

Can we test effective quantum gravity with gravitational waves?

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S. Alexander and N. Y. , “A new PPN parameter to test Chern-Simons gravity”, [hep-ph/0704.0299](#)

S. Alexander and N. Y. “Parameterized post-Newtonian expansion of Chern-Simons gravity”, *PRD***75**, 124022

S. Alexander, S. Finn and N. Y. “An observational probe of effective quantum gravity”, to be submitted soon.



A little motivation

- We would like a consistent unified quantum theory of Nature.
- Symmetry principles are useful to construct gauge theories.
- SM has CP violating interactions (eg, QCD sector), why not in gravity ?
- In the absence of full quantum gravity, consider an “effective” theory:

Effective Theory = GR + “CP-violation”

- Once we have formulated such a theory, can we test it?
 - Solar system effects? Gyroscopic precession. PPN tests.
 - Gravitational wave effects? Polarization effect. Fisher analysis.

The Chern-Simons Extension

- In analogy with QCD and inspired by CP violation, let us deform GR:

$$S = \frac{1}{16\pi G} \int d^4x \left(\sqrt{-g}R + \frac{1}{4}f[\phi] {}^*RR \right),$$

where ${}^*RR = \epsilon^{\mu\nu\alpha\beta} R^\sigma{}_{\tau\alpha\beta} R^\tau{}_{\sigma\mu\nu} / 2$ and $f[\phi]$ is some functional of some “external” dynamical field ϕ (Jackiw & Pi)

- Varying the action wrt $g_{\mu\nu}$: $\delta(f {}^*RR) \rightarrow \sqrt{-g} C_{\mu\nu} \delta g^{\mu\nu}$ and the EEs become:

$$G_{\mu\nu} + C_{\mu\nu} = 8\pi T_{\mu\nu}$$

where $C_{\mu\nu}$ is the Cotton tensor given by

$$C_{\mu\nu} \propto \left[f_{;\sigma} \epsilon^{\sigma\alpha\beta} ({}_{\mu}R_{\nu})_{\beta;\alpha} + f_{;\tau\sigma} {}^*R^\tau ({}_{\mu}{}^{\sigma}{}_{\nu}) \right]$$

- Like in QCD, we have now a **P violating effective theory** with a new scalar field ϕ (the gravitational axion).

Effective Quantum Gravity

Who ordered that?

Quantum theory suggests such a Chern-Simmons extension

- **String Theory:**

- A Chern-Simmons-like term is needed to cancel a chiral anomaly.
- **Otherwise, string theory would not be a mathematically consistent QFT**
- If quantum origin, then likely to be **suppressed** by Planck scale.
- But some scenarios postulate **enhancement** of \hat{f} via coupling to curvature and mass current.

It looks like you're trying to test string theory. Would you like me to bug you instead?

- Annoy me until my eyes bleed.
- Go away.



- **Loop Quantum Gravity:**

- **No effective Hamiltonian** in the full theory, yet. However, if ST needs it . . .

But the correction need not be quantum inspired!

- There other scenarios where the Chern-Simons correction arises from topological considerations.

Solar System Tests

The ABC of PPN

- A) Solve the EOM in your alternative theory.
 - **Expand** the modified Einstein Equations about a Minkowski background to second order in the metric perturbation (Nordtvedt & Will).
 - **Assume a perfect fluid** stress-energy source (binaries, Earth-Sun, etc).
 - Assume a **slow-motion/weak-gravity** approximation and perturbatively solve the linearized EEs: g_{00} to $\mathcal{O}(v^4)$, g_{0i} to $\mathcal{O}(v^3)$ and g_{ij} to $\mathcal{O}(v^2)$.
 - The final solution is then expressed in terms of **PPN potentials** (Green-function like integrals over the stress energy tensor.)
- B) **Construct a Super-Duper-Duper Metric**
 - Based on some assumptions, construct a family of metric solutions with PPN potentials, with the family labeled by some *PPN parameters*, eg.

$$g_{ij} = (1 + 2\gamma U) \delta_{ij}$$

- C) **Compare** solution to new theory to super-metric and **read off** PPN params.

Has that parameter been measured? → **You've tested your theory!**

Can we test CS in the Solar System?

(Alexander & Yunes)

- When we expand the Cotton tensor in the PPN framework, we find that $C_{00} = \mathcal{O}(v)^6$, C_{ij} leads to $\delta h_{ij} = \mathcal{O}(\dot{f})^2$ and

$$C_{0i} \sim -\frac{1}{4} \dot{f} \tilde{\epsilon}^{kl}{}_i \nabla^2 h_{0l,k}$$

- The only modification to the gravitational field to leading $\mathcal{O}(\dot{f})$ is then

$$\delta h_{0i} \sim 2\dot{f} (\nabla \times V)_i$$

where V_i is one of the PPN potentials (lowest-order vector potential).

- New PPN parameter!!** (such terms had not been considered before.)
- Specialize to a binary sys. of pt. ptcls. : $V^i \sim m_A v_A^i / r_A - (n_A \times J_A)^i / (2r_A^2)$

$$g_{0i}^{(1)} = -\frac{7}{2} \frac{m_1}{r_1} v_1^i - \frac{m_1}{6r_1^2} \left(v_1 - v_1^{(eff)} \right) - \frac{1}{2} n_1^i \frac{m_1}{r_1} \left(v_1^{(eff)} \cdot n_1 \right) - 2 \left(\frac{J_1^{(eff)}}{r_1^2} \times n_1 \right)^i$$

where $v_{A,eff}^i = v_A^i - 6\dot{f} J_A^i / (m_A r_A^2)$ and $J_{A(eff)}^i = J_A^i - \dot{f} m_A v_A^i$

- Axion is a fluid that is “dragged” by motion** and couples to ang. mom. \sim Kerr.

Acceleration and Frame-Dragging

- Gravitomagnetic analogy: Construct electromagnetic vectors via

$$E^i = -(\nabla\Phi)^i - \dot{A}^i/2 \quad \text{and} \quad B^i = (\nabla \times A)^i,$$

where $A^i \propto g_{0i}$. Thus, CS modifies the gravitomagnetic sector of ds^2 .

- Gravitomagnetism affects the acceleration of point particles, namely

$$\delta a^i = \frac{1}{8}\delta\dot{g}^i + \frac{1}{2}(v \times \delta\Omega)^i = -\frac{3}{2}\dot{f}(v_1 \cdot n_1)(v_1 \times n_1)^i + 1 \rightarrow 2,$$

but for circular orbits the CS correction vanishes (v perp. n).

- However, CS also affects directly the **frame-dragging of gyroscopes**, where

$$\delta\Omega^i = -\sum_A \dot{f} \frac{m_A}{r_A^3} [3(v_A \cdot n_A) n_A^i - v_A^i]$$

- Detectable? **Smith, Erickcek, Caldwell and Kamionkowski** recently expanded our analysis to account for extended objects and found that $\dot{f} < 10^{-2}$ seconds using LAGEOS.

Gravitational Waves and Birefringence

- Expand the EEs about a FRW background since redshift might enhance effect by accumulation (Alexander & Martin)

$$\square_g h_L = -i F[\dot{f}, \ddot{f}] \dot{h}_{L,i} \hat{k}^i \quad \square_g h_R = +i F[\dot{f}, \ddot{f}] \dot{h}_{R,i} \hat{k}^i$$

with solution $h_{R,L} = e^{\pm f\tau} h_{R,L}^0 \rightarrow$ the CS correction introduces an **exponential growth/decay of the L/R polarization** (“movie”).

- Pick a binary and then (Alexander, Finn & Yunes):

$$h_+ = \frac{2\mathcal{M}}{d_L} (\pi\mathcal{M}f)^{2/3} [(1 + c_i^2) C_\phi \cosh(f\tau) - 2c_i S_\phi \sinh(f\tau)]$$

$$h_\times = \frac{4\mathcal{M}}{d_L} (\pi\mathcal{M}f)^{2/3} [c_i S_\phi \cosh(f\tau) + \frac{1}{2} (1 + c_i^2) C_\phi \sinh(f\tau)]$$

where \mathcal{M} is the chirp mass, f is the GW frequency, d_L is the luminosity distance $c_i = \cos i$ with i the inclination angle and ϕ the GW phase.

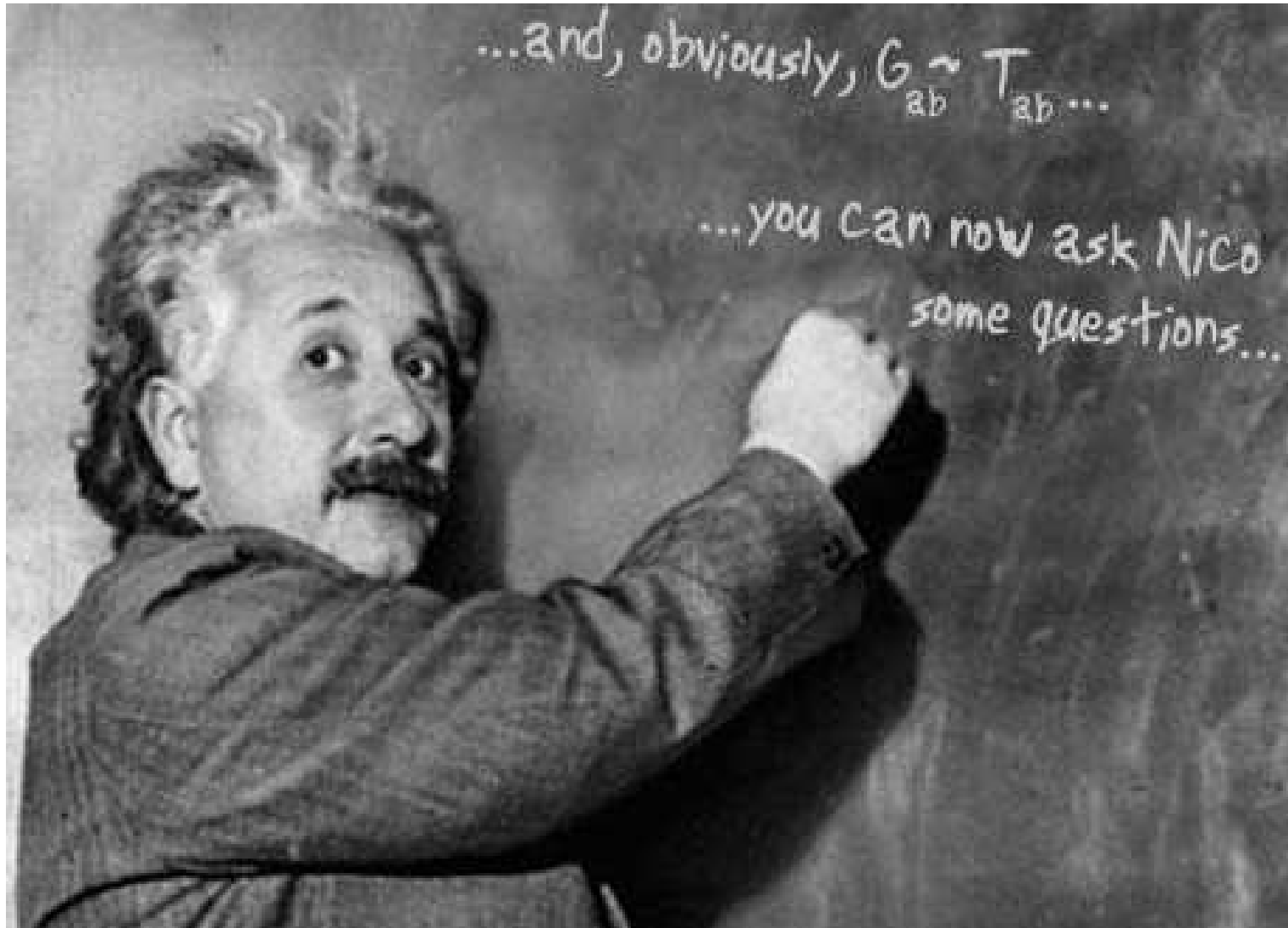
- Can we test CS correction with LISA?** Fisher analysis says YES!: $\dot{f} \lesssim 10^{-3}$ secs. (rough estimate, making several assumptions on source and instrument).

Conclusions

- Inspired by CP violation in QCD, one can construct an **effective CP-violating gravity theory**: Chern-Simons gravity.
- **Solar system effects** (frame-dragging), studied via weak-field and PPN analysis. Extending Alexander & Yunes, Smith, *et. al.* place bound with LAGEOS at the 1% level.
- **Gravitational wave effects** difference in polarization amplitude might be detectable by LISA if $\delta\phi$ is large enough. Possible bounds might be one or two orders of magnitude larger than Solar system ones.

Future Work

- **How large can \dot{f} be?** Possible enhancements via coupling with neutron currents and regions of high curvature.
- **Can we do better with LISA** by performing a more detailed Fisher analysis?
- Are there **other Solar System effects**?



THANKS