Supersymmetric R^2 Terms in 5d Supergravity

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Contents

5d Supergravity

- Minimal susy, dubbed $\mathcal{N} = \mathbf{2}$, with eight supercharges [Günaydin-Sierra-Townsend]
- M-theory on Calabi-Yau / Het on K3
- IIB on Sasaki-Einstein

5d Supergravity

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- Rich black objects with horizon topology
  - $\sim S^3$ and
  - $\sim S^2 \times S^1$
- Contains a lot of info about $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence
  - 4d $\mathcal{N} = 1$ SCFT $\longrightarrow$ eight supercharges
  - a-maximization $\longleftrightarrow$ superpotential minimization

4d $\mathcal{N} = 2$ supergravity

- [de Wit et al.] : an infinite series of higher derivative 'F-terms',
- studied black hole entropy using Wald's formula
- [Ooguri-Strominger-Vafa] : the relation with topological strings,
- detailed matching of macro- and microscopic blackhole entropy.
- What'll happen in 5d ?

  ⟶ We need supersymmetric higher derivative terms !

Today's Objective

- Coeff. of $A \wedge \mathrm{tr}\, R \wedge R$ easily determined in a particular compactification;
- not sufficient to study R^2-corrected solution.
- Let us supersymmetrize $A \wedge R \wedge R$!

- Having an off-shell formulation is helpful.
  → Superconformal Tensor Calculus.
- For 5d, it was formulated by [Kugo-Fujita-Ohashi] and by [Bergshoeff-Cucu-Derix-de Wit-Halbersma-Van Proeyen] in 2000, 2001

Contents

Approaches to Supergravity

On-shell

Off-shell

- Superspace
- Superconformal tensor calculus

Approaches to Supergravity

To construct higher derivative terms,

On-shell

Need to simultaneously modify the action and the susy tr.

Off-shell

Susy transformation independent of the action

- Superspace
- Superconformal tensor calculus

Three Steps

1st

Gauge the 5d superconformal group.

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construct an action invariant under local superconformal symmetry.

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3rd

Compensator gets a vev $\longrightarrow$
symmetry Higgsed to the ordinary Poincaré supergravity.

Gauging the Conformal group

Generators

P_a	e^a_μ	Translation
M_{ab}	ω^{ab}_μ	Rotation
D	b_μ	Dilation; Weyl tr.
K_a	f^a_μ	Special conformal

Constraints

$$\begin{aligned} \hat{R}^a_{\mu\nu}(P) &= 0, \\ e^\mu_a \hat{R}^{ab}_{\mu\nu}(M) &= 0. \end{aligned}$$

- $\omega^{ab}{}_\mu = \omega_0^{ab}{}_\mu - 2e^{[a}_\mu b^{b]}, \quad f^a_\mu = \alpha R^a_\mu + \beta e^a_\mu R$
- Consider a scalar field ϕ with Weyl weight 1
- form invariant Lagrangian

$$\begin{aligned} \mathcal{L} = e\phi^{d-3}\hat{\mathcal{D}}^a\hat{\mathcal{D}}_a\phi &= e\phi^{d-3}\hat{\mathcal{D}}^a(\mathcal{D}_a\phi - b_a\phi) \\ &\sim e\phi^{d-3}\mathcal{D}^a\mathcal{D}_a\phi + e\phi^{d-3}f^a_a\phi \end{aligned}$$

- set $\phi = 1 \longrightarrow \mathcal{L} = \sqrt{-g}R.$

Gauging 5d Superconformal Group

Generators

P_a	e_μ^a	Translation
M_{ab}	ω_μ^{ab}	Rotation
D	b_μ	Dilation; Weyl tr.
K_a	f_μ^a	Special conformal
Q_i	ψ_μ^i	Susy
S_i	ϕ_μ^i	Conformal susy
U_{ij}	V_μ^{ij}	$SU(2)$ R-symmetry

Constraints

$$\hat{R}_{\mu\nu}^a(P) = 0,$$
$$e_a^\mu \hat{R}_{\mu\nu}^{ab}(M) = 0,$$
$$\gamma^\mu \hat{R}_{\mu\nu}^i(Q) = 0.$$

Independent gauge fields

e_μ^a, b_μ, ψ_μ^i and V_μ^{ij}.

$\mathbb{W}$: Weyl Multiplet

- # bosons ≠ # fermions for e^a_μ, b_μ, ψ^i_μ, V^{ij}_μ

Components of $\mathbb{W}$

e^a_μ, b_μ, ψ^i_μ, V^{ij}_μ, and v_{ab}, χ^i, D

- $\delta \equiv \delta_Q(\epsilon) + \delta_S(\eta) + \delta_K(\xi_K) = \epsilon^i Q_i + \eta^i S_i + \xi^a_K K_a$

$$\longrightarrow \quad \delta\psi^i_\mu = \mathcal{D}_\mu \epsilon^i + \frac{1}{2} v^{ab} \gamma_{\mu ab} \epsilon^i - \gamma_\mu \eta^i$$

- Spinors are $SU(2)$-Majorana, i.e. $\chi^i = \epsilon^{ij} C (\chi^j)^*$

$\mathbb{V}$: Vector Multiplet; $\mathbb{L}$: Linear Multiplet

Components of $\mathbb{V}$

$$W_a^I, \quad M^I, \quad \Omega_i^I, \quad Y_{ij}^I$$

- Contain a gauge field W_a^I for $G = U(1)^n$, $I = 1, \ldots, n$
- Gaugino transformation:

$$\delta\Omega_i^I = -\frac{1}{4}\gamma^{ab}F_{ab}(W)\epsilon^i - \frac{1}{2}\hat{\not{\mathcal{D}}}M\epsilon^i + Y^i{}_j\epsilon^j - M\eta^i$$

Components of $\mathbb{L}$

$$L^{ij}, \quad \varphi^i, \quad E_a, \quad N$$

- Contain a conserved current $\hat{\mathcal{D}}_a E^a = 0$.
- 'dual' to vector multiplet, compare the components.

Two Formulae

$\mathbb{V} \cdot \mathbb{L}$ action formula

- $\mathbb{V}$: vector multiplet, $\mathbb{L}$: linear multiplet
  → invariant action $\mathcal{L}(\mathbb{V} \cdot \mathbb{L})$
- Supersymmetrization of $W_a E^a$.

$\mathbb{V} \times \mathbb{V}' \longrightarrow \mathbb{L}$ Embedding formula

- $\mathbb{V}, \mathbb{V}'$: vector multiplets → $\mathbb{L}[\mathbb{V}, \mathbb{V}']$: linear multiplet
- Supersymmetrization of $W_a, W'_a \rightarrow E_a = \epsilon_{abcde} F^{bc} F'^{de}$.

Action for the Vector Multiplet

- Combine $\mathbb{V}^J \times \mathbb{V}^K \longrightarrow \mathbb{L}^{JK}$ and the $\mathbb{V}^I \cdot \mathbb{L}^{JK}$ action formula.
- $E^a[\mathbb{L}^{JK}] = \epsilon^{abcde} F^J_{bc} F^K_{de}$
  $\longrightarrow$ Supersymmetrization of $\epsilon^{abcde} W^I_a F^J_{bc} F^K_{de}$!
- symmetric in I, J, K
  $\longrightarrow$ defined by a purely cubic $\mathcal{N} = c_{IJK} M^I M^J M^K$
- Total action = $\mathcal{L}_{\mathbb{V}} + \mathcal{L}_{\mathbb{H}}$;
- Terms involving D : $D(\mathcal{N} + \mathcal{A}^2/2)$
- $\mathcal{A}$ compensator: fix $\mathcal{A}^\alpha_i = \delta^\alpha_i$ $\longrightarrow$ $\mathcal{N} = 1$

Contents

Strategy

Supersymmetrize $W^I \wedge \mathrm{tr}\, R \wedge R$

- Weyl multiplet $\mathbb{W}$
  → Linear multiplet $\mathbb{L}[\mathbb{W}^2]$ with $E^a \sim \epsilon^{abcde} R^f{}_{gbc} R^g{}_{fde}$
- Use $\mathbb{V} \cdot \mathbb{L}$ action formula
  → $c_I W^I_a E^a \sim c_I W^I \,\mathrm{tr}\, R \wedge R$

- $R(Q) \xrightarrow{Q} R(M)$ so that

$$L_{ij} \xrightarrow{Q} \phi \xrightarrow{Q} \underset{R(M)R(M)}{\overset{E_a}{\Cup}}$$

- L_{ij} should be triplet, Weyl weight 3, covariant :

$$L^{ij}[\mathbb{W}^2] = i\bar{\hat{R}}_{ab}{}^{(i}(Q)\hat{R}^{abj)}(Q) + A_1 i\bar{\chi}^{(i}\chi^{j)} + A_2 v^{ab}\hat{R}_{ab}{}^{ij}(U)$$

- $\delta_S L_{ij} = 0$ fixes $A_1 = \frac{1}{12}$ and $A_2 = -\frac{4}{3}$.
- Get other components by SUSY variation !

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$$\begin{array}{ccccc} L_{ij} & \xrightarrow{Q} & \phi & \xrightarrow{Q} & E_a \\ & & \Cup & & \Cup \\ & & R(Q)R(M) & \xrightarrow{Q} & R(M)R(M) \end{array}$$

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$\mathbb{W} \times \mathbb{W} \to \mathbb{L}$

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$$\begin{array}{ccccc} L_{ij} & \xrightarrow{Q} & \phi & \xrightarrow{Q} & E_a \\ \Cup & & \Cup & & \Cup \\ R(Q)R(Q) & \xrightarrow{Q} & R(Q)R(M) & \xrightarrow{Q} & R(M)R(M) \end{array}$$

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Result

$\mathbb{W} \times \mathbb{W} \longrightarrow \mathbb{L}$ formula

$$
\begin{aligned}
L^{ij} &= i\bar{\hat{R}}_{ab}{}^{(i}(Q)\hat{R}^{abj)}(Q) + \frac{1}{12}i\bar{\chi}^{(i}\chi^{j)} - \frac{4}{3}v^{ab}\hat{R}_{ab}{}^{ij}(U), \\
\varphi^i &= \cdots, \\
E_a &= -\frac{1}{8}\epsilon_{abcde}\hat{R}^{bcfg}(M)\hat{R}^{de}{}_{fg}(M) + \frac{1}{6}\epsilon_{abcde}\hat{R}^{bcij}(U)\hat{R}^{de}{}_{ij}(U) \\
&\quad + \hat{\mathcal{D}}^b\Big(-\frac{2}{3}v_{ab}D + 2\hat{R}_{abcd}(M)v^{cd} - \frac{8}{3}\epsilon_{abcde}v^{cf}\hat{\mathcal{D}}_f v^{de} \\
&\quad - 4\epsilon_{abcde}v^c{}_f\hat{\mathcal{D}}^d v^{ef} + \frac{16}{3}v_{ac}v^{cd}v_{db} + \frac{4}{3}v_{ab}v^2\Big), \\
N &= \frac{1}{6}D^2 + \frac{1}{4}\hat{R}^{abcd}(M)\hat{R}_{abcd}(M) - \frac{2}{3}\hat{R}_{abij}(U)\hat{R}^{abij}(U) \\
&\quad - \frac{2}{3}\hat{R}_{abcd}(M)v^{ab}v^{cd} + \frac{16}{3}v_{ab}\hat{\mathcal{D}}^b\hat{\mathcal{D}}_c v^{ac} + \frac{8}{3}\hat{\mathcal{D}}^a v^{bc}\hat{\mathcal{D}}_a v_{bc} \\
&\quad + \frac{8}{3}\hat{\mathcal{D}}^a v^{bc}\hat{\mathcal{D}}_b v_{ca} - \frac{4}{3}\epsilon_{abcde}v^{ab}v^{cd}\hat{\mathcal{D}}_f v^{ef} \\
&\quad + 8v_{ab}v^{bc}v_{cd}v^{da} - 2(v_{ab}v^{ab})^2.
\end{aligned}
$$

Comments

- use $c_I \mathbb{V}^I \cdot \mathbb{L}[\mathbb{W}^2]$ action formula
- Terms involving D in the action is now

$$D(\tfrac{1}{2}\mathcal{A}^2 + c_{IJK}M^I M^J M^K) + \tfrac{1}{3}c_I M^I D^2$$

- EOM :

$$1 = c_{IJK}M^I M^J M^K - \tfrac{2}{3}c_I M^I D.$$

Contents

Supersymmetric $AdS_3 \times S^2$ solutions

- done by [Castro-Davis-Kraus-Larsen], we did AdS_5.
- Suppose the metric is (AdS_3 with radius L') $\times$ (S^2 with radius L)
- Assume scalars are constant, fluxes only in S^2

Compensatorino Variation

$$\delta\zeta^\alpha = -\gamma^{ab} v_{ab} \epsilon^j \mathcal{A}_j^\alpha + 3\mathcal{A}_j^\alpha \eta^j$$

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Gaugino Variation

$$\delta\Omega_i^I = -\frac{1}{4}\gamma^{ab} F_{ab}^I \epsilon^i - \frac{1}{2}\hat{\not{D}} M \epsilon^i + Y^i{}_j{}^I \epsilon^j - M^I \eta^i$$

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- $F_{ab}^I = -\frac{4}{3} M^I v_{ab}$, $\quad Y^I = 0$

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- $\eta^i = \frac{1}{3}\gamma\cdot v\epsilon^i$, so we have

$$\mathcal{D}_\mu\epsilon + \frac{i}{2L}\gamma_\mu\epsilon = 0 \qquad \text{for} \quad AdS_3$$
$$\mathcal{D}_\mu\epsilon + \frac{i}{L}\gamma_\mu\epsilon = 0 \qquad \text{for} \quad S^2$$

  where $L^2 = 9/(2v^{ab}v_{ab})$.
- radius of AdS_3 = $2L$, radius of S^2 =L,
- $q^I = \frac{1}{2\pi}\int F^I = LM^I$.

χ variation & D EOM

- $\delta\chi = D\epsilon^i + 4\gamma\cdot v\eta^i$, $\eta^i = \frac{1}{3}\gamma\cdot v\epsilon^i$
  → $D = \frac{2}{3}v^{ab}v_{ab} = 3/L^2$.

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Terms in the action involving D

$$D(\tfrac{1}{2}\mathcal{A}^2 + c_{IJK}M^I M^J M^K) + \tfrac{1}{3}c_I M^I D^2$$

- $M^I = q^I/L$, therefore

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Equation for L

$$L^3 = c_{IJK}q^I q^J q^K + 2c_I q^I$$

- Small black string : $c_{IJK}q^I q^J q^K = 0$, $L = 0$ classically.
- $c_I q^I \neq 0$ makes a finite horizon, $L \neq 0$
  as in 4d [Dabholkar-Kallosh-Maloney]

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Summary

- Superconformal Tensor Calculus reviewed.
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- More applications ...
- a-maximization $\longleftrightarrow$ susy condition in AdS_5 in our paper
- More detailed study of entropy correction to black strings & rings [Castro-Davis-Kraus-Larsen] and several others
- Holographic dual of heterotic strings on T^5 compactification [Dabholkar-Murthy],[Johnson], [Lapan-Simons-Strominger],[Kraus-Larsen-Shah]

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Please use our formalism !