Quark matter in neutron stars

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Conjectured QCD phase diagram

**heavy ion collisions**: chiral critical point and first-order line

**compact stars**: color superconducting quark matter core
Cooper pairing in quark matter: color superconductivity

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.

Any attractive quark-quark interaction causes pairing instability of the Fermi surface: BCS mechanism of superconductivity.

\[ \mu = E_F \]

\[ F = E - \mu N \quad \frac{dF}{dN} = 0 \]

QCD quark-quark interaction is attractive in color-antisymmetric channel:
- single gluon exchange
- instanton interaction
- strong coupling: count flux tubes
- confinement is attraction

Physical consequences of Cooper pairing
Changes low energy excitations, affecting *transport properties*.

- spontaneous breaking of global symmetries: *Goldstone bosons*, massless degrees of freedom that dominate low energy behavior. E.g. light pions, superfluidity.

- spontaneous breaking of local (gauged) symmetries: massive gauge bosons, exclusion of magnetic fields (Meissner effect).

- create a *gap in fermion spectrum*.

Adding a fermion of momentum $\vec{p}$ near the Fermi surface disrupts the condensate in that mode, i.e. breaks the Cooper pair with momenta $p$ and $-p$, costing energy $\Delta$. 

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![Graph showing energy versus momentum with a gap at $\Delta$](image-url)
Color superconducting phases

Quark Cooper pair: \( \langle q^\alpha_{ia} q^\beta_{jb} \rangle \)

Each possible BCS pairing pattern \( P \) is an \( 18 \times 18 \) color-flavor-spin matrix

\[
\langle q^\alpha_{ia} q^\beta_{jb} \rangle_{1PI} = \Delta_P P^\alpha_{i \beta j \alpha b}
\]

The attractive channel is:
- color antisymmetric [most attractive]
- spin antisymmetric [isotropic]
- \( \Rightarrow \) flavor antisymmetric

So we expect pairing between different flavors.
Calculating properties of high-density quark matter

**Lattice:** “Sign problem”—negative probabilities.

**SUSY:** Statistics crucial to quark Fermi surface.

**large $N_c$:** Large corrections. (Also gravity dual theories.)

**pert:** Applicable far beyond nuclear density.
  Neglects confinement and instantons.

**NJL:** Model, applicable at low density.
  Follows from instanton liquid model.

**EFT:** Effective field theory for lightest degrees of freedom.
  “Parameterization of our ignorance”: assume a phase, guess
  coefficients of interaction terms (or match to pert theory),
  obtain phenomenology.
Calculations using NJL or weak-coupling QCD

Guess a color-flavor-spin pairing pattern $P$; to obtain gap $\Delta_P$, calculate free energy $\Omega$ (mean-field approx typically), minimize with respect to $\Delta_P$ and impose color and electric neutrality

$$\frac{\partial \Omega}{\partial \Delta_P} = 0 \quad \frac{\partial \Omega}{\partial \mu_i} = 0$$

The pattern with the lowest $\Omega(\Delta_P)$ wins!

1. **Weak-coupling** methods. First-principles calculations direct from QCD Lagrangian, valid in the asymptotic regime, currently $\mu \gtrsim 10^6$ MeV.

2. **Nambu–Jona-Lasinio models**, ie quarks with four-fermion coupling based on instanton vertex, single gluon exchange, etc. This is a semi-quantitative guide to physics in the compact star regime $\mu \sim 400$ MeV, not a systematic approximation to QCD.

NJL gives $\Delta \sim 10–100$ MeV at $\mu \sim 400$ MeV.
The real world: $M_s$ and neutrality

In the real world (ie neutron star cores) there are three complications.

1. **Strange quark mass** is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.

2. **Neutrality requirement.** Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.

3. **Weak interaction equilibration.** In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved.

So quark matter in a compact star might be CFL, or something else: kaon-condensed CFL, 2SC, 1SC, crystalline,...
Cooper pairing vs. the strange quark mass

Unpaired

2SC pairing

CFL pairing

CFL: Color-flavor-locked phase, favored at the highest densities.

\[ \langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N} \epsilon_{ijN} \]

breaks chiral symmetry by a new mechanism: \( \langle qq \rangle \) instead of \( \langle \bar{q}q \rangle \).

2SC: Two-flavor pairing phase. May occur at intermediate densities.

\[ \langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta3} \epsilon_{ij3} \sim (rg - gr)(ud - du) \]

or: Exotic non-BCS pairing: LOFF (crystalline phase), \( p \)-wave meson condensates, single-flavor pairing (color-spin locking, \( \sim \)liq \( ^3 \)He-B).
But there are also non-uniform phases, such as the crystalline ("LOFF"/"FFLO") phase. (Alford, Bowers, Rajagopal, hep-ph/0008208)
Crystalline (LOFF) color superconductivity

When $s$-wave pairing between different flavors is stressed to near the breaking point, it may be favorable to make pairs with a net momentum, so each flavor can be close to its Fermi surface.

This yields a plane-wave condensate $\Delta(x) = \Delta_0 \exp(2i q \cdot x)$.

Two plane waves: $\Delta(x) = \Delta_0 \cos(2q \cdot x)$

With three flavors one can combine many plane waves to get crystal structures such as BCC (Rajagopal and Sharma hep-ph/0605316).
Free energy comparison of phases

Assuming $\Delta_{\text{CFL}} = 25$ MeV.

Curves for CubeX and 2Cube45z use G-L approx far from its area of validity: favored phase at $M_s^2 \sim 4\mu\Delta$ remains uncertain.

Signatures of color superconductivity in compact stars

Pairing energy \{ affects Equation of state . Hard to detect. \\ (Alford, Braby, Paris, Reddy, nucl-th/0411016) \\

Gaps in quark spectra and Goldstone bosons \{ affect Transport properties : \\
emissivity, heat capacity, viscosity (shear, bulk), conductivity (electrical, thermal). . . \\

1. Gravitational waves: r-mode instability, shear and bulk viscosity
2. Glitches and crystalline ( “LOFF” ) pairing
3. Cooling by neutrino emission, neutrino pulse at birth
r-modes: gravitational spin-down of compact stars

An r-mode is a quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.

The unstable $r$-mode can spin the star down very quickly, in days to years (Andersson gr-qc/9706075; Friedman and Morsink gr-qc/9706073; Lindblom astro-ph/0101136).

So if we see a star spinning quickly, we can infer that the interior viscosity must be high enough to damp the $r$-modes.
Constraints from r-modes

Regions above curves are forbidden because viscosity is too low to hold back the \( r \)-modes.

(Nuclear matter)

(Quark matter)

(Jaikumar, Rupak, Steiner arXiv:0806.1005)
Bulk viscosity of uniform quark matter phases

\[ \mu = 400 \text{ MeV} \quad M_s = 90 \text{ MeV} \quad \tau = 1 \text{ ms} \]

\[ \delta m \] is the kaon mass gap
\[ \delta m > 0: \text{ CFL} \]
\[ \delta m < 0: \text{ CFL}-K^0 \]

Alford, Schmitt, nucl-th/0608019; Alford, Braby, Reddy, Schäfer, nucl-th/0701067;
Manuel, Llanes-Estrada, arXiv:0705.3909;
Alford, Braby, Schmitt, arXiv:0806.0285

- Unpaired and 2SC have the largest bulk viscosity, because they have unpaired modes at Fermi surface (large phase space).
- \( K^0 \) density \( \sim \exp(-\delta \mu/T) \) drops rapidly for \( T \lesssim \delta \mu/10 \).
- \( \delta \mu = m_{K^0} - M_s^2/(2\mu) \) could be anything from negative (kaon condensation) to \( \sim 10 \) MeV.
- Superfluid modes ("phonons") alone contribute some bulk viscosity.
Glitch: star’s rotation rate suddenly increases. Thought to be due to transfer of angular momentum from core to crust as superfluid vortices unpin from some rigid structure in the star.

**Conventional picture:** pinning occurs in “inner crust” where neutron superfluid interpenetrates a lattice of nuclei, with shear modulus $\nu \sim 10^{-4}-10^{-2}$ MeV/fm$^3$.

**Alternative scenario:** pinning occurs in quark matter core in LOFF phase: superfluid and rigid crystal, shear modulus $\nu \sim 0.5-20$ MeV/fm$^3$. (Mannarelli, Rajagopal, Sharma hep-ph/0702021)

Open questions: pinning force, angular momentum transport time.
Cooling of a neutron star with quark matter core

With 2-flavor color superconductivity, and additional weak pairing of the blue quarks. Can accommodate data with masses ranging from $1.1 \, M_\odot$ to $1.7 \, M_\odot$.
Looking to the future

• Neutron-star phenomenology of color superconducting quark matter:
  – suprathermal bulk viscosity of quark matter phases \((r\text{-mode})\)
  – detailed analysis of \(r\)-mode profiles in hybrid star
  – heat capacity, conductivity and emissivity (neutrino cooling)
  – structure: nuclear-quark interface (gravitational waves?)
  – crystalline phase (glitches) (gravitational waves?)
  – **CFL**: vortices but no flux tubes

• More general questions:
  – magnetic instability of gapless phases
  – better weak-coupling calculations, include vertex corrections
  – go beyond mean-field, include fluctuations
  – solve the sign problem and do lattice QCD at high density.