

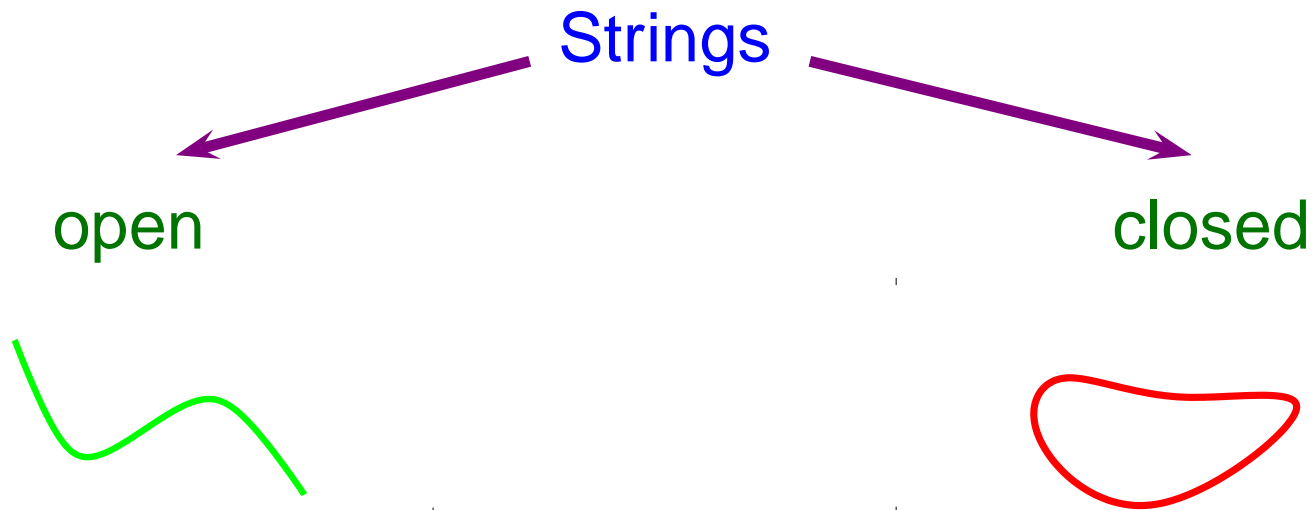
Intersecting Branes and Geometry

Oleg Lunin

University of Chicago

arXiv:0706.3396

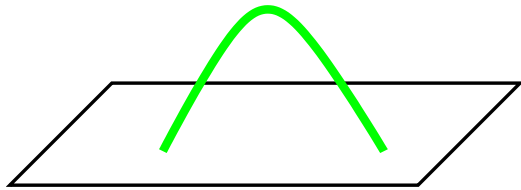
Branes in string theory



Branes in string theory

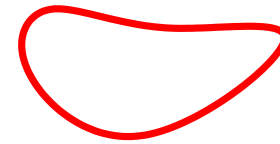
Strings

open



Dai–Leigh–Polchinski '89

closed

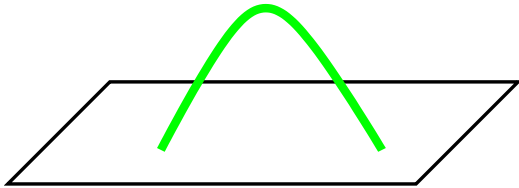


● D-brane

Branes in string theory

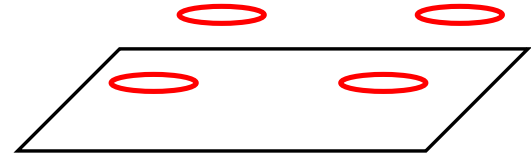
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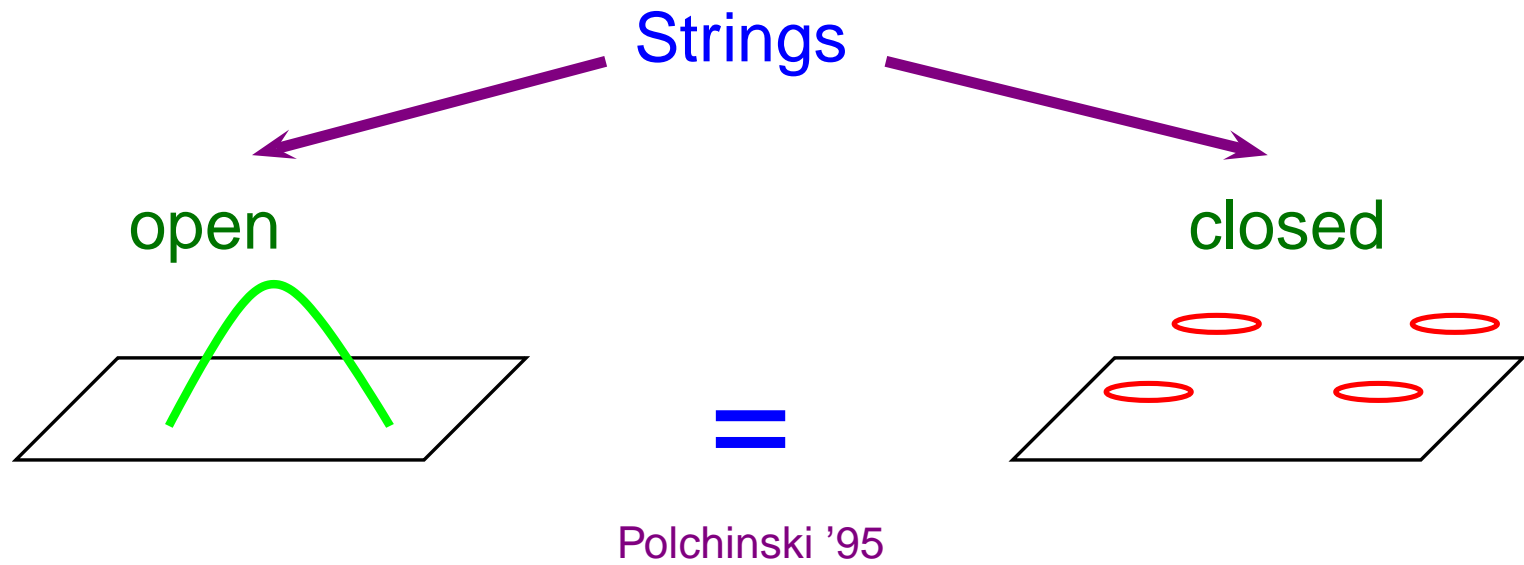


Horowitz–Strominger '91

● D–brane

● black brane (geometry)

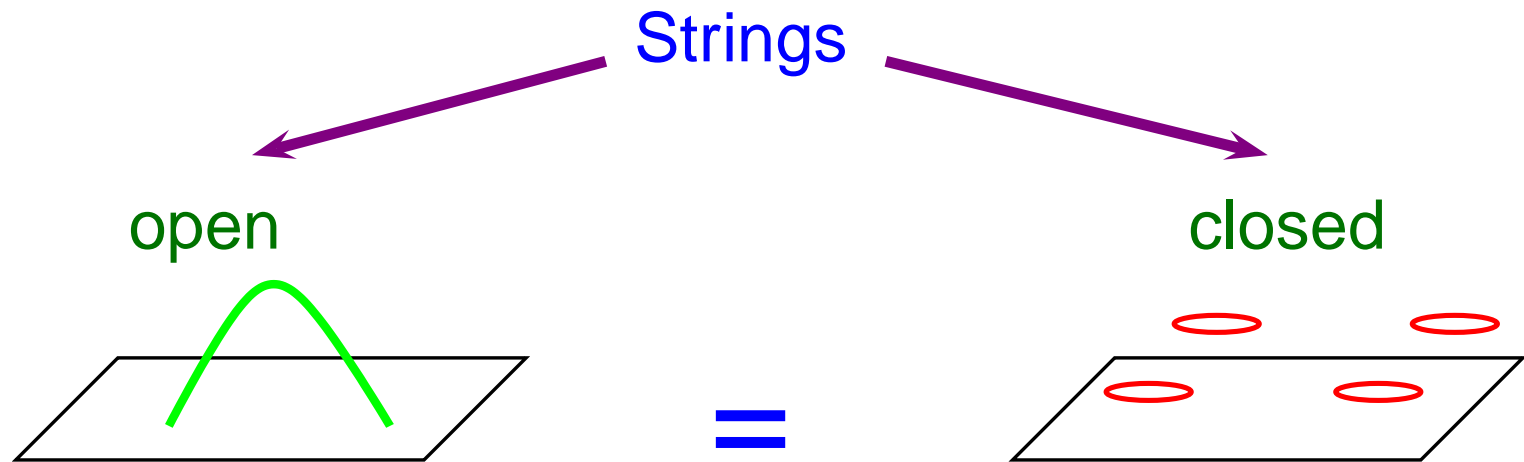
Branes in string theory



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Branes in string theory



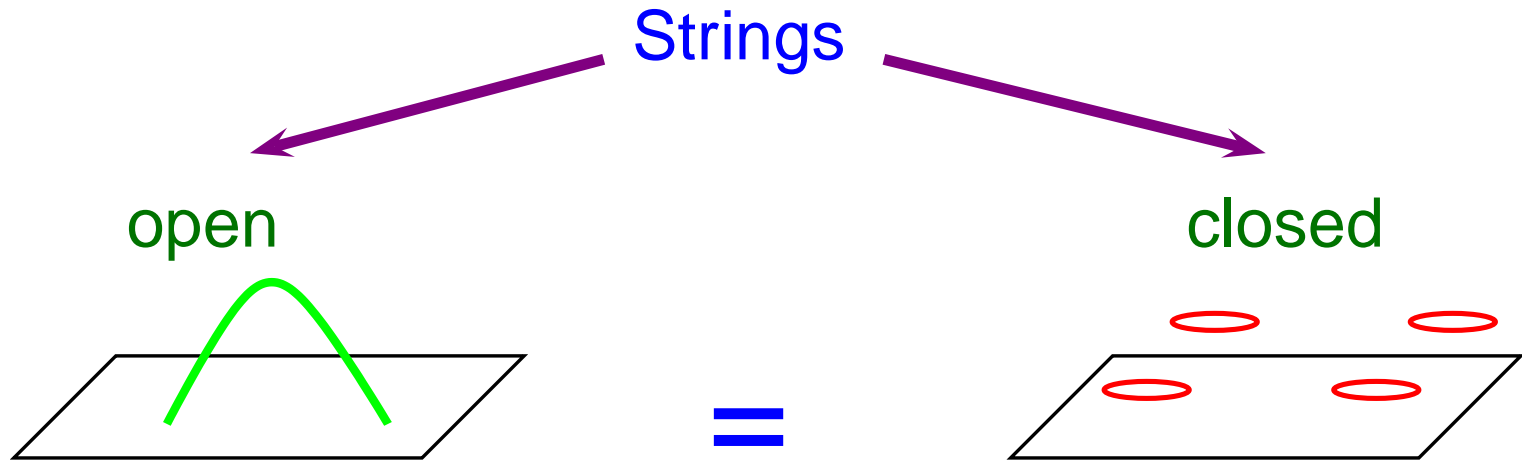
- D-brane

- effective action: DBI

- black brane (geometry)

- supergravity

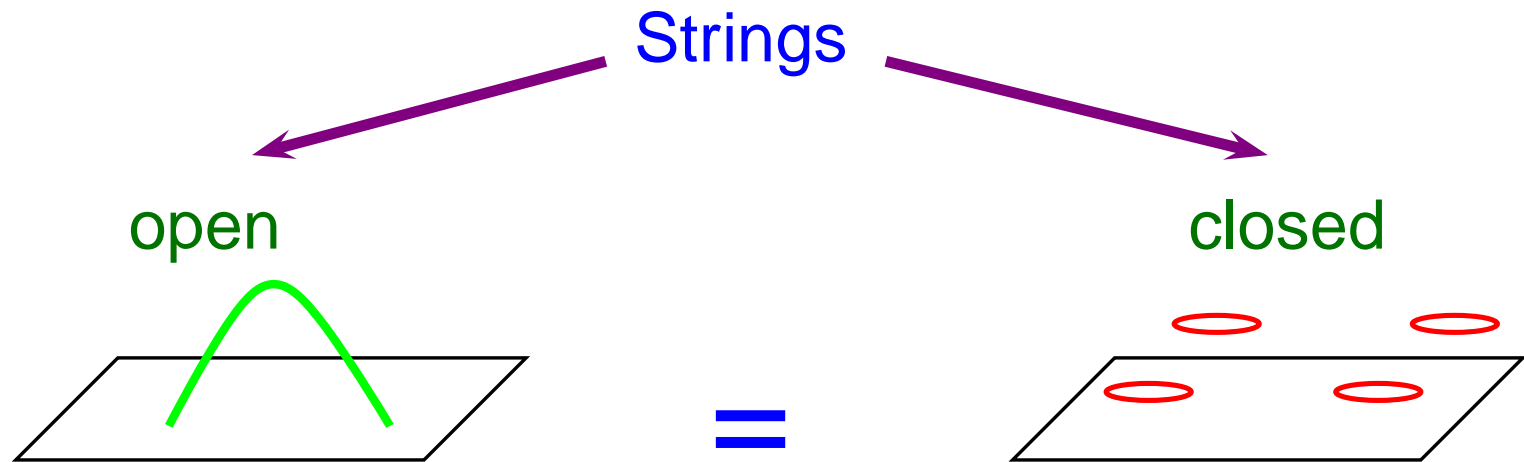
Branes in string theory



- D-brane
- effective action: DBI
- flat branes

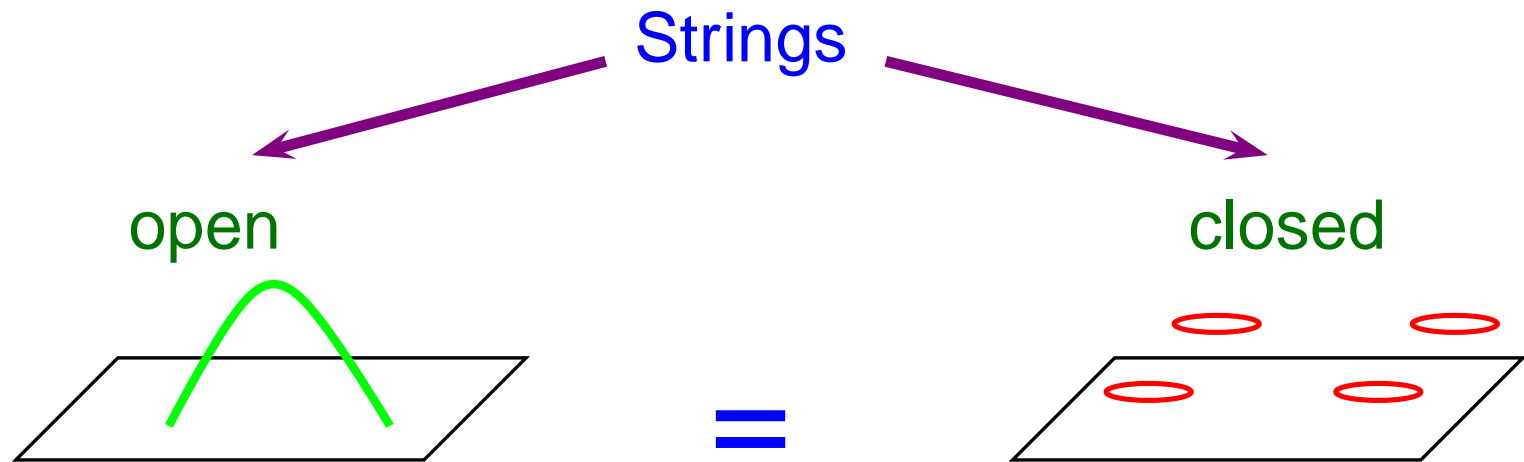
- black brane (geometry)
- supergravity
- large symmetry

Branes in string theory



- D-brane
- effective action: DBI
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- intersections
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- special solutions

Branes in string theory



- D-brane
- effective action: DBI
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 - shapes from dynamics
- black brane (geometry)
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- special solutions
 - ???

The uses of branes

- Applications to gauge theory

- low energy dynamics: super–Yang–Mills
- intersecting branes: colors and flavors
- geometric picture for Seiberg duality

Hanany–Witten '96; Witten '97

The uses of branes

- Applications to gauge theory

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Strominger–Vafa '96

- attempts to build cosmological models

KKLT '03

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● Gauge/gravity duality

- field theory = string theory
- strong/weak coupling complementarity

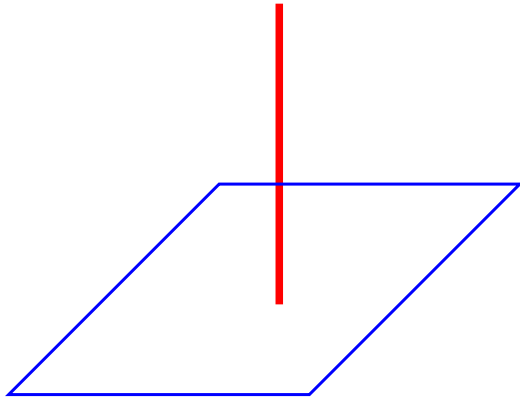
Maldacena '97

Outline

- Motivation
 - quantum gravity, gauge dynamics
 - understanding shapes on the gravity side
- Probe approximation
 - Blons in flat space
 - spikes in the presence of branes
- Solutions in supergravity
 - technique for constructing the geometries
 - local description and consistency conditions
- "DBI/SUGRA correspondence"
- Open questions

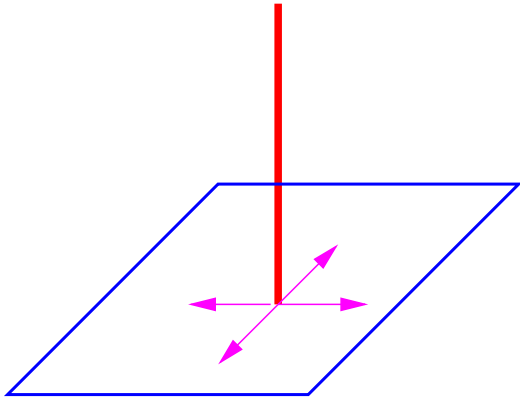
Bions in flat space

- Strings ending on a brane:



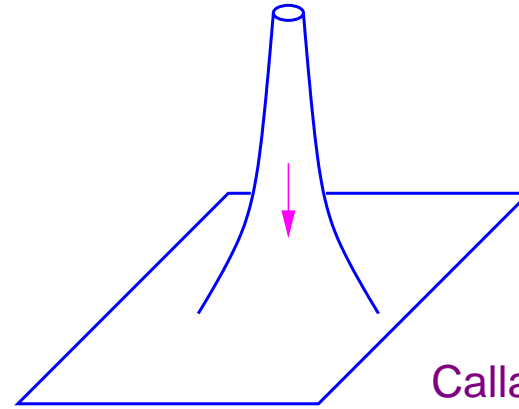
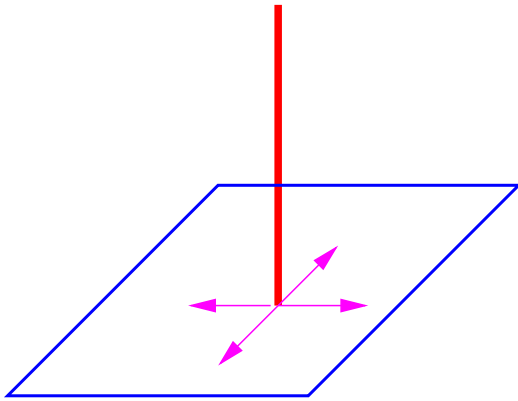
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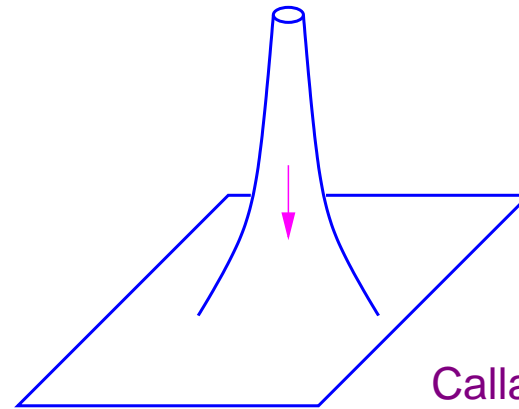
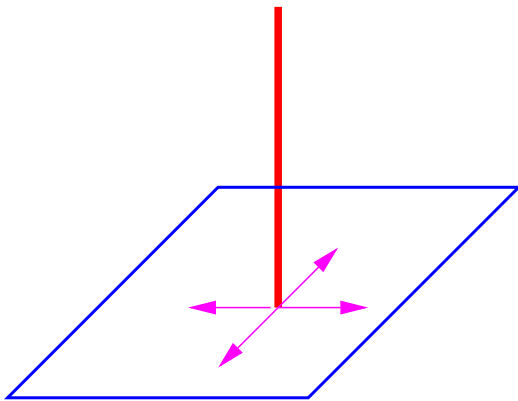
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Callan–Maldacena '97

Bions in flat space

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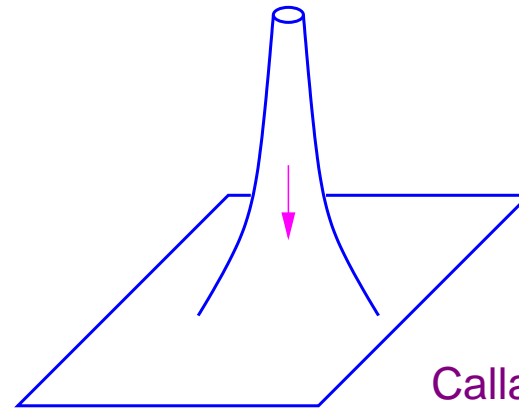
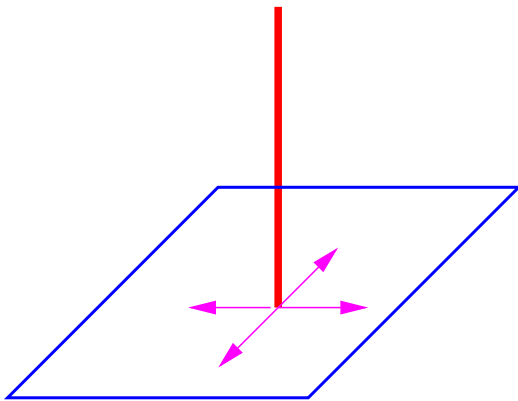
- Dirac–Born–Infeld: nonlinear electrodynamics

$$S_{DBI} = -T \int d^{p+1}\xi \sqrt{-\det(G + 2\pi\alpha' F)}$$

- induced metric \rightarrow electric field determines shape

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$$F_{ti} = \frac{1}{2\pi\alpha'} \nabla_i X, \quad \nabla^2 X = 0$$

Blons in curved space

- Spike preserves 8 supercharges

	1	2	3	4	5	6	7	8	9
$D3$	●	●	●						
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- Coupling to the RR fluxes:

$$S_p = S_{DBI} + T_p \sum \int e^{2\pi\alpha'F} \wedge P[C_q]$$

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- D3 brane probe in D3 geometry: harmonic profile
- D3 probe in D5 geometry (or vice versa):

$$-(1 + (\nabla X)^2)\partial_X H + H\nabla^2 X + 2\nabla H\nabla X = 0$$

Constructing the gravity solution

- Expected symmetries
 - eight supercharges
 - solution is static
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- Special geometry
 - $SO(3) \times SO(5) \times U(1)_t$ isometries
 - complete solution of eqns for Killing spinors

Structure of the geometry

- Local description

- two functions of 9 variables $(w, \vec{x}_3, \vec{y}_5)$:

$$\partial_w e^{-2\phi} + \Delta_{\mathbf{x}} F|_{y,w} = 0, \quad \partial_F e^{-2G-2\phi} + (\Delta_{\mathbf{y}} w)|_{x,F} = 0$$

$$e^{-2G} = \partial_w F$$

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- Boundary conditions for D3 branes

- finite dilaton, $w = \tilde{w}[\vec{y}, F - f(\vec{x})]$

$$\left(\partial_{\mathbf{x}} - \frac{\partial_{\mathbf{x}} w}{\partial_F w} \partial_F \right) \frac{\partial_{\mathbf{x}} w}{\partial_F w} = 0 \rightarrow \Delta_{\mathbf{x}} f = 0$$

- Harmonic profiles for D branes

- Positions of branes \rightarrow unique geometry

Supergravity vs DBI

- Profiles of D3 branes from SUGRA:

$$\Delta_{\mathbf{x}} f = 0$$

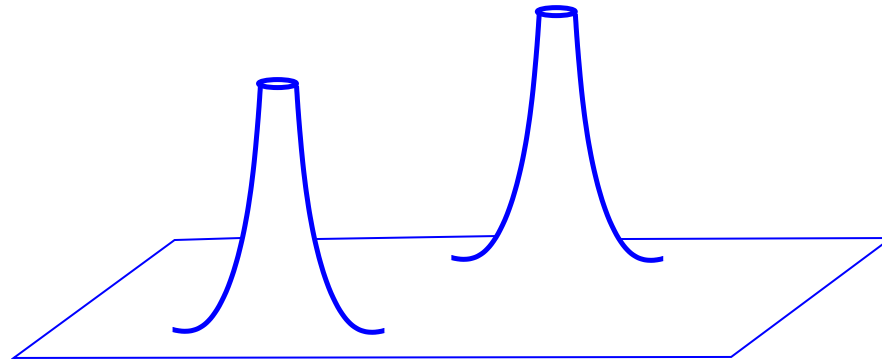
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- D3 probe in D5 geometry

$$-(1 + (\nabla X)^2) \partial_X H + H \nabla^2 X + 2 \nabla H \nabla X = 0$$

$$H = H_5(z, \vec{x})|_{z=X(\vec{x})}, \quad (\partial_z^2 + \Delta_x) H_5 = 0$$

Supergravity vs DBI

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- D3 probe in D5 geometry

$$-(1 + (\nabla X)^2) \partial_X H + H \nabla^2 X + 2 \nabla H \nabla X|_{y,w} = 0$$

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- D3 probe in D5 geometry

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- translation to the appropriate variable

$$\partial_w F = e^{2G}, \quad F_0(\vec{x}) = \int^{X(\vec{x})} H_5(z, \vec{x}) dz : \quad \Delta_x F_0|_{y,F} = 0$$

Outlook

- Shapes of branes are determined dynamically
 - open strings: solutions of DBI
 - closed strings: consistency of SUGRA
- Explicit example: 1/4–BPS D3/D5/F1 system
 - complete gravity solution
 - perfect agreement between DBI and SUGRA
- Generalizations
 - systems related by U–dualities
 - M2–M5 intersections: PST action and gravity
- Open questions
 - extension to cases with lower SUSY
 - nonrenormalization / decoupling arguments