Quasi-Local Horizon Calculations and Gauge Conditions

Yosef Zlochower (RIT)

Manuela Campanelli (RIT)

Carlos Lousto (RIT)

Badri Krishnan (AEI)

David Merritt (RIT)

IGC Inaugural Conference

August 9, 2007

Overview

- Quasi-Local Calculations
  - Angular Momentum: gr-qc/0612076, PRD 75, 064030 (2007)
  - Mass
  - **Linear Momentum: arXiv:0707.0876 [gr-qc] (2007)**
- Individual horizons spins. Orbital linear momentum
- Remnant kicks and spins
- Conclusion

Quasi-Local Energy-Momentum

$$P_{ab} = K_{ab} - K\gamma_{ab}, D_a P^{ab} = 0$$

$$\int_V \xi_b D_a P^{ab} d^3V = 0$$

$$\oint_\infty P_{ab}\xi^b d^2S^a - \sum_i \oint_{S^i} P_{ab}\xi^b d^2S^a = \int_V P^{ab} D_{(a}\xi_{b)} d^3V$$

For Killing vector ξ^a, or conformal killing vector ξ^a and maximal slicing, we get:

$$\sum_i \oint_{S^i} P_{ab}\xi^b d^2S^a = \oint_\infty P_{ab}\xi^b d^2S^a$$

ξ^a translation $\rightarrow$ linear momentum, ξ^a rotation $\rightarrow$ angular momentum. Identify $\oint_{S^i} P_{ab}\xi^b d^2S^a$ with the linear momentum/angular momentum of hole i.

Spin Direction

- How do you measure spin direction?
  - Spin direction well defined in the presence of axisymmetry
  - Isn't defined (yet?) rigorously in the absence of symmetry
  - How do you associate a vector on the horizon with a vector at infinity? Does this ever make sense?
  - What about jet direction? Can it tell us where the spin points?
  - If so, how do we tell where the jet would point?
- OK. Can we get a qualitative measure of the spin direction?
  - Can we determine if the spins precess? Yes
  - Can we determine if the spins change magnitude? Yes
  - Can we determine the final Kerr spin direction? **YES!!!**

A (semi) coordinate dependent spin

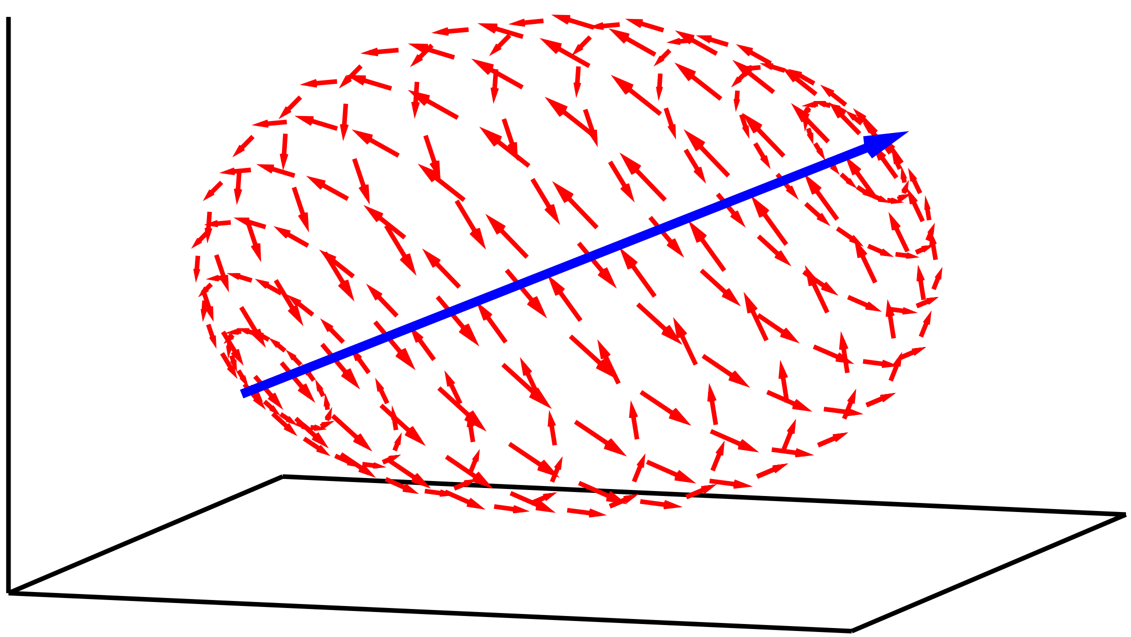
- Measure spin magnitude using Isolated Horizon formalism

$$S = \frac{1}{8\pi} \oint_{AH} (\varphi^a R^b K_{ab}) d^2V$$

φ^a is an approximate Killing vector

R^a the unit-norm to the horizon, d^2V the natural volume element

- Take coordinate direction joining the poles of the approximate Killing vector as the spin direction



Can we rely on a coordinate dependent spin?!

Of course we can't use arbitrary coordinates but . . .

- Our coordinates give good track / waveform agreement
- Good agreement of spin direction using coordinate-based rotation vectors $\xi^i_{[\ell]} = \delta_{\ell j}\delta_{mk} r^m \epsilon^{ijk}$. (supports the use of coordinate translation vectors for $\vec{P}$)
- Reasonable agreement of spin direction with PN prediction
- Good agreement between $\vec{J}_{\rm rad}$ from spin direction and waveform. gr-qc/0703061

Linear Momentum

- No translation Killing vector for binary configuration
- $\xi^a = (1,0,0), (0,1,0), (0,0,1)$ — coordinate dependent
- Sum of horizon momenta not equal to total ADM momentum in general, but works for spacetime 'close' to conformally flat and maximally sliced.
- Reasonable track / momentum agreement during orbit
- Good momentum / kick agreement

Gauge Conditions

$$\left(\partial_t - \beta^i\partial_i\right)\alpha = -2\alpha K, \partial_t\beta^a = B^a, \partial_t B^a = 3/4\partial_t\tilde{\Gamma}^a - \eta B^a.$$

- small η $(1-2)$
  - Better waveform / trajectory phase agreement
  - Better quasi-local momentum / waveform agreement
  - 'Better' spin direction
  - lower effective resolution
  - can be unstable

- medium η $(3-6)$
  - Enhanced stability
  - Good waveform accuracy

- large η $(10-)$
  - Poor track / waveform phase agreement
  - Can lose control of coordinate at late-time (grid stretching)

Configurations

Table 1: Initial data parameters for the generic binary SP6 (top), non-spinning binary Q38 (middle), and headon binary HD0 (bottom) configurations. SP6 has spins $\vec{S}_1 = (0, S, -S)$ and $\vec{S}_2 = (0, 0, 0)$, momenta $\vec{P} = \pm(P_r, P_\perp, 0)$, puncture positions $\vec{x}_1 = (x_+, d, d)$ and $\vec{x}_2 = (x_-, d, d)$. HD0 has spins $\vec{S}_1 = -\vec{S}_2 = (0, 0, S)$.

m_p/M	0.3185	d/M	0.0012817	m_1/M	0.6680
x_+/M	2.68773	P_r/M	-0.0013947	m_2/M	0.3355
x_-/M	-5.20295	$P_\perp/M$	0.10695	S/M^2	0.27941
x_1/M	1.7604572	m_1^p/M	0.718534207968	P/M	0.10682112
$x_2/$	-4.7455652	m_2^p/M	0.257487827988	J/M^2	0.69498063
m_1^H/M	7.35380191	m_2^H/M	0.27582649	L^z/M^2	0.69498063
x_1/M	-3.5000	m^p/M	0.427644	m^H/M	0.517407
x_2/M	$+$3.5000	S/M^2	0.15		

Orbital Calculations

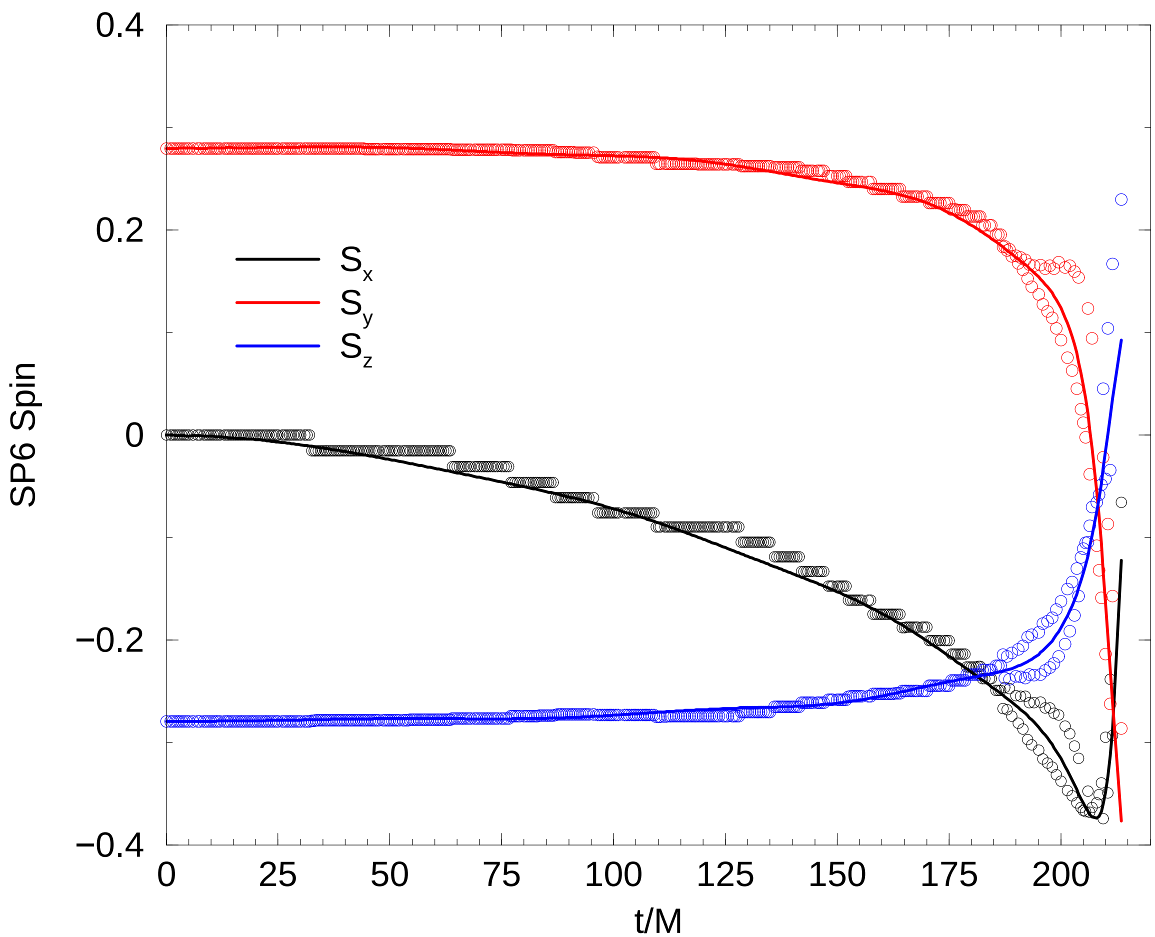


Figure 1: Spin direction for SP6 versus time.

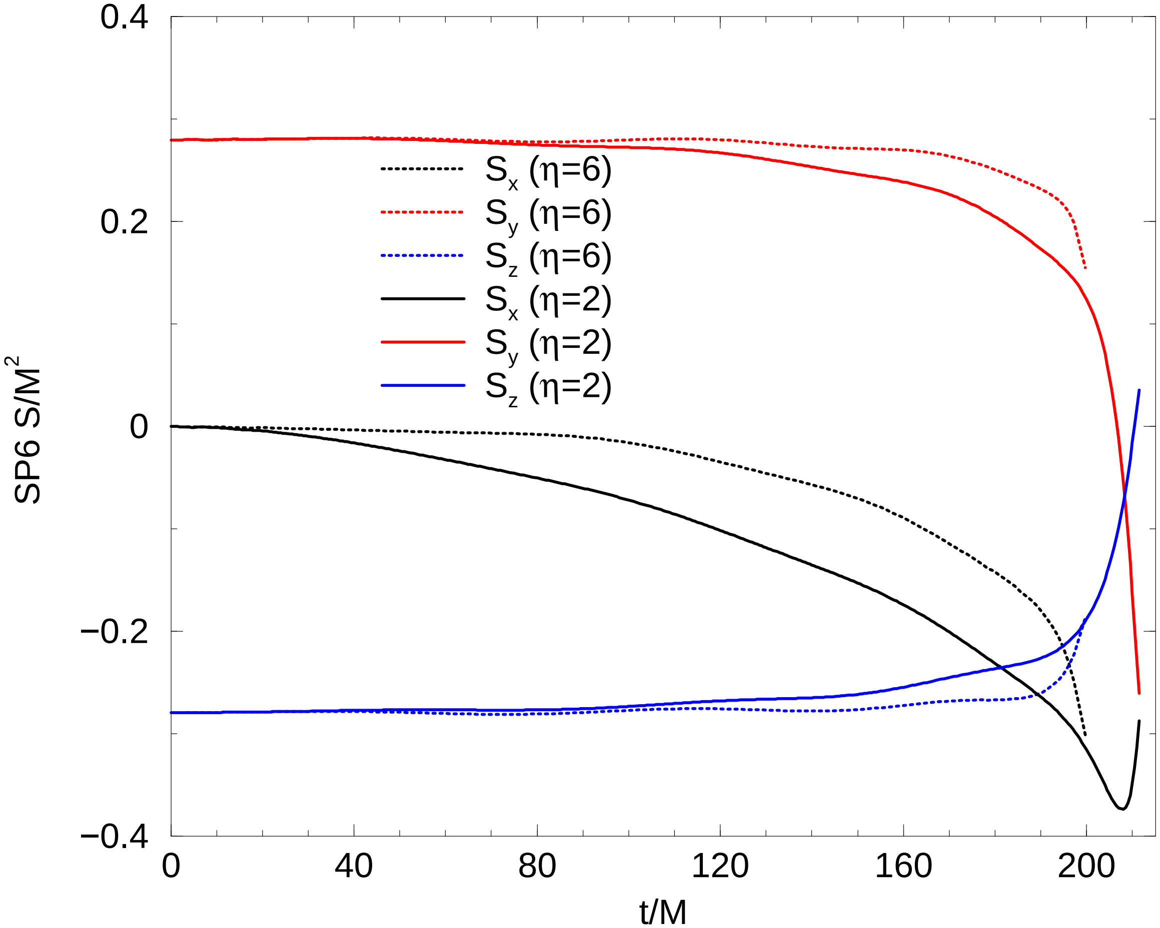


Figure 2: Spin direction for SP6 versus time and η.

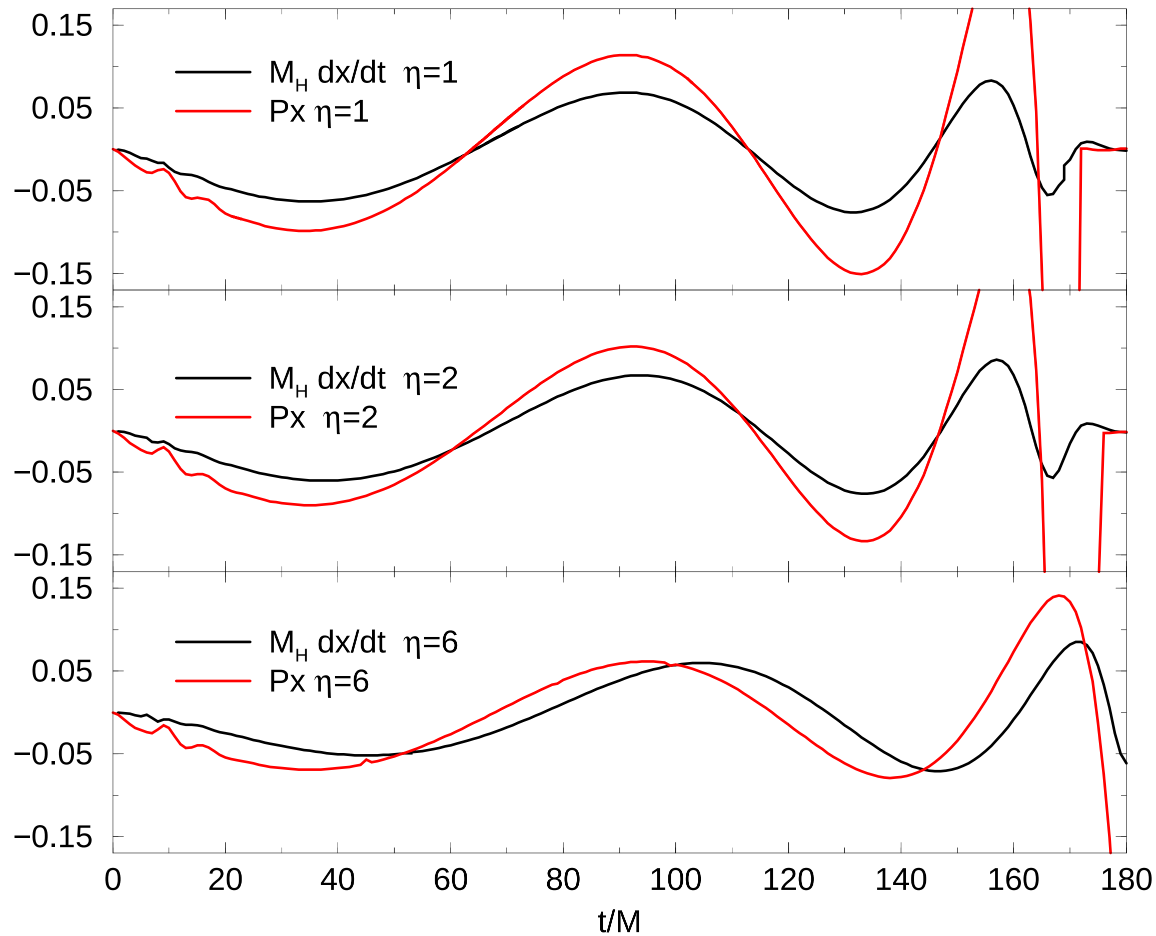


Figure 3: Orbital linear momentum for Q38 versus time and η.

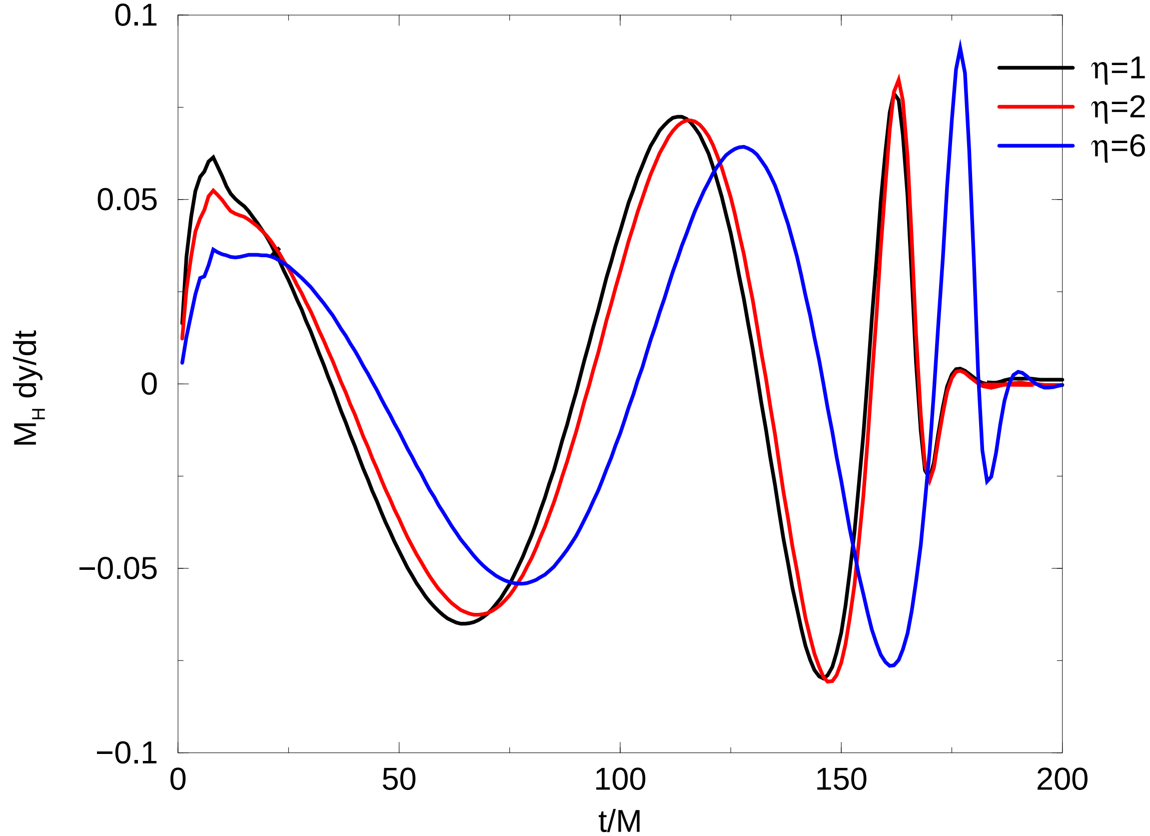


Figure 4: Orbital linear momentum for Q38 versus time and η.

Waveform Accuracy

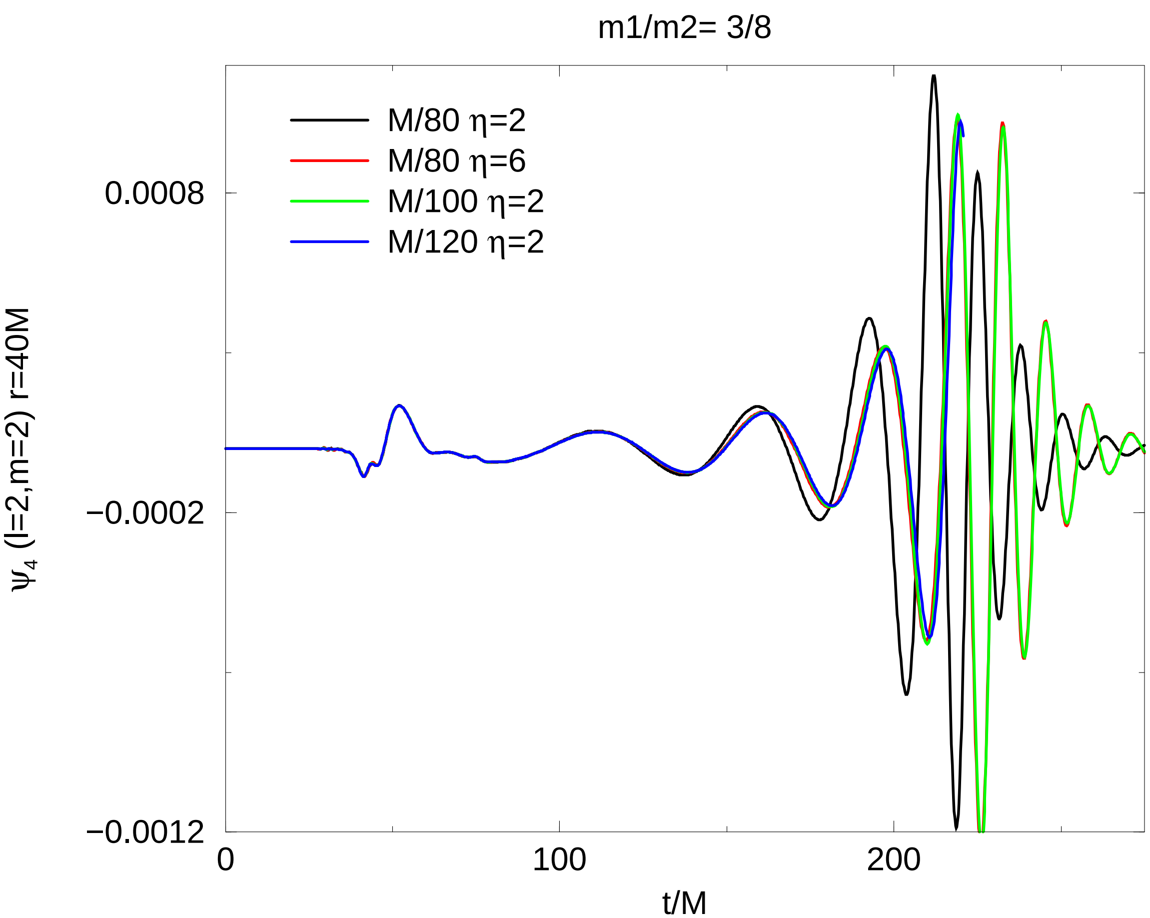


Figure 5: Q38 Waveform as a function of η and resolution.

- Precession ‘delayed’ (less realistic) for larger η.

- Reasonable agreement of spin near merger.

- Good agreement between pure coordinate spin direction, and direction from poles of the Killing vector.

- Trajectories and momentum out of phase for larger η.

- Higher effective resolution for larger η (merger delayed).

- More accurate coordinate initial coordinate trajectory for smaller η (Mdy/dt closer to P_y).

Remnant Calculation

SP6 Remnant Spin $\vec{S}_{\rm QL} = (0.040, 0.052, 0.15)$,
$\vec{S}_{\rm wave} = (0.036 \pm 0.006, 0.045 \pm 0.010, 0.155 \pm 0.009)$

HD0 remnant kick is $P_y/M = (20.4 \pm 0.5)\mathrm{km\,s^{-1}}$ (based on ψ_4).

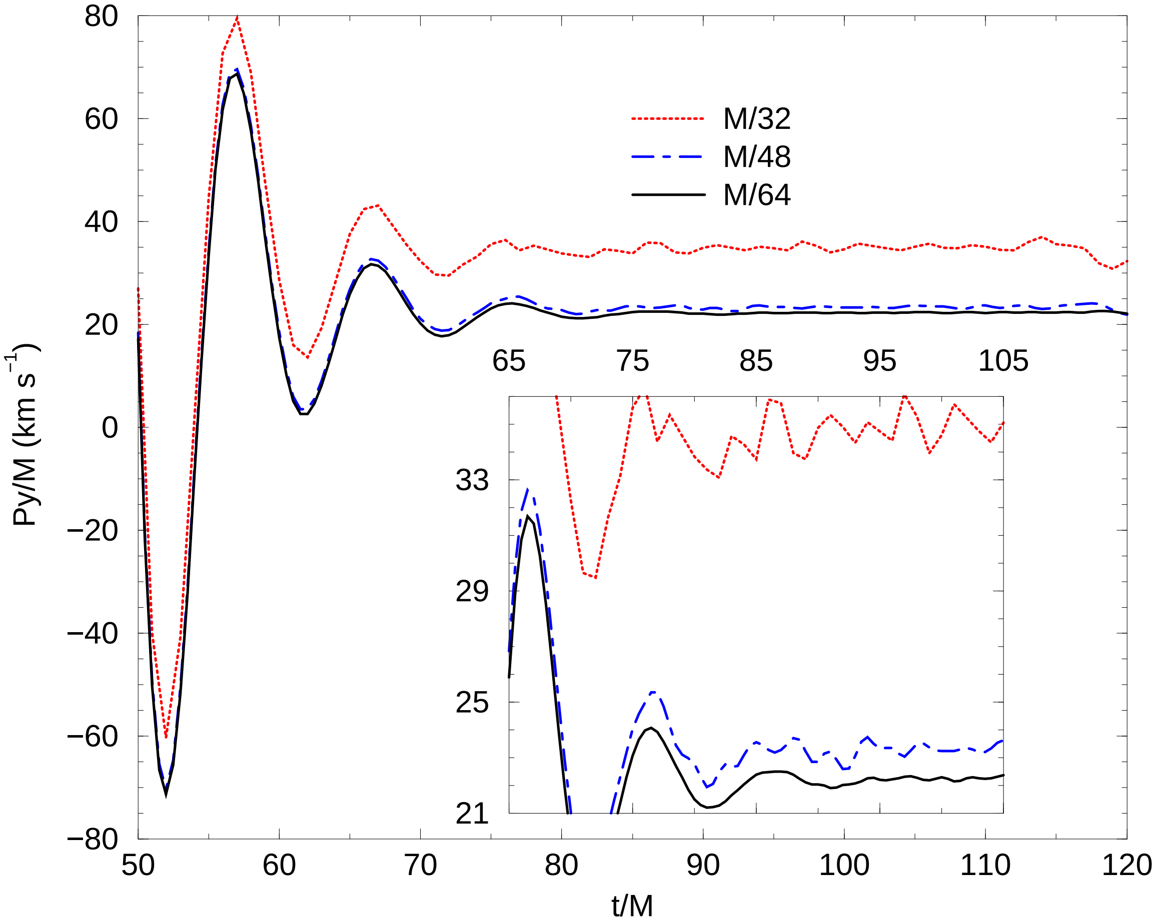


Figure 6: HD0 remnant momentum versus resolution. $(\eta = 1)$

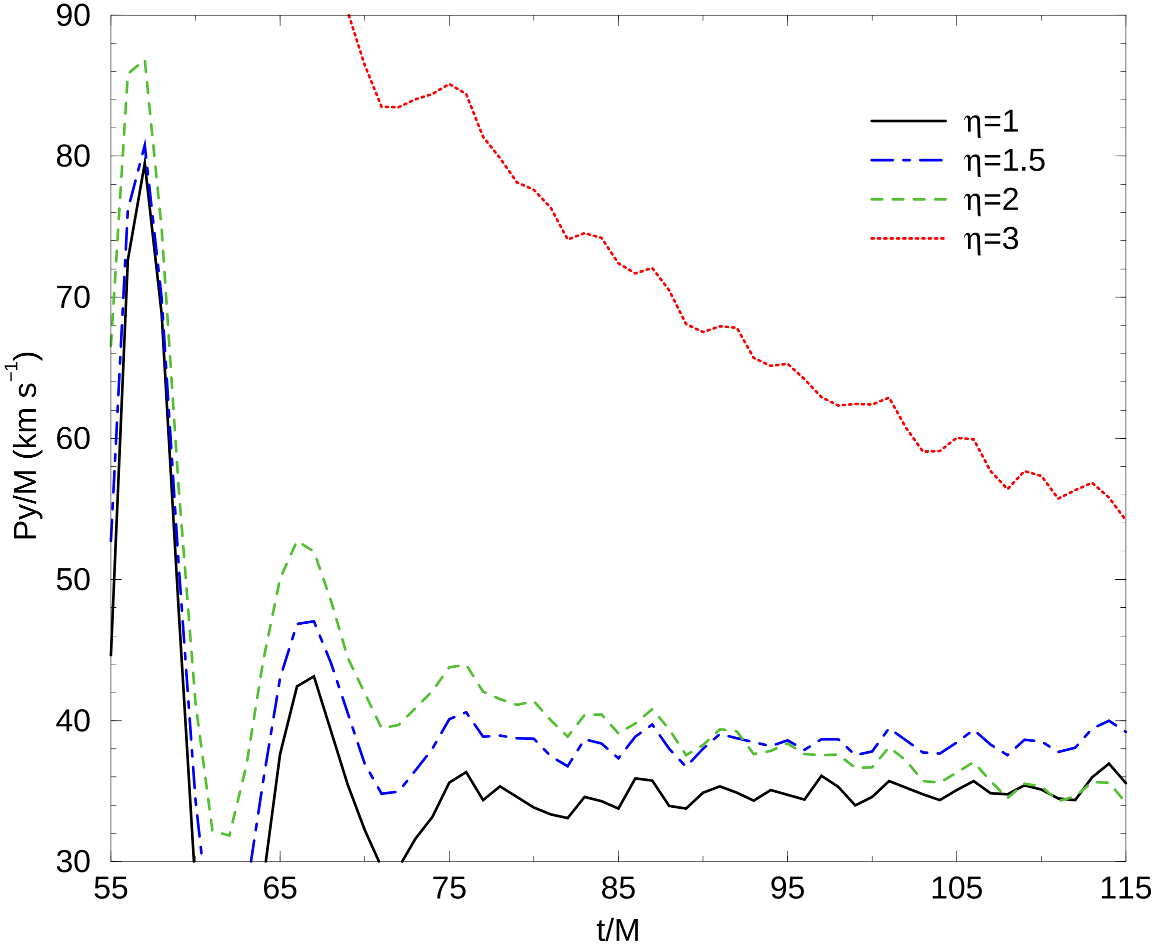


Figure 7: HD0 remnant momentum versus η ($h = M/80$).

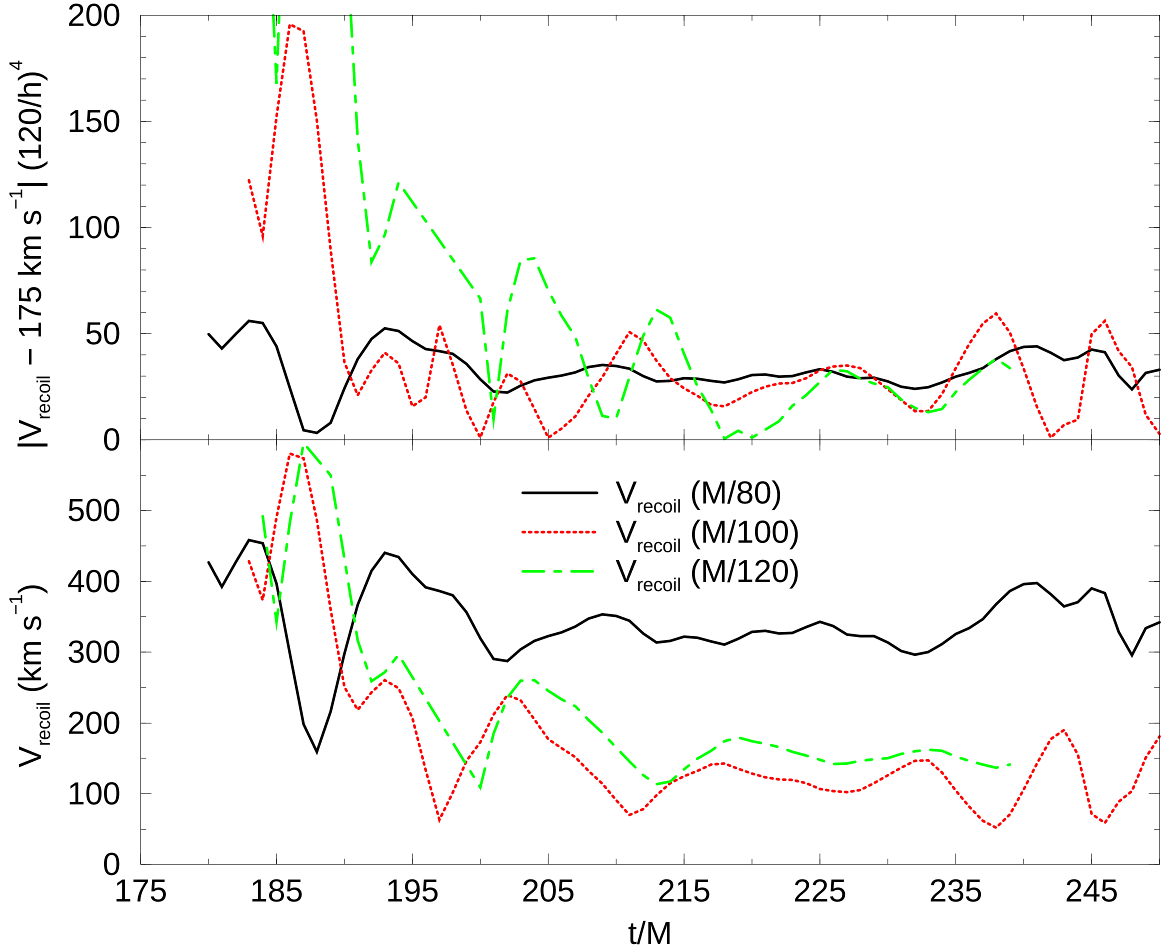


Figure 8: Q38 remnant momentum versus time ($\eta = 2$).

Conclusion

- Can get reasonable values for $\vec{S}$ and $\vec{P}$ using semi-coordinate dependent methods.
  - Reasonable values for the individual spins
  - Highly accurate values for the remnant spin
  - Reasonable values for the orbital linear momentum
  - Highly-accurate values for the remnant kick
- Need to balance value of η for good Q.L. $\vec{S}$ and $\vec{P}$ with need for overall accuracy