Bouncing Black Holes
Exercises in Dynamic Excision

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Thanks

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Overview

- Motivation / Perspective
- Methods for handing singularities
- Excision in general
- Brief History
- Excision - the Maya way
- Test Problem
- Results
Some perspective

The cover of *Science*
10 November 1995
The future?

The cover of *Science*  
27 November 2005
One component of evolution

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  - Optimized, parallel 3D FD Cauchy code
  - Your favorite formulation of equations
  - Suitable, non-pathological gauge
  - Consistent outer boundaries
  - Astrophysically relevant initial data
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  - Optimized, parallel 3D FD Cauchy code
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- **The spacetime still contains physical singularities that must be treated in some way**
Singularity-handling methods

- Singularity-avoiding slicing conditions
- Puncture methods
- Inversion symmetry
- Strange matter
- Excision
Excision in a nutshell

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- If we must impose boundary conditions:
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- Thus, we should be able to get away with murder inside the apparent horizon

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- If we must impose boundary conditions:
  - Just insist that information goes into the hole
A brief history

1987: Thornburg

- Suggests AH boundary condition
- Proposes “exclusion” in context of ID
- Based on communication w/ Unruh
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1992: Seidel & Suen
- “Singularity-proof” numerical schemes
- Introduce horizon-locking coordinates
- Introduce causal differencing
- Causal differencing quite successful in 1D
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Success in 3D:
- 1998: BBH Grand Challenge Head-on collision
- 2000: Pitt-PSU-Texas Grazing Collision (Agave)
2001: Alcubierre & Brügmann

- No causal differencing
  - Centered differencing w/ upwind on advection ($\beta^i \partial_i$)
- Extrapolation B.C. (zeroth-order r.h.s. copy)
- Cubical excision
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2001: Yo, Baumgarte, & Shapiro
- Extended to higher-order extrapolation
- Handled motion
Excision in Maya

- Carry around an extra grid variable - excision mask
- Initially, label everything as being part of computational domain (computational)
- Select an excision shape, label points inside excised
- Relabel excised points that have computational points as neighbors excised boundary
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  - Fits on a cartesian grid
  - Approximates a spheroid
  - Efficient to compute
  - Preserves symmetry of problem (e.g. octant, quadrant, bitant)
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Look to high school geometry!
The LEGO™ sphere

http://www.google.com/search?q=lego+sphere

Credit: http://www.brillig.com/lego/sphere
Excision shapes in Maya
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At each boundary point,
At the Excision Boundary

- At each boundary point,
  - Find the normal to the ideal surface
  - Find the closest approximate normal among neighbors
  - Record the grid locations \((i, j, k)\) of each point along the approximate normal
  - Record weighting coefficients \(a_n\)
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- *This suggests a data structure containing the above info.*

- *Then we have the infrastructure to later perform 1D extrapolation onto excision boundary:*

\[
f(i, j, k) = \sum_{n=1}^{N} a_n f(i_n, j_n, k_n)
\]
An example of extrapolation

\[ u^{n+1} \]

\[ u^n \]
An example of extrapolation

\[ u^{n+1} \]

\[ \rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6 \]

\[ u^n \]
An example of extrapolation
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An example of extrapolation
Populate

\[ u^{n+1} \]

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Shifted Kerr-Schild coordinates
The testbed

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KS data with no spin (IEF)
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- **Make a time-dependent spatial translation**
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\[
\begin{align*}
g_{ij} & = \delta_{ij} + \frac{2M \bar{x}_i \bar{x}_j}{\bar{r} \bar{r} \bar{r}} \\
K_{ij} & = \frac{2M}{\bar{r}^2} \left(1 + \frac{2M}{\bar{r}}\right)^{-1/2} \left[ \delta_{ij} - \frac{2\bar{r} + M \bar{x}_i \bar{x}_j}{\bar{r} \bar{r} \bar{r}} \right] \\
\alpha & = \left(1 + \frac{2M}{\bar{r}}\right)^{-1/2} \\
\beta^i & = \frac{2M}{\bar{r}} \left(1 + \frac{2M}{\bar{r}}\right)^{-1} \frac{\bar{x}^i}{\bar{r}} - \dot{x}^i_{BH}
\end{align*}
\]
Simulations

- **BSSN evolution**
- **Analytic lapse and shift** \((\alpha(t) \& \beta(t))\)
- **Excision radius:** \(1.5M\)
- **Extrapolation:** 3rd order soln
- **Outer boundary:** Analytic

- **dx^i = 0.20M**
- **dt = 0.05M**
- **Bitant symmetry**

- **Domain:**
  \(\pm10M \text{ in } x \text{ and } y,\)
  \(+7M \text{ in } z\)
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- 120\(M < T_{\text{crash}} < 140M\)
Results