Cosmological vector modes and quantum gravity effects

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References

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Cosmological Perturbations

- Linear metric perturbations around homogeneous and isotropic geometries can be split in different types.
- Scalar, vector and tensor modes (according to their transformation properties under spatial rotations).
- It is convenient!
- In linearized Einstein's equation these different modes decouple.
- Space-time coordinate transformations (gauge transformations) do not mix the modes.
- Thus they can be analyzed separately.

Scalar, Vector and Tensor modes

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- Tensor modes are essentially the gravitational waves.
- These two modes are extensively studied in literature.
- However vector modes do not receive similar attention and are often ignored.

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- These two modes are extensively studied in literature.
- However vector modes do not receive similar attention and are often ignored.
- ...and there are reasons for it.

Vector Mode Properties

- As universe expands, gauge-invariant measure of vector perturbations typically falls of as $\sim a^{-2}$ where a is scale factor. (In absence of anisotropic stress)
- Once universe becomes approximately homogeneous and isotropic *i.e.* perturbations are small then subsequent expansion leads vector mode to become even smaller.
- Thus one may be justified to ignore vector mode perturbations in an expanding cosmology!!

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- Thus one may be justified to ignore vector mode perturbations in an expanding cosmology!!
- ...but only in late times.

Vector Mode in Contracting Branch

- Gauge invariant measure of vector perturbations: $\sigma^i \sim a^{-2}$
- In backward evolution of the spacetime vector modes grow. In particular, vector modes blow up as $a \rightarrow 0$.
- Signals breakdown of linear perturbation theory.
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- This raises concern regarding robustness of the conclusions which are drawn using homogeneous assumption in small volume.
- Viewpoint of a classical cosmologist:
  - OK! vector modes grow but doesn't even homogeneous background becomes singular as $a \rightarrow 0$ (big bang)? So does breakdown of perturbation theory add any significant twist to an already twisted story?

Bouncing Models of Cosmology

- However, there are non-singular models of cosmology.
- Typically they avoid singularity by exhibiting big bounce at small volume.
- These models are invariably associated with a contracting branch.
- Conclusions regarding bounce are drawn mainly based on homogeneity assumption.
- Any such bouncing model of cosmology must show that classical growth of vector modes is suitably modified in contracting branch.

Battefeld, Brandenberger 2004

Loop Quantum Cosmology (LQC)

- Quantization of cosmological (homogeneous) models using techniques of loop quantum gravity (LQG).
- Avoid singularity by exhibiting bounce near classical singularity.
- How robust is this picture when one includes inhomogeneities?
- In particular, are vector modes better behaved with (loop) quantum gravity corrections?

Vector Modes: Cosmic Vorticity

- Metric components:

$$g_{00} = -a^2 \quad , \quad g_{0a} = a^2 S_a \quad , \quad g_{ab} = a^2 \left[\delta_{ab} + F_{a,b} + F_{b,a}\right] \; .$$

- Metric perturbations F_a and S_a are divergence-free. So they satisfy

$$\partial^a F_a = 0 \;\; ; \;\; \partial^a S_a = 0.$$

- Vector perturbations describe the vorticity of metric perturbations.

An example of Astrophysical vortex

- Milky Way Galaxy

Gauge Invariant Variable

- Gauge invariant variable:

$$\sigma^a = S^a - \dot{F}^a$$

- Perturbed stress-energy tensor

$$\delta {T^{(v)}}^i{}_a =: P(\pi_a{,}^i + \pi^i{}_{,a})$$

where P is pressure and π_a is anisotropic stress.

Classical Rate of Change

- Einstein Equation:

$$-\frac{1}{2a^4}\frac{\mathrm{d}}{\mathrm{d}\eta}\left[a^2(\sigma_a{,}^i + \sigma^i{,}_a)\right] = 8\pi GP(\pi_a{,}^i + \pi^i{,}_a)$$

- In absence of anisotropic stress, classical rate of change (for Fourier modes of vector perturbations)

$$\frac{\mathrm{d}\log\sigma^i_k}{\mathrm{d}\log a} = -2$$

Loop Quantum Gravity Corrections?

- Homogeneous models are better understood and for some models one can derive effective Hamiltonian rigorously.

  Ashtekar, Pawlowski, Singh 2006
- However, there are yet to be any formal derivation of effective Hamiltonian for inhomogeneous situations.
- In such situations, one may seek guidance within the lessons of homogeneous models.
- Such application may appear as somehow ad-hoc approach but it could give us some insights about quantum corrected vector mode dynamics.

Modified Rate of Change: Inverse Densitized Triad

- Inverse powers of the densitized triad in Hamiltonian

$$\frac{E^c_j E^d_k}{\sqrt{|\det E|}} \rightarrow \alpha(E^a_i)\frac{E^c_j E^d_k}{\sqrt{|\det E|}}$$

- Modified rate of change

$$\frac{\mathrm{d}\log\sigma^i_k}{\mathrm{d}\log a} = -2\left(1 - \frac{\bar{\alpha}'\bar{p}}{\bar{\alpha}}\right)$$

- Correction function

$$\bar{\alpha} := \alpha(\bar{p}, \delta E^a_i = 0) = 1 + c\left(\frac{\ell^2_{\mathrm{P}}}{\bar{p}}\right)^n ,$$

Modified Rate of Change: Holonomies

- Effective Hamiltonian for homogeneous models: Holonomies in places of connections:

$$\gamma \bar{k} \rightarrow \frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}}$$

- Modified rate of change due to the use of holonomies in quantum theory

$$\frac{\mathrm{d} \log \sigma^i_k}{\mathrm{d} \log a} = -\left(1 + \frac{2\bar{\mu}\gamma\bar{k}}{\sin 2\bar{\mu}\gamma\bar{k}}\right) \ .$$

Discussions

- Gauge invariant measure of vector perturbations grows in contracting branch. Signals breakdown of linear perturbations theory in small volume.
- This raises concern regarding robustness of many conclusions (such as bounce in homogeneous models) when one includes inhomogeneity.
- Typical expected (loop) quantum gravity corrections namely inverse densitized triad corrections and use of holonomies in quantum theory seem to increase the rate of change of vector perturbations.
- Questions such as whether bounce in LQC will survive if one includes inhomogeneity remains open.