DeSitter Ground States of N=2 Supergravity Theories with Symmetric Scalar Manifolds in 5 Dimensions

- Why deSitter?
- Basics of the ungauged $d = 5$ theory
- Gauging of the $d = 5$ theory
- deSitter vacua
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- Why deSitter?

Einstein's Equation:

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = 8\pi T_{ab}$$

$\Lambda > 0$: deSitter, $\Lambda < 0$: Anti-deSitter, $\Lambda = 0$: Minkowski

DeSitter Space:

$$(X^0)^2 - \sum_{i=1}^{d}(X^i)^2 = -H^{-2}$$

$$\Rightarrow \Lambda = \frac{(d-1)(d-2)}{2}H^2$$

Experimental Evidence for a positive cosmological constant Λ (Riess et al, 1998, astro-ph/9805201):

- $\Lambda \simeq 10^{-120} M^4_{planck}$

Supergravity:

The extrema of the effective potential $V(\varphi)$ of scalar fields determine the cosmological constant of the corresponding vacua.

In particular, we study the critical points of the potential for various gaugings of $d = 5, \mathcal{N} = 2$ supergravity theories coupled to vector multiplets (Maxwell-Einstein Supergravity theories or MESGT's) and/or hypermultiplets.

• Ungauged $d = 5$ MESGT

Field content of $\mathcal{N} = 2$ multiplets:

- The supergravity multiplet

$$(e_\mu^m, \psi_\mu^i, A_\mu), \qquad i = 1, 2$$

- Vector multiplet

$$(A_\mu, \lambda^i, \varphi)$$

- Hypermultiplet

$$(\zeta^A, q^X), \qquad A = 1, 2; \quad X = 1, ..., 4$$

Bosonic part of the Lagrangian:

$$\begin{aligned} \hat{e}^{-1}\mathcal{L}_{bosonic}^{\mathcal{N}=2} &= -\frac{1}{2}R - \frac{1}{4}\overset{o}{a}_{\tilde{I}\tilde{J}}\, F_{\mu\nu}^{\tilde{I}} F^{\tilde{J}\mu\nu} - \frac{1}{2}g_{XY}\partial_\mu q^X \partial^\mu q^Y \\ &\quad -\frac{1}{2}g_{\tilde{x}\tilde{y}}\partial_\mu \varphi^{\tilde{x}} \partial^\mu \varphi^{\tilde{y}} + \frac{\hat{e}^{-1}}{6\sqrt{6}} C_{\tilde{I}\tilde{J}\tilde{K}} \epsilon^{\mu\nu\rho\sigma\tau} F_{\mu\nu}^{\tilde{I}} F_{\rho\sigma}^{\tilde{J}} A_\tau^{\tilde{K}}. \end{aligned}$$

where

$$X, Y = 1, ..., 4; \qquad \tilde{I}, \tilde{J} = 0, ..., \tilde{n}; \qquad \tilde{x}, \tilde{y} = 1, ..., \tilde{n}$$

$$F_{\mu\nu}^{\tilde{I}} = 2\partial_{[\mu} A_{\nu]}^{\tilde{I}}; \qquad \hat{e} = det(e_\mu^m)$$

• Gauging of the Theory

Symmetries of the Lagrangian of $d = 5, \mathcal{N} = 2$ MESGT:

- R-symmetry: $SU(2)_R$
- Any group G of linear transformations

$$h^I \rightarrow B^I_J h^J$$

$$A^I \rightarrow B^I_J A^J$$

that leave C_{IJK} invariant.

$$B^{I'}_I B^{J'}_J B^{K'}_K C_{I'J'K'} = C_{IJK}$$

We consider MESGT's where scalar manifolds $\mathcal{M}_{VS}$ are symmetric spaces.

- Generic Jordan Family:

$$\mathcal{M}_{VS} = \frac{SO(\tilde{n}-1,1) \times SO(1,1)}{SO(\tilde{n}-1)}; \quad \tilde{n} \geq 1$$

- Magical Jordan Family:

$$\mathcal{M}_{VS} = \frac{SL(3,\mathbb{R})}{SO(3)}, \quad \frac{SL(3,\mathbb{C})}{SU(3)}, \quad \frac{SU^*(6)}{USp(6)}, \quad \frac{E_{6(-26)}}{F_4}$$

$$\tilde{n} = 5, \quad \tilde{n} = 8, \quad \tilde{n} = 14, \quad \tilde{n} = 26.$$

- Generic non-Jordan Family

$$\mathcal{M}_{VS} = \frac{SO(1,\tilde{n})}{SO(\tilde{n})}; \quad \tilde{n} \geq 1$$

- Hyperscalar manifold: $\frac{SU(2,1)_H}{U(2)}$

$\Longrightarrow$ The full symmetry group of the $d = 5, \mathcal{N} = 2$ MESGT Lagrangian coupled to a single hypermultiplet is

$$SU(2)_R \times G \times SU(2,1)_H$$

Gauging any of these isometries will generically introduce potential terms.

$$\mathcal{V}(\varphi, q) = g^2(P^{(T)}(\varphi) + \lambda P^{(R)}(\varphi, q) + \kappa P^{(H)}(q))$$

where

$$\begin{array}{lcll} P^{(T)} & = & 2W_{\tilde{x}}W^{\tilde{x}} & : \text{ Tensor coupling} \\ P^{(R)} & = & -4\vec{P}\cdot\vec{P} + 2\vec{P}^{\tilde{x}}\cdot\vec{P}^{\tilde{x}} & : \ R\text{-symmetry gauging} \\ P^{(H)} & = & 2\mathcal{N}_X\mathcal{N}^X & : \text{ Hyper-isometry gauging} \end{array}$$

and $\lambda = g_R^2/g^2, \kappa = g_H^2/g^2$.

Here

$$\begin{aligned} W_{\tilde{x}} &\equiv \tfrac{\sqrt{6}}{4} h^I K_I^{\tilde{x}} \\ \vec{P} &\equiv h^I \vec{P}_I \\ \vec{P}_{\tilde{x}} &\equiv h^I_{\tilde{x}} \vec{P}_I \\ \mathcal{N}^X &\equiv \tfrac{\sqrt{6}}{4} h^I K_I^X \end{aligned}$$

The index $\tilde{I}$ is split as $\tilde{I} = (I, M)$, $K_I^{\tilde{x}}$ and K_I^X are Killing vectors, $\vec{P}_I$ are corresponding Killing prepotentials and h^I and $h^I_{\tilde{x}} = -\sqrt{\frac{3}{2}} h^I_{,\tilde{x}}$ are elements of the very special geometry. In particular, $\mathcal{M}_{VS}$ is described by the hypersurface

$$N(h) = C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} = 1$$

of the $\tilde{n} + 1$ dimensional space

$$M = \{h^{\tilde{I}} \in \mathbb{R}^{\tilde{n}+1} | N(h) = C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} > 0\}$$

• deSitter Vacua

Example 1 (No Hypers):

Generic Jordan Family, $U(1)_R \times SO(1,1)$ gauging: (Gunaydin, Zagermann,2000)

Scalar manifold: $\mathcal{M}_{VS} = \frac{SO(\tilde{n}-1,1)\times SO(1,1)}{SO(\tilde{n}-1)}$

Cubic polynomial: $N(h) = \frac{3\sqrt{3}}{2} h^0[(h^1)^2 - (h^2)^2 - \ldots - (h^{\tilde{n}})^2]$

The constraint $N(h) = 1$ can be solved by

$$\begin{aligned} h^0 &= \frac{1}{\sqrt{3}||\varphi||^2}, & ||\varphi||^2 &= \varphi^x \eta_{xy} \varphi^y \\ h^x &= \sqrt{\frac{2}{3}}\varphi^x, & \eta_{xy} &= (+ - - \ldots), \quad x = 1, \ldots, \tilde{n} \end{aligned}$$

Positiveness of Kinetic Energy:

$$det\ \overset{o}{a}_{\tilde{I}\tilde{J}} \sim \frac{1}{||\varphi||^2}, \qquad det g_{xy} \sim \frac{1}{||\varphi||^6}$$

$$\Rightarrow ||\varphi||^2 > 0$$

The field content of the theory is

$$\begin{pmatrix} e_\mu^m \\ \Psi_\mu^i \\ A_\mu^0 \end{pmatrix}, \quad \begin{pmatrix} B_{\mu\nu}^M \\ \lambda^{iM} \\ \varphi^M \end{pmatrix}, \quad \begin{pmatrix} A_\mu^I \\ \lambda^{in} \\ \varphi^n \end{pmatrix}$$

sugra multiplet	2 tensor multiplets $M = 1, 2$	$\tilde{n} - 2$ vector multiplets $I = 3, 4, \ldots, \tilde{n}$

A^0_μ is the $SO(1,1)$ gauge field. $A_\mu[U(1)_R] = V_I A^I_\mu$ is the $U(1)_R$ gauge field. The total potential becomes

$$e^{-1}\mathcal{L}_{pot} = -g^2(P^{(T)} + \lambda P^{(R)})$$

where

$$\begin{aligned} P^{(T)} &= \tfrac{3\sqrt{6}}{16} h^I \Lambda_I^{MN} h_M h_N &= \tfrac{1}{8}\frac{\left[(\varphi^1)^2-(\varphi^2)^2\right]}{||\varphi||^6} \\ P^{(R)} &= -4C^{IJ\tilde{K}} V_I V_J h_{\tilde{K}} &= -4\sqrt{2}V_0 V_i \varphi^i ||\varphi||^{-2} + 2|V|^2||\varphi||^2 \end{aligned}$$

with $i = 3, ..., \tilde{n}$ and $|V|^2 = V_i V_i$.

The Critical Points of the Potential:

Demanding $P^{(5)}_{,\varphi^{\tilde{x}}} = 0$ one finds

$$\begin{aligned} \frac{\varphi^i_C}{||\varphi_C||^4} &= 16\sqrt{2}\lambda V_0 V_i \\ \frac{1}{||\varphi_C||^6} &= -\tfrac{1}{2}(16\sqrt{2}\lambda V_0|V|)^2 + 8\lambda|V|^2 \end{aligned}$$

with the constraints

$$\begin{aligned} |V|^2 &> 0 \\ 32\lambda(V_0)^2 &< 1. \end{aligned}$$

The value of the potential at the one parameter family of critical points becomes

$$P^{(5)}_{TOT}|_{\varphi^C} = 3\lambda||\varphi||^2|V|^2(1 - 32\lambda(V_0)^2)$$

and they correspond to deSitter vacua, which are stable (Cosemans, Smet, 2005).

	No R sym. gauging	$SU(2)_R$ gauging	$U(1)_R$ gauging
MESGT	Minkowski (supersymmetric) arbitrary $\tilde{n} \geq 0$	- $\tilde{n} \geq 4$	AdS (supersymmetric, stable) +Minkowski (nonsupersymmetric) $\tilde{n} \geq 1$
YMESGT with tensors and gauge group $SO(2)$	Minkowski (supersymmetric) $\tilde{n} \geq 3$	Minkowski (supersymmetric) $\tilde{n} \geq 6$	AdS (supersymmetric, stable + nonsupersymmetric, stable) +Minkowski (nonsupersymmetric) $\tilde{n} \geq 3$
YMESGT with tensors and gauge group $SO(1,1)$ (broken susy)	- $\tilde{n} \geq 2$	dS (stable) $\tilde{n} \geq 5$	dS (stable) $\tilde{n} \geq 3$

Table 1: Ground states of $d = 5, \mathcal{N} = 2$ supergravity *without hypermultiplets*. The columns represent different R-symmetry gaugings whereas the rows represent different tensor couplings. $\tilde{n}$ denotes the minimum number of vector multiplets that must be coupled to the theory in order to make the respective gauging possible. "-" means there are no ground states.

Example 2: (with hypers)

Generic Jordan Family, with Hyper-coupling: (Ogetbil,2006, hep-th/0612145)

Hyperscalar metric with $SU(2,1)$ isometry group:

$$ds^2 = \frac{dV^2}{2V^2} + \frac{1}{2V^2}(d\sigma + 2\theta d\tau - 2\tau d\theta)^2 + \frac{2}{V}(d\tau^2 + d\theta^2)$$

and Killing vectors (Cerelose et al, 2001, hep-th/0104056)

$$\vec{k}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{k}_2 = \begin{pmatrix} 0 \\ 2\theta \\ 0 \\ 1 \end{pmatrix}, \quad \vec{k}_3 = \begin{pmatrix} 0 \\ -2\tau \\ 1 \\ 0 \end{pmatrix}, \quad \vec{k}_4 = \begin{pmatrix} 0 \\ 0 \\ -\tau \\ \theta \end{pmatrix},$$
$$\vec{k}_5 = \begin{pmatrix} V \\ \sigma \\ \theta/2 \\ \tau/2 \end{pmatrix}, \quad \vec{k}_6 = \begin{pmatrix} 2V\sigma \\ \sigma^2 - (V + \theta^2 + \tau^2)^2 \\ \sigma\theta - \tau(V + \theta^2 + \tau^2) \\ \sigma\tau + \theta(V + \theta^2 + \tau^2) \end{pmatrix}$$
$$\vec{k}_7 = \begin{pmatrix} -2V\theta \\ -\sigma\theta + V\tau + \tau(\theta^2 + \tau^2) \\ \frac{1}{2}(V - \theta^2 + 3\tau^2) \\ -2\theta\tau - \sigma/2 \end{pmatrix}, \quad \vec{k}_8 = \begin{pmatrix} -2V\tau \\ -\sigma\tau - V\theta - \theta(\theta^2 + \tau^2) \\ -2\theta\tau + \sigma/2 \\ \frac{1}{2}(V + 3\theta^2 - \tau^2) \end{pmatrix}.$$

The Killing vectors form $SU(2,1)$ algebra under Lie brackets if they are recasted in the following combinations

$$SU(2) \quad \begin{cases} T_1 &= \frac{1}{4}(k_2 - 2k_8), \\ T_2 &= \frac{1}{4}(k_3 - 2k_7), \\ T_3 &= \frac{1}{4}(k_1 + k_6 - 3k_4), \end{cases} \qquad \frac{SU(2,1)}{U(2)} \quad \begin{cases} T_4 &= k_5, \\ T_5 &= -\frac{1}{2}(k_1 - k_6), \\ T_6 &= -\frac{1}{4}(k_3 + 2k_7), \\ T_7 &= -\frac{1}{4}(k_2 + 2k_8). \end{cases}$$
$$U(1) \quad \left\{ T_8 = \frac{\sqrt{3}}{4}(k_4 + k_1 + k_6), \right.$$

T_1, T_2, T_3 and T_8 generate the isotropy group $SU(2) \times U(1)$ of the base point $q^C = (V, \sigma, \theta, \tau) = (1, 0, 0, 0)$. The hyperscalar metric becomes diagonal at this point.

$U(1)_H$ gauging:

Potential term

$$P^{(H)} = 2 g_{XY} \mathcal{N}^X \mathcal{N}^Y$$

where

$$\mathcal{N}^X = \frac{\sqrt{6}}{4} h^I K_I^X$$

Compact Killing vectors vanish at the base point. Hence $\mathcal{N}^X|_{q^c} = 0$ and as a consequence

$$P^{(H)}|_{q^c} = \frac{\partial P^{(H)}}{\partial \varphi^I}|_{q^c} = \frac{\partial P^{(H)}}{\partial q}|_{q^c} = 0.$$

On the other hand, $P^{(R)}$ is of the form $P^{(R)} \sim f(\varphi) g(q)$, where $g(q) \geq 0$ for the generic family. Also, $g(q)$ has an extremum point at the base point of the hyperscalar manifold (i.e. $\frac{dg}{dq}|_{q^c} = 0$). This leads to

$$\frac{\partial P^{(R)}}{\partial q}|_{q^c} = \frac{\partial P^{(5)}_{TOT}}{\partial q}|_{q^c} = \frac{\partial^2 P^{(5)}_{TOT}}{\partial \varphi \partial q}|_{q^c} = 0$$

and hence the Hessian is in block diagonal form. Therefore *an additional $U(1)_H$ gauging won't change the sign of*

- *the potential at the critial point,*

- *the existing eigenvalues of the Hessian of the potential.*

What would a non-compact gauging do?

$SU(2)_R \times SO(1,1)_H$ gauging:

- $A^a_\mu, \quad a = 2,3,4$ are the $SU(2) \sim SO(3)$ gauge fields
- A linear combination $V_b A^b_\mu, \quad b = 0,1,5,6,...,\tilde{n}$ is the $SO(1,1)$ gauge field,
- T^X_4 is the non-compact Killing vector.

The total potential is

$$P^{(5)}_{TOT} = \lambda(P^{(R)} + 2\mathcal{N}_X\mathcal{N}^X)$$

where

$$\mathcal{N}^X = \frac{\sqrt{6}}{4}(h^a K^X_a + (V_b h^b)T^X_4)$$

Demanding the first derivatives of the potential vanish, one finds $\varphi^2_C = \varphi^3_C = \varphi^4_C = 0$ together with the relations

$$V_1\varphi^d = -\varphi^1 V_d \qquad \forall d = 5,6,...,\tilde{n}.$$

$$\varphi^1 = \frac{\sqrt{2}||\varphi||^4 V_0 V_1 \pm \sqrt{6||\varphi||^8 V_1^2\,(3(V_0)^2 - 8||\varphi||^6)}}{12||\varphi||^6 - 4(V_0)^2}.$$

and the constraints

$$(V_1)^2 - (V_5)^2 - ... - (V_{\tilde{n}})^2 > 0,$$

$$(V_0)^2 > \frac{8}{3}||\varphi||^6.$$

The potential evaluated at the critical point is given by

$$P^{(5)}_{TOT}|_{\phi^C} = \frac{\lambda}{4}\left(6||\varphi||^2 + \frac{\left(V_0\varphi^1 + \sqrt{2}V_1||\varphi||^4\right)^2}{(\varphi^1)^2||\varphi||^4}\right)$$

which is positive definite.

Stability: Assume

$$V_0 = 2, \qquad V_1 = 1, \qquad V_5 = ... = V_{\tilde{n}} = 0.$$

There are two critical points, given by

$$\begin{array}{lcl} \phi_1^C & : & \varphi^1 = -\frac{(\sqrt{33}-1)^{1/3}}{2^{5/6}}, \;\; \varphi^5 = ... = \varphi^{\tilde{n}} = 0, \\ \phi_2^C & : & \varphi^1 = \frac{(\sqrt{33}+1)^{1/3}}{2^{5/6}}, \quad\;\; \varphi^5 = ... = \varphi^{\tilde{n}} = 0. \end{array}$$

The values of the potential at these critical points read

$$\begin{array}{lcl} P_{TOT}^{(5)}|_{\phi_1^C} & = & \frac{3}{4}\lambda \left(\frac{3}{2}\left(69 - 11\sqrt{33}\right)\right)^{1/3}, \\ P_{TOT}^{(5)}|_{\phi_2^C} & = & \frac{3}{4}\lambda \left(\frac{3}{2}\left(69 + 11\sqrt{33}\right)\right)^{1/3} \end{array}$$

and the numerical values for the eigenvalues of the Hessian (for $\tilde{n} = 6$) are

$$\begin{array}{r} (-0.799\lambda, -0.799\lambda, -0.743\lambda, -0.686\lambda, -0.686\lambda, \\ 0.667\lambda, 1.142\lambda, 2.991\lambda, 2.991\lambda, 29.058\lambda) \end{array}$$

at ϕ_1^C and

$$\begin{array}{l} (0.843\lambda, 1.102\lambda, 1.102\lambda, 1.876\lambda, 2.186\lambda, \\ 2.186\lambda, 6.526\lambda, 7.143\lambda, 7.143\lambda, 20.441\lambda) \end{array}$$

at ϕ_2^C. Hence the second critical point is stable whereas the first one is not.

	No R sym. gauging	$SU(2)_R$ gauging	$U(1)_R$ gauging
MESGT	dS (stable)	dS (stable + unstable)	AdS (stable[b] + unstable)
YMESGT with tensors and gauge group $SO(2)$	dS (stable[1])	dS (stable + unstable)	AdS (stable[2] + unstable)
YMESGT with tensors and gauge group $SO(1,1)$	-	dS (stable)	dS (stable + unstable)

Table 2: Ground states of $d = 5, \mathcal{N} = 2$ supergravity *with one hypermultiplet* and *with noncompact* $SO(1,1)$ *gauging of the hyper sector*. The columns represent different R-symmetry gaugings whereas the rows represent different tensor couplings. Note that noncompact hyper-gauging implies broken supersymmetry. The Minkowskian ground states are not listed. “-” means there are no ground states.

- Metastability

DeSitter Metastable Vacua from $\mathcal{N} = 2$ Gauged Sugra: (Behrndt and Mahapatra, 2003)

- Supergravity coupled to *one* universal hypermultiplet
- Instanton corrections included in the hypermultiplet sector
- Abelian gauging

The unstable mode runs into a supersymmetric smooth vacuum on both sides $\Rightarrow$ Metastable state.

$\Rightarrow$ DeSitter saddle points with "locked inflation"

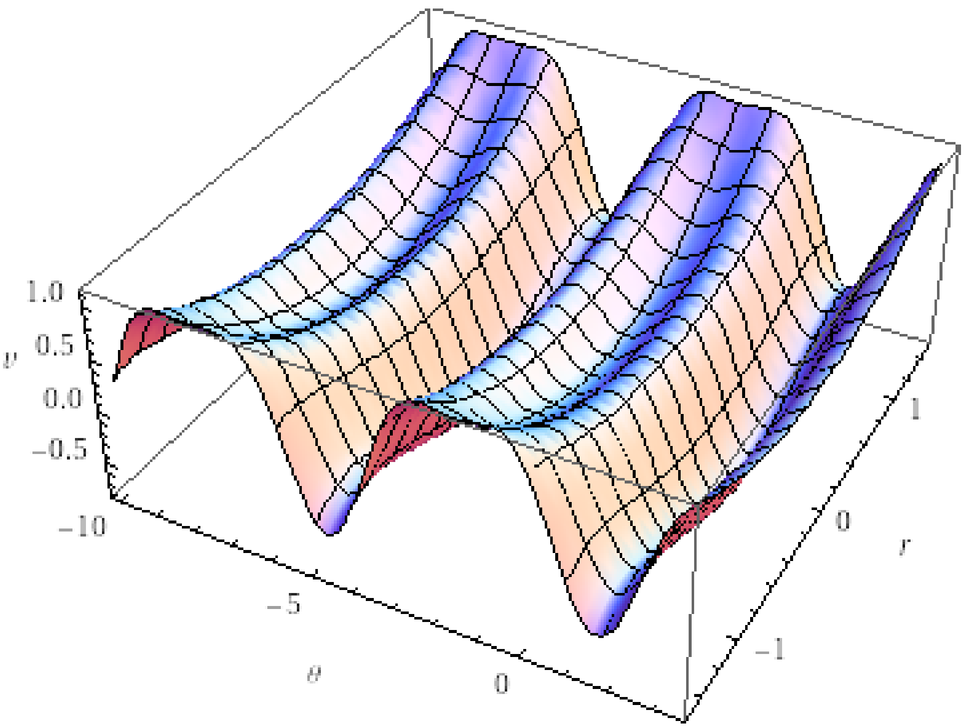

Figure 1: Potential $\mathcal{V}$ plotted as a function of hyperscalars r and θ.

• Future directions

- DeSitter ground states in $d = 4$

  Only known $\mathcal{N} = 2, d = 4$ stable deSitter vacua found by Fre, Trigiante and Van Proeyen

- Embedding in higher dimensional theories (superstring theory, M theory...).