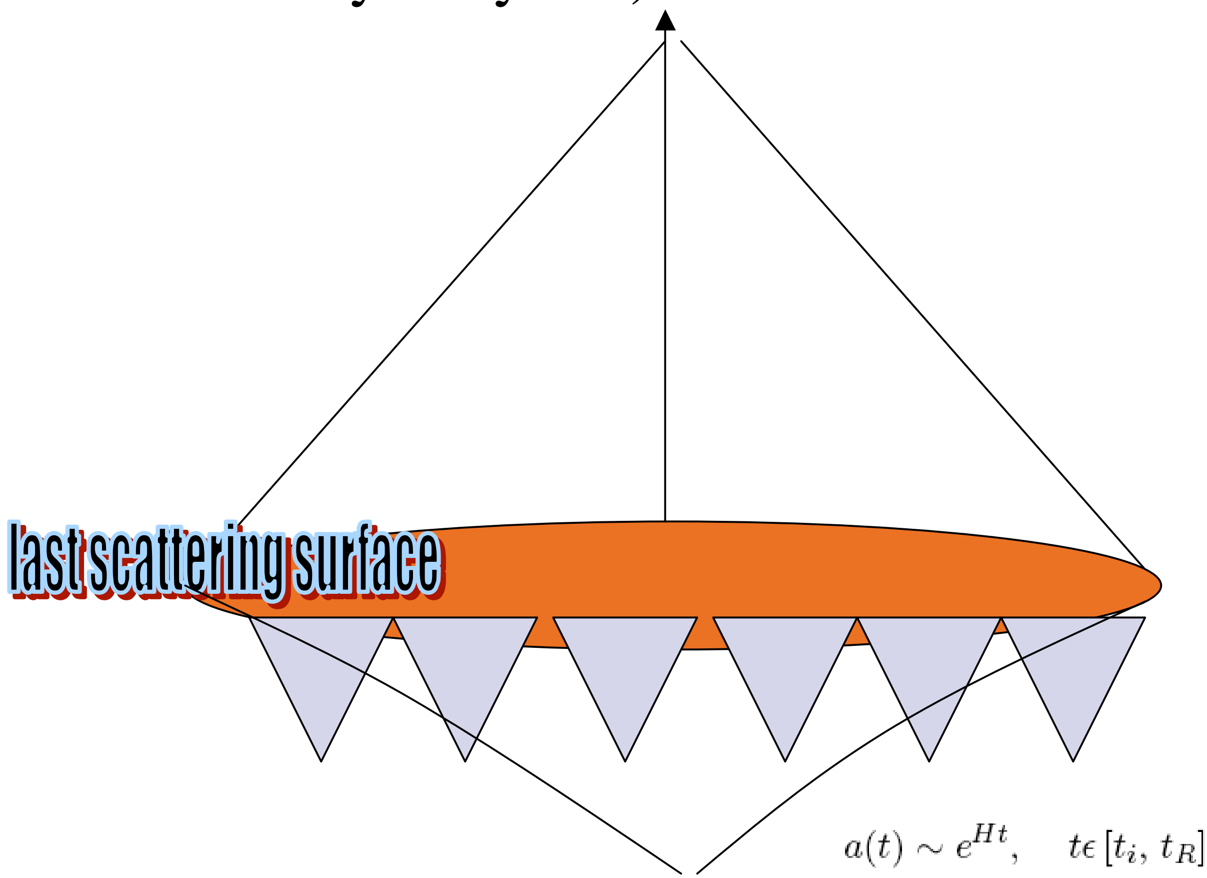
Why is Inflation Needed

A. Horizon Problem

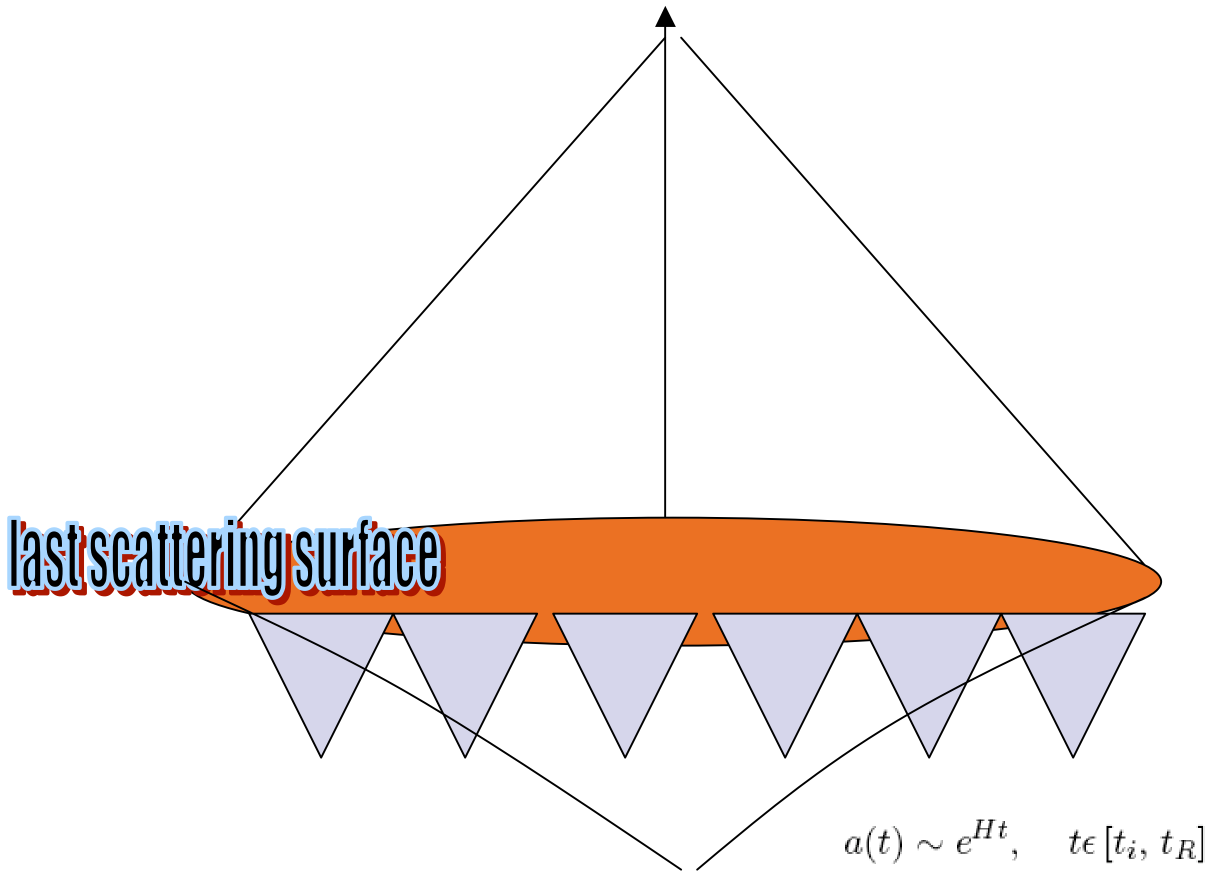


Inflation is an epoch wherein
The scale factor has positive acceleration

(Guth, Linde, Mukhanov.
Starobinsky early 80s)



Inflation is an epoch wherein
The scale factor has near exponential growth.



-The biggest problem with inflation boils down to its reliance on scalar fields.

-Borde, Guth, Vilenkin (PRL 2003) proved that scalar field driven inflation suffers geodesic incompleteness and from a big bang singularity. (modulo rare null-energy violating fluctuations)

-Scalar fields are the culprit behind the infamous fine tuning problems in inflation (and the standard model).

-How does inflation end?

-Cosmological Constant Problem.

Don’t Throw out the Baby Universe with the Bathwater!

- 1) Can we obtain inflation naturally without fundamental scalar fields?
- 2) If this is possible will we resolve the problems that currently afflict inflation
- Answer: (1) Yes

  (2) Very likely

A New Outlook: Condensates in Cosmology

(Parker, Brout, Brandenberger, Zhitnitsky, S.A, Moffat, Mbonye)

- Condensates are attractive because they can vanish in the UV and IR (superfluidity).

- They can be purely long range (macroscopic) quantum phenomena.

- They are very common across a wide range of physical phenomena.

- In the context of inflation they could drive inflation and dissapear at the end (IR) leading to a natural graceful exit.

But what exactly is this magical condensate?

The Cooper Instability

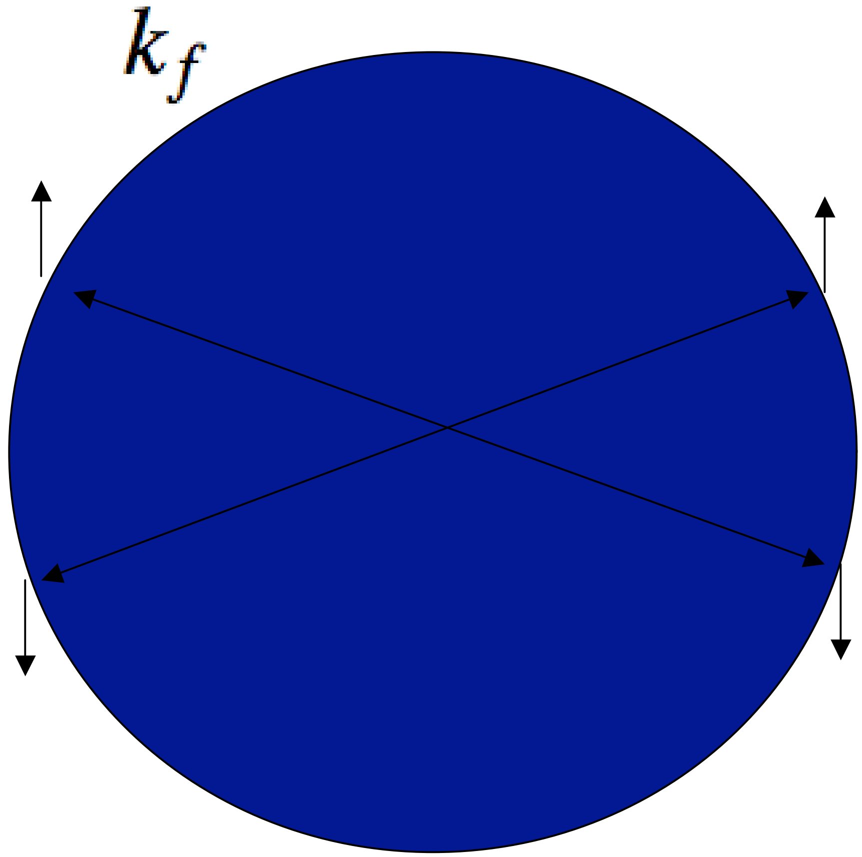
-In a conducting medium the Fermi sea is always unstable if there exist a small attractive interaction.

-Phonons screening of the electric charge giving an effective attractive interaction.

-A new ground state is found whose fundamental excitations are spin zero paired electrons (Cooper Pairs).

BCS Theory: Fundamentals

Key: Correlations of fermions across Fermi surface



SUPERCONDUCTIVITY

Fermi Surface

$$\Delta \sim e^{-1/gD(\varepsilon)}$$

True Ground State

$D(\varepsilon)$ =density of states

The perturbative vacuum (Fermi surface) is unstable, due to strongly correlated electron pairs.

There is a non-perturbative ground state with lower energy, separated by an energy gap.

WHAT IF THE PETURBATIVE INTERACTIONS OF FERMIONS WITH RESPECT TO GRAVITY IS UNSTABLE IN THE SAME WAY?

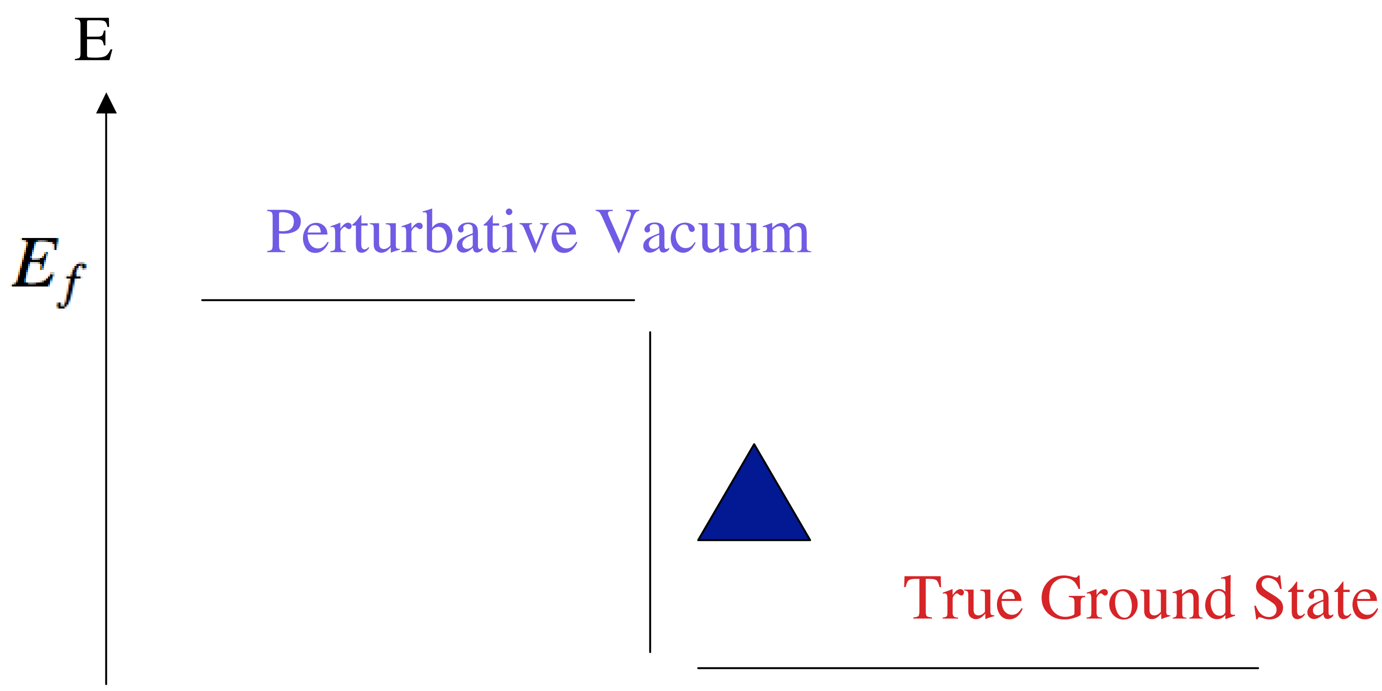
ANSWER: THIS IS PLAUSIBLE IF THERE IS

1) A FOUR FERMION INTERACTION

2) A FERMI SURFACE (CHEMICAL POTENTIAL)

3) SCREENING OF REPULSIVE INTERACTION
(NEUTRAL FERMIONS WOULD ALSO SUFFICE)

Space-time Superconductivity



The perturbative vacuum is unstable. There is a non-perturbative ground state.

A key insight in BCS theory is that the true ground state wavefunction (BCS State) is constructed by a phase coherent superposition of paired states

$$|\phi_N\rangle = \sum_{\mathbf{k}_1} \cdots \sum_{\mathbf{k}_{N/2}} g_{\mathbf{k}_1} \cdots g_{\mathbf{k}_{N/2}} \ldots a^\dagger_{\mathbf{k}_1\uparrow} a^\dagger_{-\mathbf{k}_1\downarrow} \cdots a^\dagger_{\mathbf{k}_{N/2}\uparrow} a^\dagger_{-\mathbf{k}_{N/2}\downarrow} |\varphi_0\rangle,$$

g is the weight of each of the paired states

BCS HAMILTONIAN

Bardeen, Cooper and Schrieffer considered the Hamiltonian describing the superconducting state.

$$\mathcal{H} = \sum_{\mathrm{k's',ks}} \mathcal{E}_{\mathrm{k}} \mathrm{c}^{\dagger}_{\mathrm{ks}} \mathrm{c}_{\mathrm{ks}} - \frac{1}{2} \sum_{\mathrm{k,k',s,s'}} \mathrm{V}_{\mathrm{kk'}} \mathrm{c}^{\dagger}_{\mathrm{k's'}} \mathrm{c}^{\dagger}_{\mathrm{-ks}} \mathrm{c}_{\mathrm{-k's'}} \mathrm{c}_{\mathrm{ks}}$$

k & s denote momenta and spin of fermion in fermi sea

$c^{\dagger}_{ks}$ -fermion creation operator

$V_{kk'}$ -Two body interaction potential of correlated fermions.

- This Hamiltonian has a non perturbative ground state which corresponds to pairs of fermions with opposite spin and momenta.

- The Schrodinger eq. yields an exact ground state energy with an energy gap Δ separating the condensate from the ‘free state’ perform the transformation to new operators

$$b_k = u_k c_k - v_k c^\dagger_{-k}, \quad b_{-k} = u_k c_{-k} + v_k c^\dagger_k$$

u and v -> probability of creating and destroying a Cooper Pair

The BCS state is a coherent state of Cooper Pairs:

$$|BCS> = \Pi_k (u_k^* + v_k^* c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0>$$

The |BCS> state is an eigenstate of the transformed Hamiltonian
The Amplitude (gap) needs to be determined.

This new state is the true ground state, separated from false ground state by this gap.

After transforming the Hamiltonian

We can minimize the Hamiltonian energy by diagonalizing $\mathcal{H}$, giving the condition

$$\hat{H}_k\Psi = \varepsilon_k\Psi \longrightarrow \mathcal{E}_k\left(\frac{1}{4} - x_k^2\right)^{1/2} + x_k \sum_{k'} V_{kk'}\left(\frac{1}{4} - x_{k'}^2\right)^{1/2} = 0$$

where $u_k = (\frac{1}{2} - x_k)^{1/2}$ and $v_k = (\frac{1}{2} + x_k)^{1/2}$, and $V_{kk'}$ is the interaction matrix

We define the quantity

$$\Delta_k = \sum_{k'} V_{kk'}\left(\frac{1}{4} - x_{k'}^2\right)^{1/2}$$

To get ground state, set occupation numbers $n_k = b_k^\dagger b_k$ and $n_{-k} = b_{-k}^\dagger b_{-k}$ to zero and anticommutation of b's gives a self consistent eq for the gap:

$$\Delta_k = \frac{1}{2}\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{(\mathcal{E}_{k'}^2 + \Delta_{k'}^2)^{1/2}}$$

perform the transformation to new operators

$$b_k = u_k c_k - v_k c^{\dagger}_{-k}, \quad b_{-k} = u_k c_{-k} + v_k c^{\dagger}_k$$

These new operators corresponding to breaking

Where u,v are determined by the BCS wavefunction

$$|BCS> = \Pi_k (u_k^* + v_k^* c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow})|0>$$

There will now be a transformed Hamiltonian that acts on |BCS> which is a coherent state of fermion pairs.

This new state is the true ground state, separated from false ground state by an energy gap.

In terms of the new Hamiltonian
the gap represents an exitation of quasiparticle's
energy spectrum

$$E_k = \sqrt{\xi_k^2 + \Delta^2}$$

EXACT NON-PERTURBATIVE SOLUTION

The solution to this equation is

$$\Delta = \sqrt{\Lambda} = \frac{\omega_D}{\sinh[1/VD]}$$

D = Density of states (number of states per unit frequency)
V= interaction coupling

Which has both weak and strong coupling solution.

We'll get back to this later.

The Lesson

Superconducivity is an example of a macroscopic non-perturbative phenomena which is purely quantum mechanical.

Cannot be seen by perturbation theory.

The Cooper pairs behave effectively like a scalar field but is not fundamental.

QUESTION: Can this give us inflation?

We now demonstrate that
General Relativity coupled to free
fermions have the necessary
four fermion interactions necessary
for a BCS condensate.

The Mechanism:

1) A time dependent space-time background self consistently generates a non-perturbative condensation of fermions.
2) The energy gap yields a repulsive effect which generates inflation.
3) As the universe inflates the gap contribution becomes irrelevant and inflation naturally ends.

General Relativity Covariantly Coupled to Free-Fermions

$$S_{EC}\left(e,\omega,\psi,\overline{\psi}\right) = \frac{1}{4}\int \epsilon_{abcd}\, e^a \wedge e^b \wedge R^{cd} + \frac{i}{2}\int \star\, e_a \wedge \left(\overline{\psi}\gamma^a \mathcal{D}\psi - \overline{\mathcal{D}\psi}\gamma^a\psi\right)$$

where e^a is the gravitational field, while $R^{ab} = d\omega^{ab} + \omega^a{}_{\underline{c}} \wedge \omega^{cb}$ is the Riemann curvature 2-form and ω^{ab} is the Lorentz valued spin connection 1-form.

$$\mathcal{D}\psi = d\psi - \frac{i}{4}\,\omega^{ab}\Sigma_{ab}\psi \quad \text{and} \quad \overline{\mathcal{D}\psi} = d\overline{\psi} + \frac{i}{4}\,\overline{\psi}\Sigma_{ab}\omega^{ab}$$

We now vary the action w.r.t the connection:

We find the following solution:

$$\omega_\mu^{ab} = \overset{\circ}{\omega}{}_\mu^{ab} + \frac{1}{4}\epsilon^{ab}{}_{cd} e_\mu^c J_{(A)}^d$$

First term is pure gravitational part (symmetric)
Second term is fermion current.

$$J_{(A)}^\rho = \overline{\psi}\gamma^\rho\gamma^5\psi$$

We reinsert the solutions:

$$S_{J_{(A)}-J_{(A)}}(e,\psi,\overline{\psi}) = \int d^4x \det(e)\left(-\frac{1}{2}e^{\mu}{}_{a}e^{\nu}{}_{b}\,\overset{\circ}{R}_{\mu\nu}{}^{ab} + \frac{i}{2}e^{\mu}{}_{a}\left[\overline{\psi}\gamma^{a}\,\overset{\circ}{\mathcal{D}}_{\mu}\,\psi - \overline{\overset{\circ}{\mathcal{D}}_{\mu}\,\psi}\gamma^{a}\psi\right] + \frac{3}{16}\eta_{ab}J^{a}_{(A)}J^{b}_{(A)}\right).$$

And get the much sought after universal four-fermion Interaction!

Hamiltonian

After a lot of simple algebra and using the ADM 3+1 Decomposition we arrive at the following Hamiltonian

$$H_{G+D+int} = \int d^3x\, A_t^i \left\{ \frac{1}{\kappa}[K_b, \tilde{E}^b]^i + j^i \right\}$$
$$+ N\left\{ \frac{1}{2\kappa\sqrt{q}}(R_{ab}^i - [K_a, K_b]^i)[\tilde{E}^a, \tilde{E}^b]_i + \frac{i}{2\sqrt{q}}\tilde{E}_i^a(\xi^\dagger\sigma^i D_a\xi - \rho^\dagger\sigma^i D_a\rho - c.c.)\right.$$
$$\left. + \frac{1}{2}[K_a, \tilde{E}^a]^k j_k - \frac{3}{2}\pi G\frac{\gamma^2}{\gamma^2+1}[j^2 - (-\xi^\dagger\xi + \rho^\dagger\rho)^2] + \sqrt{q}\Lambda_0 \right\}$$
$$+ N^a \left\{ \frac{2}{\kappa} D_{[a}K_{b]}^i \tilde{E}_i^b + \frac{i}{2}(\xi^\dagger D_a\xi + \rho^\dagger D_a\rho - c.c.) \right\}$$

$A_a^i = \Gamma_a^i + iK_a^i$. K is the extrinsic curvature

$\tilde{E}_i^b$ is the densitized triad, F_{ab}^i is the curvature of the restriction A_b^i to Σ

We will restrict this system to
a cosmology, ie non-vanishing Lambda

$$E^a_i = a^2\delta^a_i \quad K^i_a = a^2\dot{a}\delta^a_i \quad R^i_{ab} = 0$$

The Hamiltonian simplifies significantly.

$$\mathcal{H} = \mathcal{H}_G + \mathcal{H}_D + \mathcal{H}_{int} = -\frac{3}{\kappa}a^3 H^2 + a^3\Lambda_0 + \frac{i}{a}\left(\xi^\dagger\sigma^a\partial_a\xi - \rho^\dagger\sigma^a\partial_a\rho\right) + \frac{3\kappa}{32a^3}\frac{\gamma^2}{\gamma^2+1}\left[\xi^\dagger\xi - \rho^\dagger\rho\right]^2 = 0$$

Question: What is the self consistent solution
the fermion many body wavefunction in a time
dependent background?

$$\hat{H}(\xi) = -\hat{N}(V) + \int \frac{d^3k}{(2\pi)^3} \Big\{ (m_k + n_{-k}) \left[(u_k^2 - v_k^2) E_k + 2u_k v_k \Delta_k \right] + \Sigma_k \left[2u_k v_k E_k - (u_k^2 - v_k^2) \Delta_k \right]$$

$$+ \left[2v_k^2 E_k + E' v_k^2 - u_k v_k \Delta_k \right] \Big\} = -\hat{N}(V) + \hat{K}_1 + \hat{K}_2 + \hat{U}$$

Our Hamiltonian is very similar to the BCS one.

We can try to find the gap using the same methods.

Setting the occupation numbers equal to zero and diagonalizing H

We obtain the equation for the gap

$$\Delta_k = \alpha \int \frac{d^3k'}{(2\pi)^3} \frac{V_1'(k,k')\Delta_{k'}}{2\sqrt{E_{k'}^2 + \Delta_{k'}^2}}$$

Realization of Emergent Inflation

This mess beautifully simplifies to:

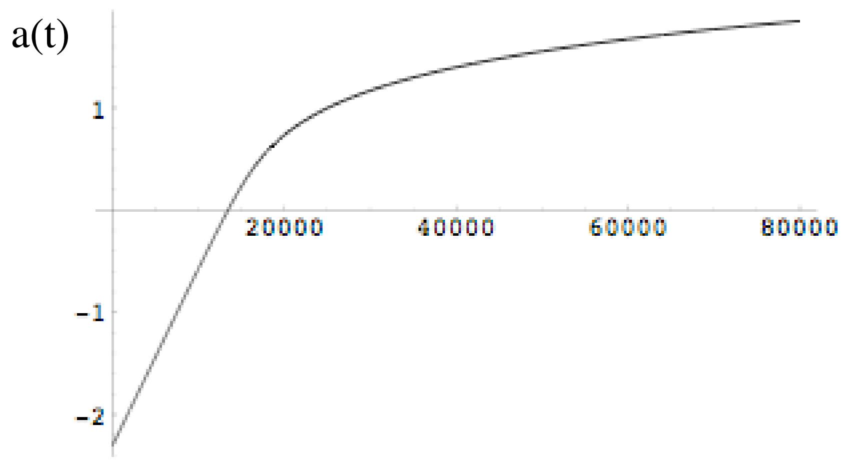
$$H^2 = \frac{1}{3M_p^2}\left[\rho_0 a^{-4} - 2\Delta^2\right]$$

$$\Delta(a) = \frac{2\hbar\omega_D}{Sinh(\frac{M^2 a^2}{k_0^2 f})}$$

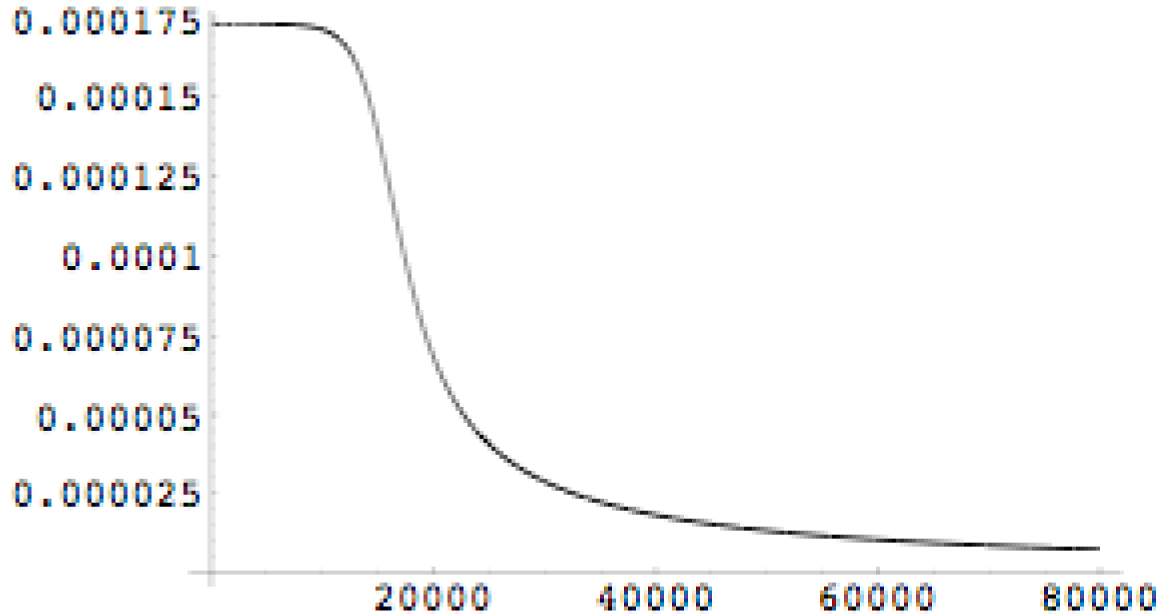
$$\ddot{\phi} + 3H\dot{\phi} + \Delta^2\phi = 0$$

EMERGENT INFLATION

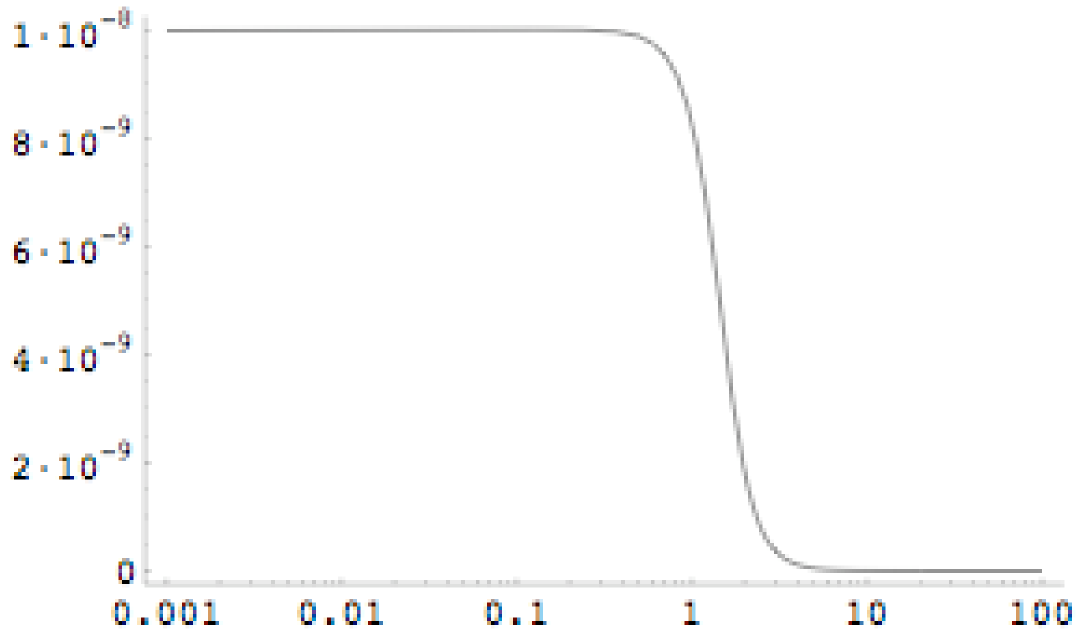
We get exponential growth which ends.



Cosmological Gap



Hubble Parameter



What is going on physically?

-The instability of the perturbative vacuum yields an emergent phenomena.

-This emergent phenomena is a large scale Phase coherence which amplifies the scale factor.

-Macroscopic phase coherence = Inflation.

-Similar to the emergent (collective) phenomena known as superconductivity.

No Multiverse!

Scale factor is not sufficient to describe correlations in Hubble radius.

Phase coherence of condensate wavefunction is tied Into the amplification of scale factor.

Therefore eternally inflating bubbles can not satisfy this Initial conditon since it is geodesically complete in past.

Do We resolve problems of scalar Field inflation?

The number of e-foldings are only sensitive
to the initial fermi energy (density of states)

The generation of perturbations on superhorizon
Scales will correspond to fluctuations in condensate
-> supressed on superhorizon scales.

Geodesically complete since scalar field is not fundamental
And initial state is Minkowski.

Conclusion and Outlook

-We have begun the first steps toward an emergent inflationary model which requires no scalar fields.

-Inflation is equivalent to a superconducting phase space-time

-The Graceful exit problem is self-consistently solved.

-We may actually have a synergy between bouncing Singularity free cosmology followed by inflation

-What are the predictions? Could it be the Higgs?

COULD THIS HAPPEN IN
THE CONTEXT OF ADS/CFT?

IS INFLATION A BOUNCING
COSMOLOGY IN DISGIUSE?

$$H^2 = \frac{1}{3M_p^2}\left[\rho_1(a) - \rho_2(a)\right]$$

$$a_b^7 \mu \exp(-2\mu a_b^3) < \frac{2\rho_0}{3\Delta_0^2}$$

$$\rho_{b,\mathtt{rad}} > \Delta_0^2 e^{-4/3}$$