On the energy of homogeneous cosmologies

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ABSTRACT

We investigate the quasi-local energy of all the homogeneous cosmological models. More specifically using the standard natural prescription we find the quasi-local energy-momentum for a large class of gravity theories with a tetrad description for all 9 Bianchi types with general gravitational sources. Using ideas related to a Hamiltonian approach we find that, with homogeneous boundary conditions, the quasi-local energy vanishes for all regions in all Bianchi class A models, and it does not vanish for any class B model. This is so not only for Einstein's general relativity but, moreover, for the whole 3-parameter class of tetrad-teleparallel theories. For the physically favored one parameter subclass, which includes the teleparallel equivalent of Einstein's theory as an important special case, the quasi-local energy for all class B models is positive.

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1. Introduction

Energy has been one of the most useful physical concepts, no less so in gravitating systems—one need only recall its utility in the Newtonian Kepler problem. Yet for the modern formulation, unlike all matter and other interaction fields in the absence of gravity, identifying a good description of the energy of gravitating systems remains the oldest and most controversial outstanding puzzle. Physically the fundamental difficulty can be understood as a consequence of Einstein's equivalence principle, from which it follows that gravity cannot be detected at a point—hence the energy-momentum of gravity is fundamentally non-local and thus inherently non-tensorial and therefore *non-covariant*. (For a good discussion of this point see Misner et al. (1973))

After Einstein proposed his gravitational energy-momentum density (Trautman 1958), several alternate prescriptions were introduced by other researchers (Papapetrou 1951; Bergman & Thompson 1953; Møller 1958; Landau & Lifshitz 1962; weinberg 1972). These investigations lead to a variety of expressions with no compelling criteria for favoring any particular one. Moreover these traditional energy-momentum *pseudotensors* are, as noted, necessarily not covariant objects, they inherently depend on the reference frame, so they cannot provide a truly physical local gravitational energy-momentum density: simply put one does not know which reference frame should be used to give the physical value. Caught between the equivalence principle and the covariance principle, the pseudotensor approach has been largely questioned, although never completely abandoned.

Because the gravitational interaction is local, some kind of local—or at least nearly local—description of energy-momentum was still sought. The modern idea is that energy-momentum is *quasi-local*, being associated with a closed surface bounding a region (for a nice review of the topic see Szabados (2004)); again there is no unique energy expression. Many definitions of quasi-local mass-energy have been proposed (Brown & York 1993), they generally give distinct results. For example Bergqvist (1992) studied several different definitions of quasi-local masses for the Kerr and Reissner-Nordström spacetimes and came to the conclusion that not even two of the examined definitions gave the same result.

Our view is that from the Hamiltonian perspective one can make sense of this situation and understand why all these otherwise perplexing choices exist and what is their physical significance. Simply put the energy of a gravitating system within a region—regarded as the value of the Hamiltonian for this system—naturally depends not only on the interior of the region but also on the boundary conditions imposed at the interface with the exterior (this should be no surprise, after all the particular solution to the field equations similarly depends on the boundary conditions). The Hamiltonian necessarily includes an integral over the 2-surface bounding the region. This Hamiltonian boundary term plays two key roles: (i) it controls the value of the Hamiltonian, and (ii) its specific form (via the requirement that the boundary term in the variation in the Hamiltonian vanish) is directly related to the selected type and value of the boundary conditions. Many of the quasi-local proposals (i.e. all those which admit a Hamiltonian representation) can be understood in these terms: their differences are simply associated with different boundary conditions. Furthermore, using a covariant Hamiltonian formalism, it has been shown that every energy-momentum pseudotensor can be associated with a particular Hamiltonian boundary term, which in turn determines a quasi-local energy-momentum (Chen & Nester 1999; Chang et al. 1999). In this sense, it has been said that the Hamiltonian quasi-local energy-momentum approach rehabilitates the pseudotensors, and dispels any doubts about the physical meaning of these energy-momentum complexes, for all their inherent ambiguities are given clear meanings.

We want to emphasize that while there are many possible Hamiltonian boundary term energy-momentum quasi-local expressions—simply because there are many conceivable boundary conditions—we nevertheless have found that in practice there is usually a particular choice suited to the task at hand. This is the case for familiar physical systems where gravity is negligible. For example in thermodynamics we have not a unique energy but rather the internal energy, the enthalpy, and the Helmholtz and Gibbs free energies—each a real physical energy adapted to a specific interface between the physical system of interest and its surroundings; similarly in classical electrostatics the physically appropriate measure of energy obtained form the work-energy relation for a finite system depends on whether one considers fixed surface charge density or fixed potential—boundary condition choices which are respectively associated with the symmetric and the canonical energy-momentum tensor.

Here we consider these Hamiltonian boundary term quasi-local energy-momentum ideas as applied to homogeneous cosmologies. Our motivation is twofold. On the one hand the utility of having a good measure of the energy of such gravitating systems—a measure with sensible answers for the ideal exactly homogeneous case but which can be applied to any perturbations thereof and even quite generally—should be obvious. We are likewise motivated by the consideration that homogeneous cosmology affords an excellent set of models where one can test the suitability of our and any other proposed quasi-local energy-momentum ideas.

Accordingly it is important to here remark on our specific choice of energy expression for these homogeneous cosmologies. We have noted *four* different approaches which lead us to exactly the same formula for energy-momentum for the homogeneous cosmologies in General Relativity (GR). (i) One may take Møller's tetrad approach to GR: among the many traditional energy-momentum expressions Møller's (1961) tetrad-teleparallel tensor is highly regarded (among other virtues it is the only classical expression that has an associated positive energy proof Nester (1989)). (ii) Another perspective is via the GR Hamiltonian: from the covariant Hamiltonian approach for GR one particular Hamiltonian boundary term stands out as being the favored expression for general applications (Chen et al. 2005). In many situations, including the present application, it reduces to Møller's tetrad-teleparallel expression. (iii) A third approach, which we favor for its simplicity, generality and straightforwardness, is to consider the general teleparallel theory, which includes Einstein's GR as a special case. For such theories, quite unlike the GR situation, all investigators have advocated a single specific expression for the energy-momentum density (this is one of the virtues of treating this whole class of theories rather than the one special case equivalent to GR), (iv) instead one could begin with homogeneous cosmology:

homogeneous cosmologies are naturally described in terms of a preferred homogeneous tetrad, then an energy-momentum expression based on the Hamiltonian for this preferred tetrad is clearly appropriate.

There have been several other studies aimed at finding the total energy of the expanding universe. An early investigation proposed that our universe may have arisen as a quantum fluctuation of the vacuum. That model predicted a universe which is homogeneous, isotropic and closed, and consists equally of matter and anti-matter. Tryon (1973) proposed that our universe must have a zero net value for all conserved quantities and presented some arguments, using a Newtonian order of magnitude estimate, favoring the fact that the net energy of our universe may indeed be zero. The general argument for this requirement is that energy can always be represented by an integral over a closed 2-surface bounding the region of interest, so if the universe has an empty boundary then the energy should vanish. Thus for closed cosmological models the total energy is necessarily zero. Years ago Misner (1963) pointed out a technical problem with the attempts at explicit demonstrations of this statement. He noted that the integrands that were being used at that time were all reference frame dependent (holonomic) pseudotensors and their associated superpotentials. None of those discussions had specifically established exactly how the particular objects behaved under changes of coordinates, but such changes were actually necessary in the calculations since the whole universe could not be covered with one coordinate patch. According to our understanding it was Wallner (1982) who first gave a clear demonstration of this important vanishing energy for closed universe requirement; his integrand (essentially Møller's) was given in terms of a globally defined frame field.

The subject of the total energy of expanding universe models was re-opened by Cooperstock & Israelit (1994); Rosen (1994); Garecki (1995) and others, using various GR energy-momentum definitions. In one of these investigations the Einstein energy-momentum pseudotensor was used to represent the gravitational energy (Rosen 1994), which led to the result that the total energy of a closed Friedman-Robertson-Walker (FRW) universe is zero. In another, the symmetric pseudotensor of Landau-Lifshitz was used (Garecki 1995). In Banerjee & Sen (1999) and Radinschi (1999), the total energy of the anisotropic Bianchi models has been calculated using different pseudotensors, leading to similar results. Recently Cooperstock & Faraoni (2003), in support of Cooperstock's proposal that gravitational energy vanishes in the vacuum, it was argued (with the aid of a non-minimally coupled scalar source) that the open, or critically open FRW universes, as well as Bianchi models evolving into de Sitter spacetimes also have zero total energy. Finally, a calculation for the closed FRW (Vargas 2004), anisotropic Bianchi (So & Vargas 2005; Salti 2006) models using the Einstein, Laudau-Lifshitz and other complexes in teleparallel gravity also led to the same conclusion.

In the present work, we examine the energy-momentum for a large class of cosmological models, more specifically all 9 Bianchi cosmological models using the quasi-local approach in the context of the general tetrad-teleparallel theory of gravity (Hayashi & Shirafuji 1979). This theory is a generalization of GR; a certain special case which is equivalent to Einstein's theory (Møller 1961; de Andrade et al. 2000) was first proposed by Møller to solve the energy localization problem. This special case has been referred to as $\mathrm{GR}_{||}$, the teleparallel equivalent to Einstein's theory (aka TEGR); it has attracted the attention of several investigators (Nester 1989; Maluf 1994). Our motive is not only to improve our understanding of these cosmological models, but also to better understand the considered gravity theories and especially to better understand the meaning and application of these quasi-local energy-momentum ideas.

Most of the previous calculations of the energy of the universe used cartesian or spherical coordinate systems, here in contrast we use only the symmetry of spacetime given by the Bianchi group. In this major extension of previous results (Vargas 2004; So & Vargas 2005)), by using the Hamiltonian formalism with *homogeneous boundary conditions* we find that the quasi-local energy *vanishes— for all regions, large or small*—for *all* Bianchi class A models and *does not* vanish for any class B model. This result is completely independent of the specific type of matter content of the universe; it depends only on the symmetry of the spacetime given by the structure constants of the Bianchi group. It is noteworthy that we find a simple physical difference related to the division of the Bianchi types into two classes.

2. Bianchi type universes

The four-dimensional spacetime manifold is foliated by homogeneous space-like hypersurfaces Σ of constant time t; homogeneity means that each spatial hypersurface has a transitive group of isometries. This 3-parameter isometry group G_3 via the classification of 3-dimensional Lie algebras led to the Bianchi classification of spatially homogeneous universes(Ellis & MacCallum 1969).

These models are characterized by homogeneous, but generally anisotropic, spatial hypersurfaces parameterized by time. In a synchronous coordinate systems, in which the axis is always normal to the hypersurfaces of homogeneity Σ, we take the spacetime orthonormal (co)frame for these cosmological models to have the form

$$\vartheta^0 = dt, \qquad \vartheta^a = h^a{}_k(t)\sigma^k, \tag{1}$$

where

$$d\sigma^k = \frac{1}{2}C^k{}_{ij}\sigma^i \wedge \sigma^j, \tag{2}$$

with $C^i{}_{jk}$ being certain constants. The associated spacetime metric then has the form

$$ds^2 = -dt^2 + g_{ij}(t)\sigma^i(x)\sigma^j(x), \tag{3}$$

where $g_{ij} := \delta_{ab} h^a{}_i h^b{}_j$ is a spatial 3-metric which depends only upon time; for our analysis it need not be diagonal.

There are 9 Bianchi types distinguished by the particular form of the structure constants $C^k{}_{ij}$ (Ellis & MacCallum 1969; Kramer et al. 1980). They fall into 2 special classes: class A (types I, II, VI_0, VII_0, VIII, IX) have $A_k := C^i{}_{ki} \equiv 0$ and class B (types III, IV, V, VI_h, VII_h) are characterized by $A_k \neq 0$. For our needs here we really do not need any more details regarding these types. For our later comparison with other approaches we note that the respective scalar curvatures are vanishing for Type I, positive for Type IX, and negative for all the other types. Although the general idea of these models is homogeneous but non-isotropic, it should be mentioned that certain special cases can be isotropic, specifically isotropic Bianchi I, V, IX are, respectively, isometric to the usual Friedmann-Robertson-Walker (FRW) models: $k = 0, -1, +1$.

3. tetrad-teleparallel theory

The Bianchi cosmological models are naturally expressed in terms of a frame. So an appropriate theoretical framework is the tetrad/teleparallel formulation. For teleparallel theories the curvature vanishes, consequently there is a preferred frame in which the connection also vanish. The basic variable is then this preferred tetrad, most conveniently represented as the (co)frame $\vartheta^\alpha = e^\alpha{}_j dx^j$. In our calculations we find it convenient to also use the dual co-frame $\eta^{\alpha\beta\cdots} := *(\vartheta^\alpha \wedge \vartheta^\beta \wedge \ldots)$. Since the connection coefficients vanish in this frame the associated teleparallel torsion field is simply $T^\alpha := d\vartheta^\alpha$.

A Lagrangian 4-form $\mathcal{L} = \mathcal{L}(\vartheta^\alpha, T^\alpha)$ gives the tetrad field equations. The conjugate momentum field

$$\tau_\mu := \frac{\partial \mathcal{L}}{\partial T^\mu} = \frac{1}{2} \tau_\mu{}^{\alpha\beta} \eta_{\alpha\beta}, \tag{4}$$

in order to have quasi-linear second order field equations is taken to be a linear combination, $\tau = \kappa^{-1}(a_1 T_{\rm ten} + a_2 T_{\rm vec} + a_3 T_{\rm axi})$, of the tensor, vector, and axivector irreducible parts of the teleparallel torsion; there is thus a 3-parameter class of such theories Szabados (2004). The general theory determines a preferred frame. However the special parameter choice $4a_3 = a_2 = -2a_1$ is distinguished by having local Lorentz gauge freedom. With $a_1 = -1$ and $\kappa := 8\pi G/c^4$ this model is $\mathrm{GR}_{||}$, the teleparallel equivalent of Einstein's GR (aka TEGR), which was first proposed by Møller to solve the GR energy localization problem.

4. Hamiltonian quasi-local boundary expression

As mentioned in the introduction, for Einstein's GR many energy-momentum expressions (reference frame dependent pseudotensors) have been proposed (the most famous ones were proposed by Einstein, Landau-Lifshitz, Papapetrou, Bergmann-Thompson, Møller, and Weinberg). It should be emphasized that, despite much effort and many nice results, there is *no consensus* as to which, if any, is best. The Hamiltonian approach certainly helps. From that perspective the quasi-local energy-momentum is determined by the boundary term in the Hamiltonian (Chen & Nester 1999; Chang et al. 1999). Although (at least formally) the formalism allows for an infinite number of Hamiltonian boundary expressions (including all the superpotentials that generate the pseudotensors), the ambiguities have been tamed: each expression has a geometrically and physically clear significance associated with the boundary conditions (Nester 2004) determined from the variation of the Hamiltonian. Nevertheless there are (at least formally) an infinite number of possible boundary conditions.

On the other hand, the situation for the tetrad/teleparallel theory is in sharp contrast. Investigators (Møller 1961; de Andrade et al. 2000; Hayashi & Shirafuji 1979; Nester 1989; Maluf 1994; Blagojević & Vasilić 2001; Kawai & Toma 1991) essentially agree as to the expression for the energy-momentum within a volume V:

$$P_\mu(V) := \oint_{\partial V} \tau_\mu, \tag{5}$$

unlike GR there is no ambiguity. Thus our consideration of this general tetrad/teleparallel class of theories yields two benefits: (i) it allows us to get a result of great generality, applying to this whole class of theories, (ii) moreover, when specialized to TEGR, it determines a *unique* preferred energy-momentum expression for GR. This expression of Møller has generally been recognized as one of the best, perhaps the best, description of the gravitational energy-momentum for GR.

5. The Bianchi energy calculation

The energy-momentum integral over the boundary of a region at a fixed time t is

$$P_\mu(V) := \oint_{\partial V} \tau_\mu = \oint_{\partial V} \frac{1}{2} \tau_\mu{}^{\alpha\beta} \eta_{\alpha\beta}. \tag{6}$$

In this Bianchi cosmology case the components of $\tau_\mu{}^{\alpha\beta}$ and $T^\mu{}_{\alpha\beta}$ (in the preferred teleparallel frame) are functions of time alone—dependence on the spatial coordinates shows up only in the teleparallel coframe ϑ^μ via the σ^i. Consequently, in detail

$$\begin{aligned} P_\mu(V) &= \frac{1}{2}\tau_\mu{}^{\alpha\beta}(t) \oint_{\partial V} \eta_{\alpha\beta} = \frac{1}{2}\tau_\mu{}^{\alpha\beta}(t) \int_V d\eta_{\alpha\beta} = \frac{1}{2}\tau_\mu{}^{\alpha\beta}(t) \int_V d\vartheta^\gamma \wedge \eta_{\alpha\beta\gamma} \\ &= \frac{1}{2}\tau_\mu{}^{\alpha\beta}(t) \frac{1}{2} T^\gamma{}_{\lambda\delta}(t) \delta^{\kappa\lambda\delta}_{\alpha\beta\gamma} \int_V \eta_\kappa = \frac{1}{2}\tau_\mu{}^{\alpha\beta}(t)[T^\kappa{}_{\alpha\beta}(t) + T^\gamma{}_{\gamma\alpha}(t)\delta^\kappa_\beta - T^\gamma{}_{\gamma\beta}(t)\delta^\kappa_\alpha]\delta^0_\kappa V \\ &= \frac{1}{2}\tau_\mu{}^{\alpha\beta}(t)[T^0{}_{\alpha\beta}(t) + T^\gamma{}_{\gamma\alpha}(t)\delta^0_\beta - T^\gamma{}_{\gamma\beta}(t)\delta^0_\alpha]V. \end{aligned} \tag{7}$$

Note that the quasi-local energy momentum is given by an integral over the 2-dimensional boundary of the region, yet it turns out to be simply proportional to the size of the included 3-dimensional volume (not its shape); it is also noteworthy that there is no dependence on the location.

To properly appreciate our results here, one should note that most previous calculations of energy for cosmological models considered only the isotropic Friedman-Robertson-Walker models or one or at most a few Bianchi types (and they usually confined their results to diagonal metrics). They generally used *holonomic* energy-momentum expressions, which correspond to quite different boundary conditions. Typically they considered some ball of constant radius around the origin or only the whole space. The expressions used were not manifestly homogenous; the results were shape dependent, not simply proportional to volume.

Continuing with our calculation, we obtain the torsion tensor components:

$$T^0{}_{\alpha\beta} = 0, \qquad T^\gamma{}_{b\gamma} = T^c{}_{bc} = C^j{}_{kj} h^k{}_c = A_k h^k{}_b, \qquad T^c{}_{0b} = \dot{h}^c{}_j h^j{}_b. \tag{8}$$

Consequently the quasi-local energy-momentum is

$$P_\mu(V) = \tau_\mu{}^{\alpha\beta}(t) T^\gamma{}_{\beta\gamma}(t) \delta^0_\alpha V = \tau_\mu{}^{0b}(t) T^c{}_{bc}(t) V = \tau_\mu{}^{0b}(t) A_k h^k{}_b(t) V, \tag{9}$$

which vanishes for all class A models.

For class B models we need

$$\tau_\mu{}^{0b} = \kappa^{-1}(a_1 T_{\rm ten} + a_2 T_{\rm vec} + a_3 T_{\rm axi})_\mu{}^{0b} = \kappa^{-1}\left[a_1 T + (a_2 - a_1) T_{\rm vec} + (a_3 - a_1) T_{\rm axi}\right]_\mu{}^{0b}, \tag{10}$$

where $\kappa = 8\pi G/c^4$. For energy we need just the $\mu = 0$ component:

$$\kappa \tau_0{}^{0b} = (a_2 - a_1) T_{\rm vec0}{}^{0b} = (a_2 - a_1)\frac{1}{3} T^c{}_c{}^b = (a_2 - a_1)\frac{1}{3} A_j h^j{}_a \delta^{ab}. \tag{11}$$

Hence the energy within any volume V at time t is

$$P_0(V) = \tau_0{}^{0b}(t) A_k h^k{}_b(t) V = \kappa^{-1}(a_2 - a_1)\frac{1}{3} A_j h^j{}_a \delta^{ab} A_k h^k{}_b V = \frac{a_2 - a_1}{3\kappa} A_j A_k g^{jk}(t) V. \tag{12}$$

From various investigations it has been found that certain parameter restrictions should be imposed. It particular it is generally considered that we must require the so-called *viable* condition (Hehl 2002), $2a_1 + a_2 = 0$. Moreover normalization to the Newtonian limit gives $a_1 = -1$. This leads us to the one parameter teleparallel theory, also known as NGR (New General Relativity (Hayashi & Shirafuji 1979)). If these parameter conditions are satisfied we have, for all viable cases (including the very special case of GR, $a_1 + 2a_3 = 0$), *positive energy*

$$P_0(V) = \kappa^{-1} A_j A_k g^{jk}(t) V(t) > 0 \tag{13}$$

for all regions in class B models.

We note that to find the explicit value of the "linear momentum" $P_k(V)$ one needs

$$\tau_c{}^{0b} = \kappa^{-1} \left[a_1 T + (a_2 - a_1) T_{\rm vec} + (a_3 - a_1) T_{\rm axi} \right]_c{}^{0b}, \tag{14}$$

which is linear (but not very neat) in $T^c{}_{0b} = \dot{h}^c{}_j h^j{}_b$.

6. Discussion and conclusions

We obtained some insight into the issues from looking to the energy of homogeneous cosmologies, considering the questions: what energy should be associated with a region of the universe? Does the total energy of a closed universe vanish? Is the energy of an open universe positive? Does the energy of empty flat space vanish?

Specifically, using the natural prescription we found the quasi-local energy-momentum for the general tetrad-teleparallel theory (which includes Einstein's GR as an important special case) for all 9 Bianchi types—with general (but of course homogeneous) gravitational sources. From the Hamiltonian approach we find that, with *homogeneous* boundary conditions, the quasi-local energy vanishes for all regions in all Bianchi class A models, and it does not vanish for any class B model. This is the case for the whole 3-parameter class of tetrad-teleparallel theories, including the teleparallel equivalent of Einstein's theory, which has positive energy for all class B models.

Let us note that all the cosmologies in the Bianchi class A models can be compactified, so this result is *consistent* with the requirement of vanishing energy for a closed universe, whereas the class B models cannot be compactfied in general. However there are a few special exceptions. Nevertheless these exceptions are not problematical, while in certain cases the metric geometry can be compactified, but this cannot be done in such a way that the *frames* match up to give a globally defined smooth frame. Indeed a proof of this Ashetkar & Samuel (1991) used a calculation virtually identical to ours above without noting the energy interpretation.

Note that our result holds, in particular, for *all* types of material sources including *dark matter* and *dark energy* (either as a cosmological constant or as some kind of unusual field like *quintessence* appearing as a part of the energy-momentum tensor). Also contrary to our initial expectation, there is no correlation between the energy and the sign or vanishing of the scalar curvature and depends only on the symmetry of the spacetime given by structure constants of the Bianchi group. Recently, it has been shown by Faraoni and Cooperstock that the open, or critically open FRW universes, as well as Bianchi models evolving into de Sitter spacetimes also have zero total energy. Our calculations confirms and generalizes their result to all Bianchi A classes, while for the B classes ????

Our result of course applies to the special case of isotropic models. In those cases, the *homogenous* result can be contrasted with the usual "isotropic-about-one-point" formulations, where the energy (given by various expressions) for spherically symmetric regions of the closed FRW, first increases and then decreases to zero as the radius reaches the antipode. The different "localizations of energy" for the same physical situation provides a good example of the effect of different boundary conditions.

Finally, although we have referred to the spatial components of P_μ as "momentum" this is just a name. More precisely it is the conserved quantity associated with spatial displacements described by constant components along the spatial legs of the preferred tetrad. Since these spatial displacements do not commute it is not so clear to what degree these conserved quasi-local quantities actually resemble the familiar linear momentum of flat space.

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