# Quantum Extensions of Classically Singular Spacetimes – The CGHS Model

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# CGHS Model

Action: $$S(g,\phi,f) = \frac{1}{2G}\int d^2x\sqrt{|g|}\left[e^{-2\phi}\left(R + 4\nabla^a\phi\nabla_a\phi + 4\kappa^2\right) + G\nabla^a f\nabla_a f\right]$$

•Free Field Equation for f $\Box f = 0$

•Dilaton is completely determined by stress energy due to f.

$$g^{ab} = e^{-2\rho}\eta^{ab} =: \Omega\eta^{ab} = \Phi\Theta^{-1}\eta^{ab}$$

•Field Redefinitions: $\Phi = e^{-2\phi}$ $\Theta = e^{2\rho-2\phi}$

•Equations of motion:

$$\begin{aligned} \partial_+\partial_-\Phi + \kappa^2\Theta &= 2GT_{+-} \\ \Phi\partial_+\partial_-\ln\Theta &= 2GT_{+-} \end{aligned}$$

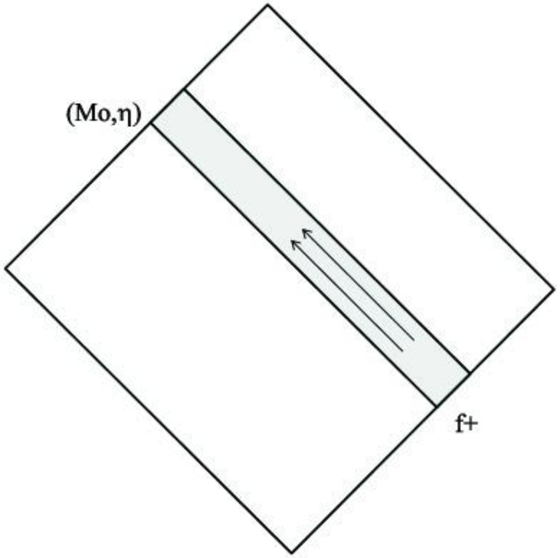
•Boundary Conditions:

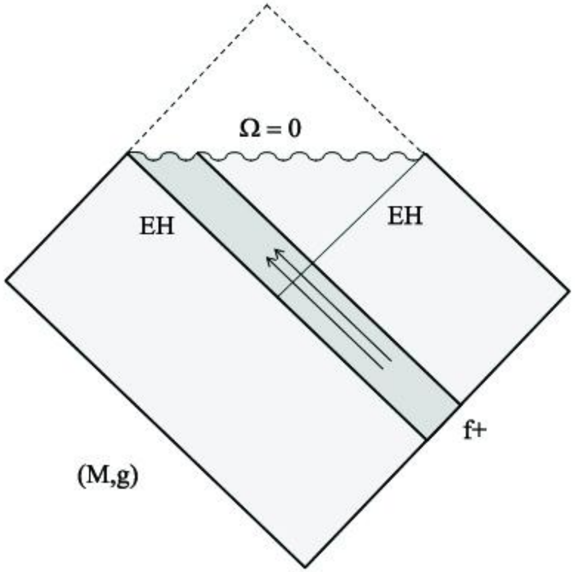
$$\begin{aligned} -\partial_+^2\Phi + \partial_+\Phi\partial_+\ln\Theta &= 2GT_{++} \\ -\partial_-^2\Phi - \partial_-\Phi\partial_-\ln\Theta &= 2GT_{--} \end{aligned}$$

Callan, Giddings, Harvey, and Strominger (1992)

# BH Collapse Solutions in CGHS

- Black Hole Solutions
- Physical spacetime has a singularity.
- True DOFs in $f_+$ and $f_-$
- Hawking Effect $T = \frac{\kappa\hbar}{2\pi}$



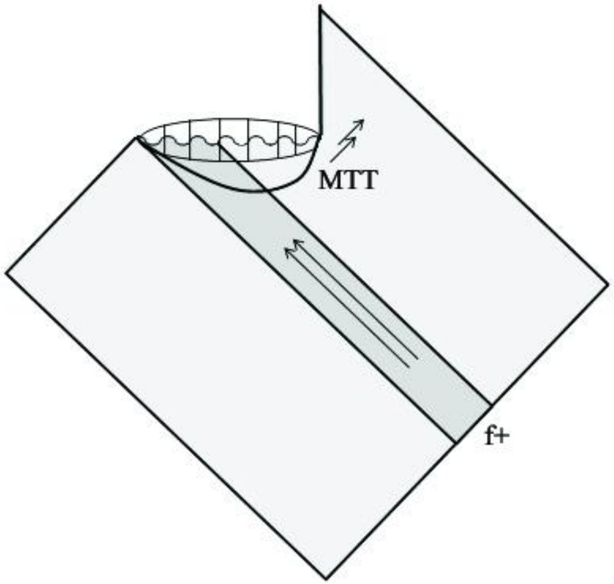

Giddings and Nelson (1992)

# Numerical Work

•Incorporated the backreaction into an effective term in the action

$$S_{eff} = S_D + \frac{\hbar}{96\pi} \int d^2x R \Box^{-1} R$$

•Equations discretized and solved numerically.  Evolution breaks down at the singularity and near the endpoint of evaporation.




Lowe (1993), Piran and Strominger (1993)

# Quantum Theory

Operator Equations:

$$\partial_+\partial_-\hat{\Phi} + \kappa^2\hat{\Theta} = 2G\hat{T}_{+-}$$
$$\hat{\Phi}\partial_+\partial_- \ln\hat{\Theta} = 2G\hat{T}_{+-}$$

\+ Boundary conditions

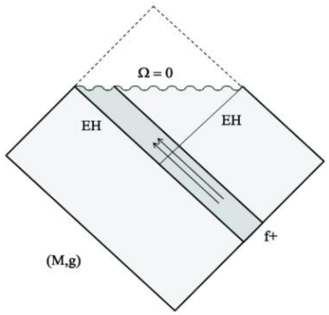
- For $\Phi$ an operator valued distribution and $\Theta$ a positive definite operator field
- $\hat{\Omega} = \hat{\Phi}\hat{\Theta^{-1}}$ well defined everywhere even though $\langle\hat{\Omega}\rangle$ may vanish
- Requires that we be able to specify $\hat{T}_{+-}(\hat{\Theta}, \hat{\Phi})$
  (work in progress)
- Even without $\hat{T}_{+-}(\hat{\Theta}, \hat{\Phi})$ one can proceed by making successive approximations to the full quantum theory

# Bootstrapping

- $0^{th}$ order – Put $\hat{T}_{+-} = 0$

  Compute $\langle \hat{g}^{ab} \rangle = \langle \hat{\Omega} \rangle \eta^{ab}$ in the state $|0_-\rangle \otimes |C_{f_+}\rangle$

  This yields the BH background.




- $1^{st}$ order – Interpret the vacuum on the $\mathcal{I}_R^+$ of the BH background metric, this is precisely the Hawking effect.

- $2^{nd}$ order - Semiclassical gravity (mean field approximation) :

  Ignore fluctuations in $\Phi$ and $\Theta$, but not f.

$$\partial_+\partial_-\langle\hat{\Phi}\rangle + \kappa^2\langle\hat{\Theta}\rangle \;=\; 2G\langle\hat{T}_{+-}\rangle$$
$$\langle\hat{\Phi}\rangle\partial_+\partial_- \ln\langle\hat{\Theta}\rangle \;=\; 2G\langle\hat{T}_{+-}\rangle$$

  Use $\langle\hat{T}_{+-}\rangle$ determined from the trace anomaly.

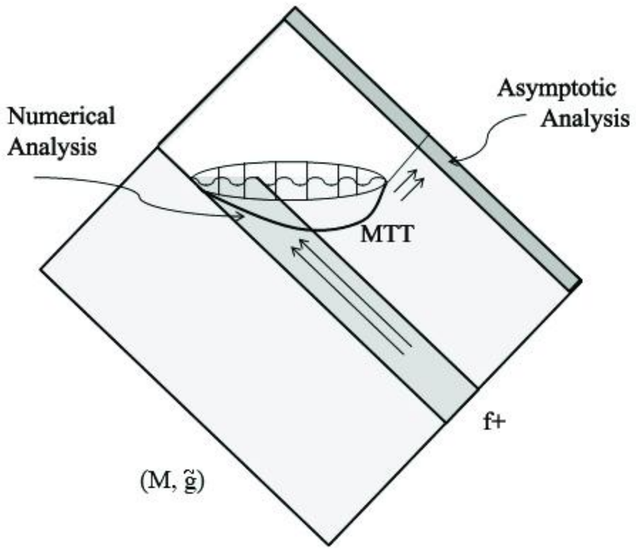
  Agreement with analytic solution near $\mathcal{I}_R^+$ obtained by asymptotic analysis. Equations of motion are the ones in the numerical work.

# Asymptotic Analysis

•Expand $\Phi$ and $\Theta$ in inverse powers of $x^+$.

•Idea: We should have a decent control of what is going on near $\mathscr{I}_R^+$ since curvatures and fluxes there are small

$$\partial_- M_B = -\langle \hat{T}_{--} \rangle$$




•The Hawking flux and the Bondi mass go to 0 and the physical metric approaches the flat one.

• $\mathscr{I}_R^+$ for the physical metric agrees with background $\mathscr{I}_R^+$ .

# Summary

- To $0^{th}$ order we recover the black hole background.
- To higher orders in the truncation, not only do we recover the Hawking effect but we obtain a self-consistent system of semiclassical equations.
- $\mathcal{I}_R^+$ of the semiclassical metric coincides with that of the background metric thus there is no information loss.
- The pure state resembles a thermal state in the past of $\mathcal{I}_R^+$ of the $1^{st}$ order physical metric.
- The quantum spacetime is larger than the classical spacetime.

# Future Work

- Need to extend QFT on CST to QFT on QG. $\mathcal{I}_R^+$
- Although the result on information loss only depends on the size of work remains to be done on when the information comes out.