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Quantum Space-times

Beyond the Continuum of Minkowski and Einstein

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Summary. In general relativity space-time ends at singularities. The big bang is considered as the Beginning and the big crunch, the End. However these conclusions are arrived at by using general relativity in regimes which lie well beyond its physical domain of validity. Examples where detailed analysis is possible show that these singularities are naturally resolved by quantum geometry effects. Quantum space-times can be vastly larger than what Einstein had us believe. These non-trivial space-time extensions enable us to answer of some long standing questions and resolve of some puzzles in fundamental physics. Thus, a century after Minkowski's revolutionary ideas on the nature of space and time, yet another paradigm shift appears to await us in the wings.

1.1 Introduction

A hundred years ago Hermann Minkowski fused space and time into a smooth 4-dimensional continuum. Remarkably, this continuum —the Minkowski space-time— still serves as the arena for all non-gravitational interactions both in classical *and* quantum physics. Time is no more absolute. Whereas in Newtonian physics there is a unique 3-plane through each space-time point representing space, now there is a unique cone, spanned by light rays passing through that point. The constant time plane curls up into a 2 sheeted cone that separates the region which is causally connected with the point from the region which is not. This causality dictates the propagation of physical fields in classical physics, and the commutation relations and uncertainty relations between field operators in quantum physics. With the demise of absolute simultaneity, Newtonian ideas are shattered. The world view of physics is dramatically altered.

However, as in Newtonian physics, there is still a fixed space-time which serves as the arena for all of physics. It is the stage on which the drama of evolution unfolds. Actors are particles and fields. The stage constrains what the actors can do. The Minkowski metric dictates the field equations and restricts the forms of interaction terms in the action. But the actors cannot influence the stage; Minkowskian geometry is immune from change. To incorporate the gravitational force, however, we had to abandon this cherished paradigm. We follow Einstein and encode gravity in the very geometry of space-time. Matter curves space-time. The space-time metric is no longer fixed. There is again a dramatic paradigm shift. However, we continue to retain one basic feature of Newtonian and Minkowskian frameworks: space-time is still represented by a smooth continuum.

This is not uncommon: New paradigms are often created by abandoning one key feature of the older paradigm but retaining another. But global coherence of the description of Nature is a huge burden and such a strategy often leads to new tensions. For example, to achieve compatibility between mechanics and Maxwellian electrodynamics, Einstein abandoned absolute simultaneity but retained the idea that space and time are fixed, unaffected by matter. The strategy worked brilliantly. Not only was the new mechanics compatible with Maxwell's theory but it led to deep, unforeseen insights. Energy and mass are simply two facets of the same physical attribute, related by $E = mc^2$; electric and magnetic fields $\mathbf{E}, \mathbf{B}$ are but two projections of an electromagnetic field tensor F_{ab}; in a quantum theory of charged particles, each particle must be accompanied by an anti-particle with opposite charge. However, the new mechanics flatly contradicted basic tenets of Newton's theory of gravitation. To restore coherence of physics, one has to abandon the idea that space-time is fixed, immune to change. One had to encode gravity into the very geometry of space-time, thereby making this geometry dynamical.

Now the situation is similar with general relativity itself. Einstein abandoned the tenet that geometry is inert and made it a physical entity that interacts with matter. This deep paradigm shift again leads to unforeseen consequences that are even more profound. Thanks to this encoding, general relativity predicts that the universe began with a big bang; that heavy stars end their lives through a gravitational collapse to a black hole; that ripples in the space-time curvature propagate as gravitational waves carrying energy-momentum. However, general relativity continues to retain the Newtonian and Minkowskian premise that space-time is a smooth continuum. As a consequence, new tensions arise.

In Newtonian or Minkowskian physics, a given physical field could become singular at a space-time point. This generally implied that the field could not be unambiguously evolved to the future of that point. However, this singularity had no effect on the global arena. Since the space-time geometry is unaffected by matter, it remains intact. Other fields could be evolved indefinitely. Trouble was limited to the one field which became ill behaved. However, because gravity is geometry in general relativity, when the gravitational field becomes singular, the continuum tares and the space-time itself ends. There is no more an arena for other fields to live in. All of physics, as we know it, comes to an abrupt halt. Physical observables associated with both matter and geometry simply diverge signalling a fundamental flaw in our description of Nature. This is the new quandary.

When faced with deep quandaries, one has to carefully analyze the reasoning that led to the impasse. Typically the reasoning is flawed, possibly for subtle reasons. In the present case the culprit is the premise that general relativity —with its representation of space-time as a smooth continuum— provides an accurate description of Nature arbitrarily close to the singularity. For, general relativity completely ignores quantum effects and, over the last century, we have learned that these effects become important in the physics of the small. They should in fact be *dominant* in parts of the universe where matter densities become enormous. Thus there is no reason to trust the predictions of general relativity near space-time singularities. Classical physics of general relativity does come to a halt at the big-bang and the big crunch. But this is not an indication of what *really* happens because use of general relativity near singularities is an extrapolation which has no physical justification whatsoever. We need a theory that incorporates not only the dynamical nature of geometry but also the ramifications of quantum physics. We need a quantum theory of gravity, a new paradigm.

These considerations suggest that singularities of general relativity are perhaps the most promising gates to physics beyond Einstein. They provide a fertile conceptual and technical ground in our search of the new paradigm. Consider some of the deepest conceptual questions we face today: the issue of the Beginning and the end End; the arrow of time; and the puzzle of black hole information loss. Their resolutions hinge on the true nature of singularities. In my view, considerable amount of contemporary confusion about such questions arises from our explicit or implicit insistence that singularities of general relativity are true boundaries of space-time; that we can trust causal structure all the way to these singularities; that notions such as event horizons are absolute even though changes in the metric in a Planck scale neighborhood of the singularity can move event horizons dramatically or even make them disappear altogether [1].

Over the last 2-3 years several classically singular space-times have been investigated in detail through the lens of loop quantum gravity (LQG) [2, 3, 4]. This is a non-perturbative approach to the unification of general relativity and quantum physics in which one takes Einstein's encoding of gravity into geometry seriously and elevates it to the quantum level. One is thus led to build quantum gravity using *quantum* Riemannian geometry [5, 6, 7, 8]. Both geometry and matter are *dynamical* and described *quantum mechanically* from the start. In particular, then, there is no background space-time. The kinematical structure of the theory has been firmly established for some years now. There are also several interesting and concrete proposals for dynamics (see, in particular [2, 3, 4, 9]). However, in my view there is still considerable ambiguity and none of the proposals is fully satisfactory. Nonetheless, over the last 2-3 years, considerable progress could be made by restricting oneself to subcases where detailed and explicit analysis is possible [10, 11, 12, 13, 14, 15]. These 'mini' and 'midi' superspaces are well adapted to analyze the deep conceptual tensions discussed above. For, they consider the most interesting of classically singular space-times —Friedman-Robertson-Walker (FRW) universes with the big bang singularity and black holes with the Schwarzschild-type singularity— and analyze them in detail using symmetry reduced versions of loop quantum gravity. In all cases studied so far, classical singularities are naturally resolved and *the quantum space-time is vastly larger than what general relativity had us believe.* As a result, there is a new paradigm to analyze the old questions.

The purpose of this article is to summarize these developments, emphasizing the conceptual aspects[1] from an angle that, I hope, will interest not only physicists but especially philosophers and historians of science. We will see that some of the long standing questions can be directly answered, some lose their force in the new paradigm while others have to be rephrased.

This chapter is organized as follows. In section 1.2 I will discuss cosmological singularities and in 1.3 the black hole singularities. In each case I will discuss examples of fundamental open issues and explain their status in the corresponding models. We will see that quantum geometry has unexpected ramifications that either resolve or significantly alter the status of these issues. Finally in section 1.4 I will summarize the outlook and discuss some of the fresh challenges that the new paradigm creates.

1.2 Quantum Nature of the Big Bang

1.2.1 Issue of the Beginning and the End

Over the history of mankind, cosmological paradigms have evolved in interesting ways. It is illuminating to begin with a long range historical perspective by recalling paradigms that seemed obvious and most natural for centuries only to be superseded by radical shifts.

Treatise on Time, the Beginning and the End date back at least twenty five centuries. Does the flow of time have an objective, universal meaning beyond human perception? Or, is it fundamentally only a convenient, and perhaps merely psychological, notion? Did the physical universe have a finite beginning or has it been evolving eternally? Leading thinkers across cultures meditated on these issues and arrived at definite but strikingly different answers. For example, in the sixth century BCE, Gautama Buddha taught that 'a period of time' is a purely conventional notion, time and space exist only in relation to our experience, and the universe is eternal. In the Christian thought, by contrast, the universe had a finite beginning and there was debate whether time represents 'movement' of bodies or if it flows only in the soul. In the fourth century CE, St. Augustine held that time itself started with the world.

Founding fathers of modern Science from Galileo to Newton continued to accept that God created the universe. Nonetheless, their work led to a

[1] Thus I will not include any derivations but instead provide references where the details can be found.

radical change of paradigm. Before Newton, boundaries between the absolute and the relative, the true and the apparent and the mathematical and the common were blurry. Newton rescued time from the psychological *and* the material world and made it objective and absolute. It now ran uniformly from the infinite past to the infinite future. This paradigm became a dogma over centuries. Philosophers often used it to argue that the universe itself *had* to be eternal. For, as Immanuel Kant emphasized, otherwise one could ask "what was there before?"

General relativity toppled this Newtonian paradigm in one fell swoop. Now the gravitational field is encoded in space-time geometry. Since geometry is a dynamical, physical entity, it is now perfectly feasible for the universe to have had a finite beginning —the big-bang— at which not only matter but *space-time itself* is born. If space is compact, matter *as well as space-time* end in the big-crunch singularity. In this respect, general relativity took us back to St. Augustine's paradigm but in a detailed, specific and mathematically precise form. In semi-popular articles and radio shows, relativists now like to emphasize that the question "what was there before?" is rendered meaningless because the notions of 'before' requires a pre-existing space-time geometry. We now have a new paradigm, a new dogma: In the Beginning there was the Big Bang.

But as I pointed out in section 1.1, general relativity is incomplete and there is no reason to trust its predictions near space-time singularities. We must fuse it with quantum physics and let the new theory tell us what happens when matter and geometry enter the Planck regime.

1.2.2 Some key questions

If the smooth continuum of Minkowski and Einstein is only an approximation, on the issue of the origin of the universe we are now led to ask:

- How close to the big-bang does a smooth space-time of general relativity make sense? Inflationary scenarios, for example, are based on a space-time continuum. Can one show from some first principles that this is a safe approximation already at the onset of inflation?
- Is the big-bang singularity naturally resolved by quantum gravity? This possibility led to the development of the field of quantum cosmology in the late 1960s. The basic idea can be illustrated using an analogy to the theory of the hydrogen atom. In classical electrodynamics the ground state energy of this system is unbounded below. Quantum physics intervenes and, thanks to a non-zero Planck's constant, the ground state energy is lifted to a finite value, $-me^4/2\hbar^2 \approx -13.6\text{eV}$. Since it is the Heisenberg uncertainly principle that lies at the heart of this resolution and since the principle must feature also in quantum gravity, one is led to ask: Can a similar mechanism resolve the big-bang and big crunch singularities of general relativity?

- Is a new principle/ boundary condition at the big bang or the big crunch essential? The most well known example of such a boundary condition is the 'no boundary proposal' of Hartle and Hawking [16]. Or, do quantum Einstein equations suffice by themselves even at the classical singularities?
- Do quantum dynamical equations remain well-behaved even at these singularities? If so, do they continue to provide a deterministic evolution? The idea that there was a pre-big-bang branch to our universe has been advocated in several approaches, most notably by the pre-big-bang scenario in string theory [17] and ekpyrotic and cyclic models [18, 19] inspired by the brane world ideas. However, these are perturbative treatments which require a smooth continuum in the background. Therefore, their dynamical equations break down at the singularity whence, without additional input, the pre-big-bang branch is not joined to the current post-big-bang branch by a deterministic evolution. Can one improve on this situation?
- If there is a deterministic evolution, what is on the 'other side'? Is there just a quantum foam from which the current post-big-bang branch is born, say a 'Planck time after the putative big-bang'? Or, was there another classical universe as in the pre-big-bang and cyclic scenarios, joined to ours by deterministic equations?

Clearly, to answer such questions we cannot start by assuming that there is a smooth space-time in the background. But already in the classical theory, it took physicists several decades to truly appreciate the dynamical nature of geometry and to learn to do physics without recourse to a background. In quantum gravity, this issue becomes even more vexing.[2]

For simple systems, including Minkowskian field theories, the Hamiltonian formulation generally serves as the royal road to quantum theory. It was therefore adopted for quantum gravity by Dirac, Bergmann, Wheeler and others. But absence of a background metric implies that the Hamiltonian dynamics is generated by constraints [21]. In the quantum theory, physical states are solutions to quantum constraints. All of physics, including the dynamical content of the theory, has to be extracted from these solutions. But there is no external time to phrase questions about evolution. Therefore we are led to ask:

- Can we extract, from the arguments of the wave function, one variable which can serve as *emergent time* with respect to which the other arguments 'evolve'? If not, how does one interpret the framework? What are the physical (i.e., Dirac) observables? In a pioneering work, DeWitt proposed that the determinant of the 3-metric can be used as an 'internal' time [22]. Consequently, in much of the literature on the Wheeler-DeWitt (WDW) approach to quantum cosmology, the scale factor is assumed to play the

[2] There is a significant body of literature on issue; see e.g., [20] and references therein. These difficulties are now being discussed also in the string theory literature in the context of the AdS/CFT conjecture.

role of time, although sometimes only implicitly. However, in closed models the scale factor fails to be monotonic due to classical recollapse and cannot serve as a global time variable already in the classical theory. Are there better alternatives at least in the simple setting of quantum cosmology? If not, can we still make physical predictions?

Finally there is an ultraviolet-infrared tension.

- Can one construct a framework that cures the short-distance difficulties faced by the classical theory near singularities, while maintaining an agreement with it at large scales?

By their very construction, perturbative and effective descriptions have no problem with the second requirement. However, physically their implications can not be trusted at the Planck scale and mathematically they generally fail to provide a deterministic evolution across the putative singularity. Since the non-perturbative approaches often start from deeper ideas, it is conceivable that they could lead to new structures at the Planck scale which modify the classical dynamics and resolve the big-bang singularity. But once unleashed, do these new quantum effects naturally 'turn-off' sufficiently fast, away from the Planck regime? The universe has had some *14 billion years* to evolve since the putative big bang and even minutest quantum corrections could accumulate over this huge time period leading to observable departures from dynamics predicted by general relativity. Thus, the challenge to quantum gravity theories is to first create huge quantum effects that are capable of overwhelming the extreme gravitational attraction produced by matter densities of some 10^{105} gms/cc near the big bang, and then switching them off with extreme rapidity as the matter density falls below this Planck scale. This is a huge burden!

These questions are not new; some of them were posed already in the late sixties by quantum gravity pioneers such as Peter Bergmann, Bryce DeWitt, Charles Misner and John Wheeler [21, 22, 23]. However, the field reached an impasse in the late eighties. Fortunately, this status-quo changed significantly over the last decade with a dramatic inflow of new ideas from many directions. In the next two subsections, I will summarize the current status of these issues in loop quantum cosmology.

1.2.3 FRW models and the WDW theory

Almost all phenomenological work in cosmology is based on the k=0 homogeneous and isotropic Friedmann Robertson Walker (FRW) space-times and perturbations thereof. For concreteness, I will focus on FRW model in which the only matter source is a scalar massless field.[3] I will consider k=0 (or

[3] Our discussion will make it clear that it is relatively straightforward to allow additional fields, possibly with complicated potentials.

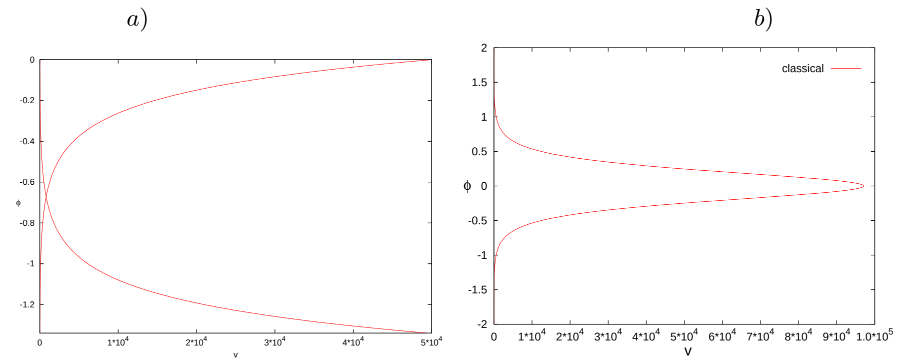

Fig. 1.1. *a*) Classical solutions in k=0, $\Lambda = 0$ FRW models with a massless scalar field. Since $p_{(\phi)}$ is a constant of motion, a classical trajectory can be plotted in the v-ϕ plane, where v is the volume (essentially in Planck units). There are two classes of trajectories. In one the universe begins with a big-bang and expands and in the other it contracts into a big crunch. *b*) Classical solutions in the k=1, $\Lambda = 0$ FRW model with a massless scalar field. The universe begins with a big bang, expands to a maximum volume and then undergoes a recollapse to a big crunch singularity. Since the volume is double valued in any solution, it cannot serve as a global time coordinate in this case. The scalar field on the other hand does so both in the k=0 and k=1 cases.

spatially flat) as well as k=1 (spatially closed) models with or without a cosmological constant (of either sign). Conceptually, these models are interesting for our purpose because *every* of their classical solutions has a singularity (see Fig 1.1). Therefore a natural singularity resolution without external inputs is highly non-trivial. In light of the spectacular observational inputs over the past decade, the k=0 model is the one that is phenomenologically most relevant. However as we will see, because of its classical recollapse, the k=1 model offers a more stringent viability test for the quantum cosmology.

In the classical theory, one considers one space-time at a time and although the metric of that space-time is dynamical, it enables one to introduce time coordinates that have direct physical significance. However in the quantum theory —and indeed already in the phase space framework that serves as the stepping stone to quantum theory— we have to consider all possible homogeneous, isotropic space-times. In this setting one can introduce a natural foliation of the 4-manifold each leaf of which serves as the 'home' to a spatially homogeneous 3-geometry. However, unlike in non-gravitational theories, there is no preferred physical *time variable* to define evolution. A natural strategy is to use part of the system as an 'internal' clock with respect to which the rest of the system evolves. This leads one to Leibnitz's *relational time*. Now, in any spatially homogeneous model with a massless scalar field ϕ, the conjugate momentum $p_{(\phi)}$ is a constant of motion, whence ϕ is monotonic along any dynamical trajectory. Thus, in the classical theory, it serves as a global clock (see Fig 1.1). Questions about evolution can thus be phrased as: "If the curvature or matter density or an anisotropy parameter is such and such when $\phi = \phi_1$ what is it when $\phi = \phi_2$?" What is the situation in the quantum theory? There is no a priori guarantee that a variable which serves as a viable time parameter in the classical theory will continue to do so in the quantum theory. Whether it does so depends on the form of the Hamiltonian constraint. For instance as Fig 1.1a shows, in the k=0 model without a cosmological constant, volume (or the scale factor) is a global clock along any classical trajectory. But the form of the quantum Hamiltonian constraint [24] in loop quantum gravity is such that it does not serve this role in the quantum theory. The scalar field, on the other hand, continues to do so (also in the k=1 case and with or without a cosmological constant).[4]

Because of the assumption of spatial homogeneity, in quantum cosmology one has only a finite number of degrees of freedom. Therefore, although the conceptual problems of quantum gravity remain, there are no field theoretical infinities and one can hope to mimic ordinary text book quantum mechanics to pass to quantum theory.

However, in the k=0 case, because space is infinite, homogeneity implies that the action, the symplectic structure and Hamiltonians all diverge since they are represented as integrals over all of space. Therefore, in any approach to quantum cosmology —irrespective of whether it is based on path integrals or canonical methods— one has to introduce an elementary cell $\mathcal{C}$ and restrict all integrals to it. In actual calculations, it is generally convenient also to introduce a fiducial 3-metric ${}^{o}q_{ab}$ (as well as frames ${}^{o}e_i^a$ adapted to the spatial isometries) and represent the physical metric q_{ab} via a scale factor a, $\;q_{ab} = a^2\, {}^{o}q_{ab}$. Then the geometrical dynamical variable can be taken to be either a, or the oriented volume v of the fiducial cell $\mathcal{C}$ as measured by the physical frame e_i^a, where v is positive if e_i^a has the same orientation as ${}^{o}e_i^a$ and negative if the orientations are opposite. (In either case the physical volume of the cell is $|v|$.) In this chapter I will use v rather than the scale factor. Note, however, physical results cannot depend on the choice of the fiducial $\mathcal{C}$ or ${}^{o}q_{ab}$.[5] In the k=1 case, since space is compact, a fiducial cell is unnecessary and the dynamical variable v is then just the physical volume of the universe.

[4] If there is no massless scalar field, one could still use a suitable matter field as a 'local' internal clock. For instance in the inflationary scenario, because of the presence of the potential the inflaton is not monotonic even along classical trajectories. But it is possible to divide dynamics into 'epochs' and use the inflaton as a clock locally, i.e., within each epoch [25]. There is considerable literature on the issue of internal time for model constrained systems [20] (such as a system of two harmonic oscillators where the total energy is constrained to be constant [26]).

[5] This may appear as an obvious requirement but unfortunately it is often overlooked in the literature. The claimed physical results often depend on the choice of $\mathcal{C}$ and/or ${}^{o}q_{ab}$ although the dependence is often hidden by setting the volume v_o of $\mathcal{C}$ with respect to ${}^{o}q_{ab}$ to 1 (in unspecified units) in the classical theory.

With this caveat out of the way, one can proceed with quantization. Situation in the WDW theory can be summarized as follows. This theory emerged in the late sixties and was analyzed extensively over the next decade and a half [21]. Many of the key physical ideas of quantum cosmology were introduced during this period [22, 23] and a number of models were analyzed. However, since a mathematically coherent approach to quantization of full general relativity did not exist, there were no guiding principles for the analysis of these simpler, symmetry reduced systems. Rather, quantization was carried out following 'obvious' methods from ordinary quantum mechanics. Thus, in quantum kinematics, states were represented by square integrable wave functions $\Psi(v, \phi)$, where v represents geometry and ϕ, matter; and operators $\hat{v}, \hat{\phi}$ acted by multiplication and their conjugate momenta by ($-i\hbar$ times) differentiation. With these choices The Hamiltonian constraint takes the form of a differential equation that must be satisfied by the physical states[27]:

$$\partial_\phi^2 \underline{\Psi}(v,\phi) = \underline{\Theta}_o \underline{\Psi}(v,\phi) \;:=\; -12\pi G \,(v\partial_v)^2\, \Psi(v,\phi) \tag{1.1}$$

for k=0, and

$$\partial_\phi^2 \underline{\Psi}(v,\phi) = -\underline{\Theta}_1 \underline{\Psi}(v,\phi) \;:=\; -\underline{\Theta}_o \underline{\Psi}(v,\phi) - G\,C\,|v|^{\frac{4}{3}}\, \underline{\Psi}(v,\phi)\;, \tag{1.2}$$

for k=1, where C is a numerical constant. *In what follows* $\underline{\Theta}$ *will stand for either* $\underline{\Theta}_o$ *or* $\underline{\Theta}_1$. In the older literature, the emphasis was on finding and interpreting the WKB solutions of these equations (see, e.g., [28]). However, near the singularity, the WKB approximation fails and we need an exact quantum theory.

The exact theory can be readily constructed [24, 27]. Note first that the form of (1.1) and (1.2) is the same as that of a Klein-Gordon equation in a 2-dimensional static space-time (with a ϕ-independent potential in the k=1 case), where ϕ plays the role of time and v of space. This suggests that we think of ϕ as the relational time variable with respect to which v, the 'true' degree of freedom, evolves. A systematic procedure based on the so-called group averaging method [29] (which is applicable for a very large class of constrained systems) then leads us to the physical inner product between these states. Not surprisingly it coincides with the expression from the Klein-Gordon theory in static space-times.

The physical sector of the final theory can be summarized as follows. The physical Hilbert space $\mathcal{H}_{\rm phy}$ in the k=0 and k=1 cases consists of 'positive frequency' solutions to (1.1) and (1.2) respectively. A complete set of observables is provided by the momentum $\hat{p}_{(\phi)}$ and the relational observable $|\hat{v}||_\phi$ representing the volume at the 'instant of time ϕ':

$$\hat{p}_{(\phi)} = -i\hbar\partial_\phi \quad \text{and} \quad \hat{V}|_\phi = e^{i\underline{\Theta}\,(\phi-\phi_o)}\,|v|\,e^{-i\underline{\Theta}\,(\phi-\phi_o)} \tag{1.3}$$

There are *Dirac* observables because their action preserves the space of solutions to the constraints and are self-adjoint on the physical Hilbert space *a*) *b*)

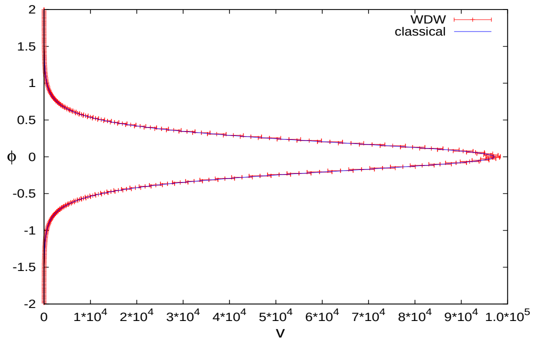

Fig. 1.2. Expectation values (and dispersions) of $|\hat{v}|_\phi$ for the WDW wave function in the k=1 model. The WDW wave function follows the classical trajectory into the big-bang and big-crunch singularities. (In this simulation, the parameters were: $p_\phi^\star = 5000$, and $\Delta p_\phi / p_\phi^\star = 0.02$.)

$\mathcal{H}_{\rm phy}$. With the exact quantum theory at hand, we can ask if the singularities are naturally resolved. More precisely, from $\hat{p}_{(\phi)}$ and $\hat{V}|_\phi$ we can construct observables corresponding to matter density $\hat{\rho}$ (or space-time scalar curvature $\hat{R}$). Since the singularity is characterized by divergence of these quantities in the classical theory, in the quantum theory we can proceed as follows. We can select a point (v_o, ϕ_o) at a 'late time' ϕ_o on a classical trajectory of Fig 1.1 —e.g., now, in he history of our universe— when the density and curvature are *very* low compared to the Planck scale, and construct a semi-classical state which is sharply peaked at v_o at $\phi = \phi_o$. We can then evolve this state *backward* in time. Does it follow the classical trajectory? To have the correct 'infra-red' behavior, it must, until the density and curvature become very high. What happens in this 'ultra-violet' regime? Does the quantum state remain semi-classical and follow the classical trajectory into the big bang? Or, does it spread out making quantum fluctuations so large that although the quantum evolution does not break down, there is no reasonable notion of classical geometry? Or, does it remain peaked on some trajectory which however is so different from the classical one that, in this backward evolution, the the universe 'bounces' rather than being crushed into the singularity? Or, does it ... Each of these scenarios provides a distinct prediction for the ultra-violet behavior and therefore for physics in the deep Planck regime.[6]

[6] Sometimes apparently weaker notions of singularity resolution are discussed. Consider two examples [30]. One may be able to show that the wave function vanishes at points of the classically singular regions of the configuration space. However, if the *physical* inner product is non-local in this configuration space —as the group averaging procedure often implies— such a behavior of the wave function would not imply that the probability of finding the universe at these configura-

It turns out that the WDW theory leads to similar predictions in both k=0 and k=1 cases [24, 27, 31]. They pass the infra-red tests with flying colors (see Fig 1.2). But unfortunately the state follows the classical trajectory into the big bang (and in the k=1 case also the big crunch) singularity. Thus the first of the possibilities listed above is realized. The singularity is not resolved because expectation values of density and curvature continue to diverge in epochs when their classical counterparts do. The analogy to the hydrogen atom discussed in section 1.2.2 fails to be realized.

1.2.4 Loop quantum cosmology: New quantum mechanics

For a number of years, the failure of the WDW theory to naturally resolve the big bang singularity was taken to mean that quantum cosmology cannot, by itself, shed any light on the quantum nature of the big bang. Indeed, for systems with a finite number of degrees of freedom we have the von Neumann uniqueness theorem which guarantees that quantum kinematics is unique. The only freedom we have is in factor ordering and this was deemed insufficient to alter the status-quo provided by the WDW theory.

The situation changed dramatically in LQG. Here, a well established, rigorous kinematical framework *is* available for full general relativity [5, 2, 3, 4]. If one mimics it in symmetry reduced models, one is led to a quantum theory which is *inequivalent to that of the WDW theory already at the kinematic level.* Quantum dynamics built in this new arena agrees with the WDW theory in 'tame' situations but differs dramatically in the Planck regime, leading to a natural resolution of the big bang singularity.

But what about the von Neumann uniqueness theorem? The theorem states that 1-parameter groups $U(\lambda)$ and $V(\mu)$ satisfying the Weyl commutation relations[7] admit (up to isomorphism) a unique irreducible representation by unitary operators on a Hilbert space $\mathcal{H}$ in which $U(\lambda)$ and $V(\mu)$ are weakly continuous in the parameters λ and μ. By Stone's theorem, weak continuity is the necessary and sufficient condition for $\mathcal{H}$ to admit self adjoint operators $\hat{x}, \hat{p}$ such that $U(\lambda) = e^{i\lambda\hat{x}}$ and $V(\mu) = e^{i\mu\hat{p}}$. Therefore assumption of the von Neumann theorem are natural in non-relativistic quantum mechanics and we are led to a unique quantum kinematics. However, in full loop quantum gravity, x is analogous to the gravitational connection and $U(\lambda)$ to its holonomy. One can again construct an abstract algebra using holonomies and operators conjugate to connections and ask for its representations satisfying

tions is zero. The second example is that the wave function may become highly non-classical. This by itself would not mean that the singularity is avoided unless one can show that the expectation values of a family of Dirac observables which become classically singular remain finite there.

[7] These are: $U(\lambda V(\lambda) = e^{i\lambda\mu}V(\mu)U(\lambda)$ and can be obtained by setting $U(\lambda) = e^{i\lambda\hat{x}}$ and $V(\mu) = e^{i\mu\hat{p}}$ in the standard Schrödinger theory. Given a representation $U(\lambda)$ is said to be *weakly continuous* in λ if its matrix elements between any two fixed quantum states are continuous in λ.

natural assumptions the most important of which is the diffeomorphism invariance dictated by background independence. There is again a uniqueness theorem [32]. However, in the representation that is thus singled out, holonomy operators —analogs of $U(\lambda)$— fails to be weakly continuous whence there are no operators corresponding to connections! Furthermore, a number of key features of the theory —such as the emergence of a quantum Riemannian geometry in which there is fundamental discreteness— can be traced back to this unforeseen feature. Therefore, upon symmetry reduction, although we have a finite number of degrees of freedom, it would be incorrect to just mimic Schröddinger quantum mechanics and impose weak continuity. When this assumption is dropped, the von Neumann theorem is no longer applicable and *we have new quantum mechanics* [33].

Thus, the key difference between LQC and the WDW theory lies in the fact that while one does not have reliable quantum kinematics in the WDW theory, there is a well developed and rigorous framework in LQG which, furthermore, is *unique*! If we mimic it as closely as possible in the symmetry reduced theories, we are led to a new kinematic arena, distinct from the one used in the WDW quantum cosmology. LQC is based on this arena.

1.2.5 LQC: Dynamics

It turns out WDW dynamics is not supported by the new arena because, when translated in terms of gravitational connections and their conjugate momenta, it requires that there be an operator corresponding to the connection itself. Therefore one has to develop quantum dynamics ab-initio on the new arena. The result is that the differential operator $\underline{\Theta}_o = -12\pi G\,(v\partial_v)^2$ in Eqs (1.1) and (1.2) is now replaced by a second order *difference* operator in v, where the step size is dictated by the 'area gap' of LQG, i.e., the lowest non-zero eigenvalue of the area operator in LQG. There is a precise sense in which the Wheeler-DeWitt equations result as the limits of LQC equations when the area gap is taken to zero, i.e., when the Planck scale discreteness of quantum geometry determined by LQG is neglected. We will now see that this discreteness is completely negligible at late times but plays a crucial role in the Planck scale geometry near singularities.

The LQC dynamics has been analyzed using three different methods.

- Numerical solutions of the exact quantum equations [34, 24, 27, 31]. A great deal of effort was spent in ensuring that the results are free of artifacts of simulations, do not depend on the details of how semi-classical states are constructed and hold for a wide range of parameters.

- Effective equations [35, 27, 31]. These are differential equations which include the leading quantum corrections. The asymptotic series from which these contributions were picked was constructed rigorously but is based on assumptions whose validity has not been established. Nonetheless the effective equations approximate the exact numerical evolution of semi-classical states extremely well.

- Exactly soluble, but simplified model in the k=0 case [36, 37]. The simplification is well controlled [37]. This analysis has provided some results which provide an analytical understanding of numerical results and also several other results which are not restricted to states which are semi-classical at late times. In this sense the analysis shows that the overall picture is robust within these models.

I will provide a global picture that has emerged from these investigations, first for the k=1 model without the cosmological constant Λ and for the k=0 case for various values of Λ.

Recall that in classical general relativity, the k=1 closed universes start out with a big bang, expand to a maximum volume $V_{\max}$ and then recollapse to a big-crunch singularity. Consider a classical solution in which $V_{\max}$ is astronomically large —i.e., on which the constant of motion $p_{(\phi)}$ takes a large value $p^\star_{(\phi)}$— and consider a time ϕ_o at which the volume $v^\star$ of the universe is also large. Then there are well-defined procedures to construct states $\Psi(v, \phi)$ in the *physical Hilbert space* which are sharply peaked at these values of observables $\hat{p}_{(\phi)}$ and $\hat{V}_{\phi_o}$ at the 'time' ϕ_o. Thus, at 'time' ϕ_o, the quantum universe is well approximated by the classical one. What happens to such quantum states under evolution? As emphasized earlier, there are infra-red and ultra-violet challenges:
i) Does the state remain peaked on the classical trajectory in the low curvature regime? Or, do quantum geometry effects accumulate over the cosmological time scales, causing noticeable deviations from classical general relativity? In particular, is there a recollapse and if so does the value $V_{\max}$ of maximum volume agree with that predicted by general relativity [38]?
ii) What is the behavior of the quantum state in the Planck regime? Is the big-bang singularity resolved? What about the big-crunch? If they are both resolved, what is on the 'other side'?

Numerical simulations show that the wave functions do remain sharply peaked on classical trajectories in the low curvature region also in LQC. But there is a radical departure from the WDW results in the strong curvature region. The WDW evolution follows classical dynamics all the way into the big-bang and big crunch singularities (see Fig 1.2). In LQC, by contrast, *the big bang and the big crunch singularities are resolved and replaced by big-bounces* (see Fig 1.3). In these calculations, the required notion of semi-classicality turns out to be surprisingly weak: these results hold even for universes with $a_{\max} \approx 23\ell_{\rm Pl}$ and the 'sharply peaked' property improves greatly as $a_{\max}$ grows.

More precisely, numerical solutions have shown that the situation is as follows. (For details, see [31].)

- The trajectory defined by the expectation values of the physical observable $\hat{V}|_\phi$ in the full quantum theory is in good agreement with the trajectory

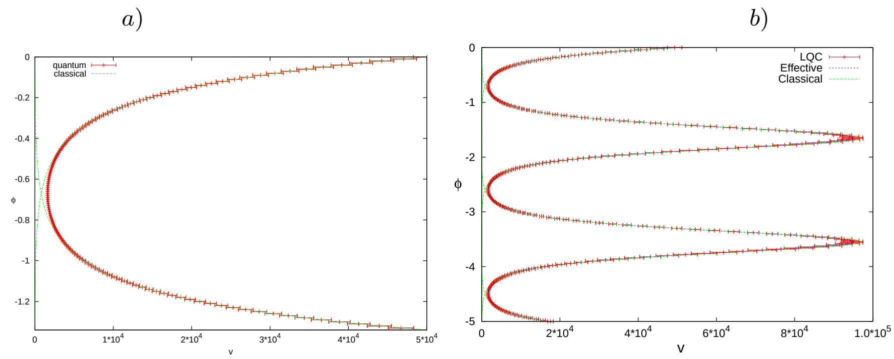

Fig. 1.3. In the LQC evolution of models under consideration, the big bang and big crunch singularities are replaced by quantum bounces. Expectation values and dispersion of $|\hat{v}|_\phi$, are compared with the classical trajectory and the trajectory from effective Friedmann dynamics. The classical trajectory deviates significantly from the quantum evolution at Planck scale and evolves into singularities. The effective trajectory provides an excellent approximation to quantum evolution at all scales.
a) The k=0 case. In the backward evolution, the quantum evolution follows our post big-bang branch at low densities and curvatures but undergoes a quantum bounce at matter density $\rho \sim 0.82\rho_{\rm PL}$ and joins on to the classical trajectory that was contracting to the future. *b*) The k=1 case. The quantum bounce occurs again at $\rho \sim 0.82\rho_{\rm Pl}$. Since the big bang and the big crunch singularities are resolved the evolution undergoes cycles. In this simulation $p^\star_{(\phi)} = 5 \times 10^3$, $\Delta p_{(\phi)}/p^\star_{(\phi)} = 0.018$, and $v^\star = 5 \times 10^4$.

defined by the classical Friedmann dynamics until the energy density ρ in the matter field is about two percent of the Planck density. In the classical solution, scalar curvature and the matter energy density keep increasing on further evolution, eventually leading to a big bang (respectively, big crunch) singularity in the backward (respectively, forward) evolution, where $v \to 0$. The situation is very different with quantum evolution. As the density and curvature increases further, quantum geometry effects become dominant creating an effective repulsive force which rises very quickly, overwhelms classical gravitational attraction, and causes a bounce at $\rho \sim 0.82\rho_{\rm Pl}$, thereby resolving the past (or the big bang) and future (or the big crunch) singularities. There is thus a cyclic scenario depicted in Fig 1.3.

- The volume of the universe takes its minimum value $V_{\rm min}$ at the bounce point. $V_{\rm min}$ scales linearly with $p_{(\phi)}$: [8]

$$V_{\rm min} = \Big(\frac{4\pi G\gamma^2 \Delta}{3}\Big)^{\frac{1}{2}}\, p_{(\phi)} \;\approx (1.28 \times 10^{-33}\,{\rm cm})\; p_{(\phi)} \tag{1.4}$$

[8] Here and in what follows, numerical values are given in the classical units $G = c = 1$. In these units $p_{(\phi)}$ has the same physical dimensions as $\hbar$ and the numerical value of $\hbar$ is $2.5 \times 10^{-66} {\rm cm}^2$.

Consequently, $V_{\rm min}$ can be *much* larger than the Planck size. Consider for example a quantum state describing a universe which attains a maximum radius of a megaparsec. Then the quantum bounce occurs when the volume reaches the value $V_{\rm min} \approx 5.7 \times 10^{16}\,{\rm cm}^3$, *some* 10^{115} *times the Planck volume.* Deviations from the classical behavior are triggered when the density or curvature reaches the Planck scale. The volume can be very large and is not the relevant scale for quantum gravity effects.

• After the quantum bounce the energy density of the universe decreases and the repulsive force dies quickly when matter density reduces to about two percept of the Planck density. The quantum evolution is then well-approximated by the classical trajectory. On subsequent evolution, the universe recollapses both in classical and quantum theory at the value $V = V_{\rm max}$ when energy density reaches a minimum value $\rho_{\rm min}$. $V_{\rm max}$ scales as the 3/2-power of $p_{(\phi)}$:

$$V_{\rm max} = (16\pi G/3\ell_o^2)^{3/4}\, p_{(\phi)}^{3/2} \;\approx\; 0.6\, p_{(\phi)}^{3/2} \tag{1.5}$$

Quantum corrections to the classical Friedmann formula $\rho_{\rm min} = 3/8\pi G a_{\rm max}^2$ are of the order $O(\ell_{\rm Pl}/a_{\rm max})^4$. For a universe with $a_{\rm max} = 23\ell_{\rm Pl}$, the correction is only one part in 10^5. For universes which grow to macroscopic sizes, classical general relativity is essentially exact near the recollapse.

• Using ideas from geometrical quantum mechanics [39], one can obtain certain effective classical equations which incorporate the leading quantum corrections [35, 31]. While the classical Friedmann equation is $(\dot a/a)^2 = (8\pi G/3)\,(\rho - 3/8\pi G a^2)$, the effective equation turns out to be

$$\left(\frac{\dot a}{a}\right)^2 = \tfrac{8\pi G}{3}\,(\rho - \rho_1(v))\;\left[f(v) - \frac{\rho}{\rho_{\rm crit}}\right] \tag{1.6}$$

where ρ_1 and f are specific functions of v with $\rho_1 \sim 3/8\pi G a^2$. Bounces occur when $\dot a$ vanishes, i.e. at the value of v at which the matter density equals $\rho_1(v)$ or $f(v) = \rho/\rho_{\rm crit}$. The first root $\rho(v) = \rho_1(v)$ corresponds to the classical recollapse while the second root, $f(v) = \rho/\rho_{\rm crit}$, to the quantum bounce. Away from the Planck regime, $f \approx 1$ and $\rho/\rho_{\rm crit} \approx 0$. Bounces occur when $\dot a$ vanishes, i.e. at the value of v at which the matter density equals $\rho_1(v)$ or $\rho_2(v)$.

• For quantum states under discussion, the density $\rho_{\rm max}$ is well approximated by $\rho_{\rm crit} \approx 0.82\rho_{\rm Pl}$ up to terms $O(\ell_{\rm Pl}^2/a_{\rm min}^2)$, independently of the details of the state and values of $p_{(\phi)}$. (For a universe with maximum radius of a megaparsec, $\ell_{\rm Pl}^2/a_{\rm min}^2 \approx 10^{-76}$.) The density $\rho_{\rm min}$ at the recollapse point also agrees with the value $(3/8\pi G a_{\rm max}^2)$ predicted by the classical evolution to terms of the order $O(\ell_{\rm Pl}^4/a_{\rm max}^4)$. Furthermore the scale factor $a_{\rm max}$ at which recollapse occurs in the quantum theory agrees to a very good precision with the one predicted by the classical dynamics.

• The trajectory obtained from effective Friedmann dynamics is in excellent agreement with quantum dynamics *throughout the evolution.* In particular, the maximum and the minimum energy densities predicted by the effective description agree with the corresponding expectation values of the density operator $\hat{\rho} \equiv \widehat{p_{(\phi)}^2/2|p|^3}$ computed numerically.

- The state remains sharply peaked for a *very large number of 'cycles'.* Consider the example of a semi-classical state with an almost equal relative dispersion in $p_{(\phi)}$ and $|v|_\phi$ and peaked at a large classical universe of the size of a megaparsec. When evolved, it remains sharply peaked with relative dispersion in $|v|_\phi$ of the order of 10^{-6} *even after* 10^{50} *cycles of contraction and expansion!* Any given quantum state eventually ceases to be sharply peaked in $|v|_\phi$ (although it continues to be sharply peaked in the constant of motion $p_{(\phi)}$). Nonetheless, the quantum evolution continues to be deterministic and well-defined for an infinite number of cycles. This is in sharp contrast with the classical theory where the equations break down at singularities and there is no deterministic evolution from one cycle to the next.

This concludes the summary of our discussion of the k=1 model. An analogous detailed analysis has been carried out also in the k=0 model, again with a free massless scalar field [34, 24, 27, 37]. In this case, if the cosmological constant Λ vanishes, as Fig 1.1 shows, classical solutions are of two types, those which start out at the big-bang and expand out to infinity and those which start out with large volume and contract to the big crunch singularity. Again, in this case while the WDW solution follows the classical trajectories into singularities, the LQC solutions exhibit a big bounce. The LQC dynamics is again faithfully reproduced by an effective equation: the Friedmann equation $(\dot{a}/a)^2 = (8\pi G\,\rho/3)$ is replaced just by $(\dot{a}/a)^2 = (8\pi G\,\rho/3)\,(1-\rho/\rho_{\rm crit})$. The quantum correction $\rho/\rho_{\rm crit}$ is completely negligible even at the onset of the standard inflationary era. Quantum bounce occurs at $\rho = \rho_{\rm crit}$ and the critical density is again given by $\rho_{\rm crit} \approx 0.82\rho_{\rm Pl}$. Furthermore, one can show that the spectrum of the density operator *on the physical Hilbert space* admits a finite upper bound $\rho_{\rm sup}$. By plugging values of constants in the analytical expression of this bound, one finds $\rho_{\rm sup} = \rho_{\rm crit}$! If $\Lambda > 0$, there are again two types of classical trajectories but the one which starts out at the big-bang expands to an infinite volume in finite value $\phi_{\rm max}$ of ϕ. (The other trajectory is a 'time reverse' of this.) Because the ϕ 'evolution' is unitary in LQC, it yields a natural extension of the classical solution beyond $\phi_{\rm max}$. If $\Lambda < 0$, the classical universe undergoes a recollapse. This is faithfully reproduced by the LQC evolution. Since both the big-bang and the big-crunch singularities are resolved, the LQC evolution leads to a cyclic universe as in the k=1 model. Thus, in all these cases, the principal features of the LQC evolution are robust, including the value of $\rho_{\rm crit}$.

Let us summarize the overall situation. In simple cosmological models, all the questions raised in section 1.2.2 have been answered in LQC in remarkable detail. The scalar field plays the role of an internal or emergent time and enables us to interpret the Hamiltonian constraint as an evolution equation. The matter momentum $\hat{p}_{(\phi)}$ and 'instantaneous' volumes $\hat{V}|_\phi$ form a complete set of Dirac observables and enable us to ask physically interesting questions. Answers to these questions imply that the big bang and the big crunch singularities are naturally replaced by quantum bounces. On the 'other side' of the bounce there is again a large universe. General relativity is an excellent approximation to quantum dynamics once the matter density falls below a couple of percent of the Planck density. Thus, LQC successfully meets both the 'ultra-violet' and 'infra-red' challenges. Furthermore results obtained in a number of models using distinct methods re-enforce one another. One is therefore led to take at least the qualitative findings seriously: *Big bang is not the Beginning nor the big crunch the End.* Quantum space-time appears to be vastly larger than what general relativity had us believe!

1.3 Black Holes

The idea of black holes is quite old. Already in 1784, in an article in the Proceedings of the Royal Society John Mitchell used the formula for escape velocity in Newtonian gravity to argue that light can not escape from a body of mass M if it is compressed to a radius $R = 2GM/c^2$. He went on to say

> *if there should exist in nature any [such] bodies .... we could have no information from sight; yet if any other luminous bodies should happen to revolve around them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones with some degree of probability.*

Remarkably, it is precisely observations of this type that have now led us to the conclusion that there is a 3.4 million solar mass black hole in the center of our galaxy! In the second volume of Exposition du systèm du Monde published in 1798, the Marquis de Laplace came to the same conclusion independently and was more confident of the existence of black holes:

> *there exist, in the immensity of space, opaque bodies as considerable in magnitude, and perhaps equally as numerous as stars.*

While in many ways these observations are astonishingly prescient, the underlying reasoning is in fact incorrect. For, if light (which is assumed to be corpuscular in this argument) from a distant source were to impinge on such an object, it would bounce back and by Newtonian conservation laws it would reach the point from which it came. Distant observers should therefore be able to see these objects. Indeed, if all speeds —including that of light— are relative as in Newtonian mechanics, there can really be no black holes. The existence of black holes requires both gravity and an absolute speed of light; general relativity is essential.

1.3.1 Horizons

To capture the intuitive notion that black hole is a region from which signals can not escape to the asymptotic part of space-time, one needs a precise definition of future infinity. The standard strategy is to use Penrose's conformal boundary $\mathcal{I}^+$ [40]. It is a future boundary: No point of the physical space-time lies to the future of any point of $\mathcal{I}^+$. It has topology $\mathbb{S}^2 \times \mathbb{R}$ and it is null (assuming that the cosmological constant is zero). In Minkowski space-time, one can think of $\mathcal{I}^+$ as the 'final resting place' of all future directed null geodesics. More precisely, the chronological past $I^-(\mathcal{I}^+)$ of $\mathcal{I}^+$ is entire Minkowski space.[9]

Given a general asymptotically flat space-time (M, g_{ab}), one first finds the chronological past $I^-(\mathcal{I}^+)$ of $\mathcal{I}^+$. If it is not the entire space-time, then there is a region in (M, g_{ab}) from which one cannot send causal signals to infinity. When this happens, one says that the space-time admits a black hole. More precisely, *Black-hole region* $\mathbb{B}$ of (M, g_{ab}) is defined as

$$\mathbb{B} = M \; - \; I^-(\mathcal{I}^+) \tag{1.7}$$

where the right side is the set of points of M which are not in $I^-(\mathcal{I}^+)$. The boundary $\partial\mathbb{B}$ of the black hole region is called the *event horizon* (EH) and is denoted by E [41]. $I^-(\mathcal{I}^+)$ is often referred to as the asymptotic region and e is the boundary of this region within physical space-time.

Event horizons and their properties have provided a precise arena to describe black holes and their dynamics. In particular, we have the celebrated result of Hawking's [42, 41]: assuming energy conditions, the area $a_{\rm hor}$ of an EH cannot decrease under time evolution. The area $a_{\rm hor}$ is thus analogous to thermodynamic entropy. There are other laws governing black holes which are in equilibrium (i.e. stationary) and that make transitions to nearby equilibrium states due to influx of energy and angular momentum. They are similar to the zeroth and the first law of thermodynamics and suggest that the surface gravity κ of stationary black holes is the analog of thermodynamic temperature. These analogies were made quantitative and precise by an even deeper result Hawking obtained using quantum field theory in a black hole background [43]: black holes radiate quantum mechanically as though they are black bodies at temperature $T = \kappa\hbar/2\pi$. Their entropy is then given by $S = a_{\rm hor}/4\ell_{\rm Pl}^2$. Not surprisingly these results have led to a rich set of insights and challenges over the last 35 years.

However, the notion of an EH also has two severe limitations. First, while the notion neatly captures the idea that asymptotic observers can not 'look into' a black hole, it is too global for many applications. For example, since

[9] $I^-(\mathcal{I}^+)$ is the set of all points in the physical space-time from which there is a future directed time-like curve to a point on $\mathcal{I}^+$ in the conformally completed space-time. The term 'chronological' refers to the use of time-like curves. A curve which is everywhere time-like or null is called 'causal'.

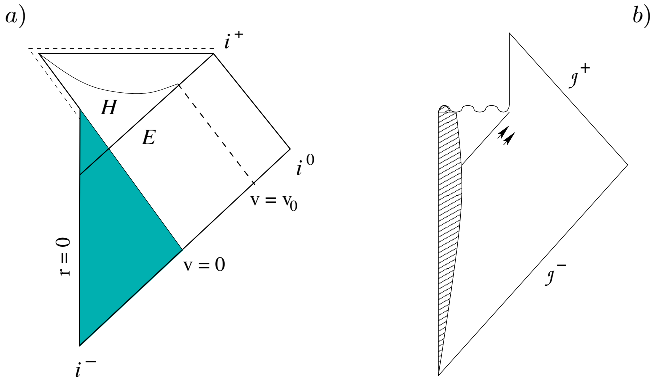

Fig. 1.4. *a*) A Vaidya solution: Collapse of a spherical null fluid to form a black hole. The null fluid radiation starts at the retarded time $v = 0$ and ends at $v = v_0$. Space time is flat in the past of $v = 0$ and Schwarzschild to the future of $v = v_0$. The dynamical horizon H starts out space-like and joins on to the null event horizon at $v = v_0$. The event horizon first forms and grows in the flat part of space-time. *b*) Conjectured Penrose diagram of an evaporating black hole: A black hole forms by stellar collapse and evaporates due to Hawking radiation. Due to back reaction, the singularity loses its strength as we move right along the wiggly line, and finally disappears. Nonetheless because there is still a piece of space-like singularity in the future, $\mathcal{I}^+$ does not constitute the full future boundary of space-time, leading to information loss.

it refers to null infinity, it can not be used in spatially compact space-times. Asymptotic flatness and the notion of $\mathcal{I}^+$ is used also in other contexts, in particular to discuss gravitational radiation in full, non-linear general relativity [40]. However, there $\mathcal{I}^+$ is used just to facilitate the imposition of boundary condition and make notions such as '$1/r^n$-fall-off' precise. Situation with EHs is quite different because they refer to the *full chronological past of* $\mathcal{I}^+$. As a consequence, by changing the geometry in a small —say Planck scale region— around the singularity, one can change the EH dramatically and even make it disappear [1]! As I explained in section 1.1, there is no reason to trust classical general relativity very close to the singularity. If the singularity is resolved due to quantum effects, there may be no longer an EH. What then is a black hole? If the notion continues to be meaningful, can we still associate with it entropy in absence of an EHs?

The second limitation is that the notion is teleological; it lets us speak of a black hole *only after we have constructed the entire space-time*. Thus, for example, an EH may well be developing in the room you are now sitting *in anticipation* of a gravitational collapse that may occur in this region of our galaxy a million years from now. Indeed, as Fig 1.4a shows, EHs can form and grow *even in flat space-time* where there is no influx of matter of radiation. How can we then attribute direct physical significance to the growth of their area? Clearly, when astrophysicists say that they have discovered a black hole in the center of our galaxy, they are referring to something much more concrete and quasi-local than an EH.

Over the last five years, quasi-local horizons were introduced to improve on this situation [44, 45, 46, 47]. The idea is to use the notion of marginally trapped surfaces. Consider a space-like 2-sphere in Minkowski space and illuminate it instantaneously. Then there are two light fronts, one traveling outside the sphere and expanding continuously and the other traveling inside and contracting. Now, if the 2-sphere were placed in a strong gravitational field, both these light fronts could contract. Then light would be trapped and the sphere would not be visible from outside. These two situations are separated by the marginal case where one light front would be contracting and the area of the other would neither decrease not increase. Such 2-surfaces are said to be *marginally trapped* and their world tubes represent quasi-local horizons. More precisely, a marginally trapped tube (MTT) is a 3-manifold which is foliated by a family of marginally trapped 2-spheres. If it is space-like, the area of the marginally trapped surfaces increases to the future and the MTT is called a *dynamical horizon* (DH). Heuristically it represents a growing black hole. If the MTT is null, it is called an *isolated horizon* (IH) and represents a black hole in equilibrium. In Fig 1.4a a DH H forms due to gravitational collapse of infalling null fluid, grows in area with the in-fall and settles down to an IH which coincides with the future part of the EH E. Note that the definitions of MTT, DH and IH are all quasi-local. In particular, they are not teleological; you can be rest assured that none of these quasi-local horizons exists in the room you are now sitting in!

There is however a significant drawback: lack of uniqueness. Although partial uniqueness results exist [48], in general we cannot yet associate a unique DH with a generic, growing black hole. But this weakness is compensated in large measure by the fact that interesting results hold for *every* DH. In particular, not only does the direct analog of Hawking's area theorem hold on DHs, but there is a precise *quantitative* relation between the growth of area of a DH and the amount of energy falling into it [45, 46]. Therefore, in striking contrast with EHs, we can associate a direct physical significance to the growth in area of DHs. This and other quantitative relations have already made DHs very useful in numerical simulations of black hole formation and mergers [47]. Finally, since they refer only to the space-time geometry in their immediate vicinity, the existence and properties of these horizons are insensitive to what happens near the singularity. Thus, quantum gravity modifications in the space-time geometry in the vicinity of the classical singularity would have no effect on these horizons.[10]

[10] One might wonder: Don't the singularity theorems essentially guarantee that if there is an MTT there must be a singularity? Recall however that the theorems also assume classical Einstein's equations and certain energy conditions. Both these assumptions would be violated in quantum gravity. Therefore, it is per-

Conceptually these quasi-local horizons are also useful in quantum considerations. Let us first consider equilibrium situations. In loop quantum gravity, there is a statistical mechanical derivation of the entropy associated with any isolated horizon [49, 2]. These cover not only the familiar stationary black holes but also hairy black holes as well as cosmological horizons. Next, consider dynamics. During the collapse, the MTT is space-like and we have a DH. But once the in-fall of matter ends, the mass of the black hole must decrease and the horizon area must shrink. In this phase the MTT is time-like and so there is no obstruction at all for leakage of matter from inside the MTT to the outside region.

To summarize, black holes were first described using EHs. While this description has led to important insights, they also have some important limitations in the dynamical context. The more recent quasi-local horizons provide concepts and tools that are more directly useful both in numerical relativity and quantum gravity.

1.3.2 Hawking radiation and information loss

Consider a spherically symmetric gravitational collapse depicted in Fig 1.4a. Once the black hole is formed, space-time develops a new, future boundary at the singularity, whence one can not reconstruct the geometry and matter fields by evolving the data *backward* from future null infinity, $\mathcal{I}^+$. Thus, whereas an appropriately chosen family of observers near $\mathcal{I}^-$ has full information needed to construct the entire space-time, no family of observers near $\mathcal{I}^+$ has such complete information. In this sense, black hole formation leads to information loss. Note that, contrary to the heuristics often invoked, this phenomenon is not directly related to black hole uniqueness results: it occurs even when uniqueness theorems fail, as with 'hairy' black holes or in presence of matter rings non-trivially distorting the horizon. The essential ingredient is the future singularity which can act as the sink of information.

A natural question then is: what happens in quantum gravity? Is there again a similar information loss? Hawking's [43] celebrated work of 1974, mentioned in section 1.3.1, analyzed this issue in the framework of quantum field theory in curved space-times. In this approximation, three main assumptions are made: i) the gravitational field can be treated classically; ii) one can neglect the back-reaction of the spontaneously created matter on the space-time geometry; and iii) the matter quantum field under investigation is distinct from the collapsing matter, so one can focus just on spontaneous emission. Under these assumptions, at late times there is a steady emission of particles to $\mathcal{I}^+$ and the spectrum is thermal at a temperature dictated by the surface gravity of the final black hole. In particular, pure states on $\mathcal{I}^-$ evolve to mixed states on $\mathcal{I}^+$. However, this external field approximation is too crude;

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fectly feasible for MTTs to exist even though the (quantum) space-time has no singularities.

in particular it violates energy conservation. To cure this drawback, one can include back-reaction. A detailed calculation is still not available. However, following Hawking [43], one argues that, as long as the black hole is large compared to the Planck scale, the quasi-stationary approximation should be valid. Then, by appealing to energy conservation and the known relation between the mass and the horizon area of *stationary* black holes, one concludes that the area of the EH should steadily decrease.[11] This then leads to black hole evaporation depicted in figure 1.4b [42]. If one does not examine space-time geometry but uses instead intuition derived from Minkowskian physics, one may be surprised that although there is no black hole at the end, the initial pure state has evolved in to a mixed state. Note however that even after the inclusion of back reaction, in this scenario *there is still a final singularity, i.e., a final boundary in addition to* $\mathcal{I}^+$. Therefore, it is not at all surprising that, in this approximation, information is lost —it is still swallowed by the final singularity. Thus, provided figure 1.4b is a reasonable approximation of black hole evaporation and one does not add new input 'by hand', then pure states must evolve in to mixed states.

The question then is to what extent this diagram is a good representation of the physical situation. The general argument in the relativity community has been the following. Figure 1.4b should be an excellent representation of the actual physical situation as long as the black hole is much larger than the Planck scale. Therefore, problems, if any, are associated *only* with the end point of the evaporation process. It is only here that the semi-classical approximation fails and one needs full quantum gravity. Whatever these 'end effects' are, they deal only with the Planck scale objects and would be too small to recover the correlations that have been steadily lost as the large black hole evaporated down to the Planck scale. Hence pure states must evolve to mixed states and information is lost.

Tight as this argument seems, it overlooks two important considerations. First, one would hope that quantum theory is free of infinities whence figure 1.4b can not be a good depiction of physics near the *entire singularity* — not just near the end point of the evaporation process. Second, as we saw in section 1.3.1, the EH is a highly global and teleological construct. Since the structure of the *quantum* space-time could be very different from that of figure 1.4b near (and 'beyond') the singularity, the causal relations implied by the presence of the EH of figure 1.4b is likely to be quite misleading [1]. Indeed, using the AdS/CFT conjecture, string theorists have argued that the evolution must be unitary and information is not lost. However, since the crux of that argument is based on the boundary theory (which is conjectured to be equivalent to string theory in the bulk), this line of reasoning does not provide a direct *space-time description* of how and why the information is recovered. Where does the above reasoning of relativists, fail? How must it be corrected?

[11] This does not contradict the area law because the energy conditions used in its derivation are violated by the quantum emission.

I believe that answer to these question lies in the fact that, because of singularity resolution, the quantum space-time is larger than the classical [50]. In support of this view, in the next two sections I will use a 2-dimensional black hole to argue that the loss of information is not inevitable even in space-time descriptions favored by relativists.

1.3.3 CGHS black holes

Let us begin with the spherical collapse of a massless scalar field f in 4 space-time dimensions resulting in a black hole. Because of spherical symmetry, it is convenient to factor out by the 2-spheres of symmetry and pass to the $r-t$ plane. Let us express the 4-dimensional space-time metric ${}^4g_{ab}$ as:

$$ {}^4g_{ab} = g_{ab} + \frac{e^{-2\phi}}{\kappa^2}\, s_{ab}\,, $$

where we have introduced a constant κ with dimensions of inverse length and set $r = e^{-\phi}/\kappa$. Then the (symmetry reduced) Einstein Hilbert action becomes

$$ S(g,\phi,\kappa) = \frac{1}{2G}\int d^2x\,\sqrt{|g|}\,\left[e^{-2\phi}\,(R + \mathbf{2}\nabla^a\phi\nabla_a\phi + \mathbf{2}\mathbf{e}^{-\mathbf{2}\phi}\kappa^2)\, + G\,\mathbf{e}^{-\phi}\nabla^a f\nabla_a f\right] \tag{1.8} $$

where R is the scalar curvature of the 2-metric g. This theory is very rich especially because the well-known critical phenomena. The classical equations cannot be solved exactly. However, an apparently small modification of this action —changes in the bold coefficients (compare (1.8) and (1.9))— gives a 2-dimensional theory which *is* exactly soluble classically. There is again a black hole formed by gravitational collapse and it evaporates by Hawking radiation. This is the Callen, Giddings, Harvey, Strominger (CGHS) model [51] and it arose upon symmetry reduction of a low energy action motivated by string theory. Because it has many of the qualitative features of the 4-d theory but is technically simpler, the model attracted a great deal of attention in the 90s (for reviews, see e.g., [52]). Here, the basic fields are again a 2-dimensional metric g of signature -+, a geometrical scalar field ϕ, called the dilaton, and a massless scalar field f. The action is given by:

$$ S(g,\phi,f) := \frac{1}{2G}\int d^2x\,\sqrt{|g|}\,\left[e^{-2\phi}\,(R + \mathbf{4}\nabla^a\phi\nabla_a\phi + \mathbf{4}\kappa^2) + G\,\nabla^a f\nabla_a f\right] \tag{1.9} $$

We will analyze this 2-d theory in its own right.

Recall that imposition of spherical symmetry in 4-d general relativity implies that the gravitational field is completely determined by matter —the true degrees of freedom are all contained in the matter. The same is true in the CGHS model. Furthermore, in the CGHS case there is a simplification: the equation of motion of f is just $\Box_{(g)} f = 0$; dynamics of f is decoupled from ϕ. In 2 dimensions, the physical metric g^{ab} is conformally related to a flat

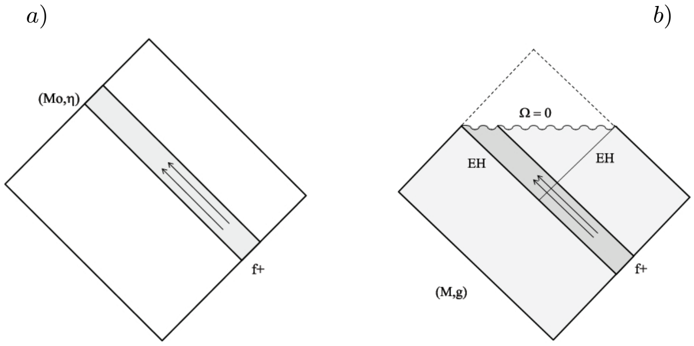

Fig. 1.5. *a*) A typical solution for the f_+ mode in Minkowski space. *b*) When interpreted in terms of the physical metric g, a black hole has formed because of the gravitational collapse of f_+. The physical space-time M is a proper subset of M_o but the subset realized depends on the solution f_+. Therefore already in the classical Hamiltonian theory, the kinematical arena is provided by M_o.

metric $g^{ab} = \Omega\eta^{ab}$ and conformal invariance of the wave equation implies that $\Box_{(g)} f = 0$ if and only if $\Box_{(\eta)} f = 0$. Therefore, we can fix a fiducial flat metric η, parameterize g by Ω and determine f by solving the wave equation on the 2-dimensional Minkowski space (M_o, η). Finally, let us set $\Phi = e^{-2\phi}$ and write the conformal factor Ω as $\Omega = \Theta^{-1}\Phi$. The passage $(g, \phi, f) \longrightarrow (\Theta, \Phi, f)$ just corresponds to a convenient choice of field redefinitions.

Since $\Box_{(\eta)} f = 0$, we know $f = f_+(z^+) + f_-(z^-)$, where $f_\pm$ are arbitrary smooth functions of their arguments and $z^\pm$ are the advanced and retarded coordinates of η (i.e., $\eta_{ab} = -\partial_{(a} z^+ \partial_{b)} z^-$). Given f, the equations of motion for Θ and Φ (together with appropriate boundary conditions) determine the classical solution completely. To display it, it is simplest to use coordinates $x^\pm$ given by:

$$\kappa x^+ = e^{\kappa z^+} \quad \text{and} \quad \kappa x^- = -e^{-\kappa z^-} .$$

Then, for any given $f_\pm$, the solution is given by

$$\begin{aligned}&\Theta = -\kappa^2 x^+ x^- \quad \text{and}\\ &\Phi = \Theta - \frac{G}{2}\int_0^{x^+} \mathrm{d}\bar{x}^+ \int_0^{\bar{x}^+} \mathrm{d}\bar{\bar{x}}^+ \,(\partial f_+/\partial\bar{\bar{x}}^+)^2 \;-\; \frac{G}{2}\int_0^{x^-} \mathrm{d}\bar{x}^- \int_0^{\bar{x}^-} \mathrm{d}\bar{\bar{x}}^- \,(\partial f_-/\partial\bar{\bar{x}}^-)^2 \,.\end{aligned} \tag{1.10}$$

The black hole sector of interest is obtained by setting $f_- = 0$ as in Fig 1.5a and letting f_+ collapse. (Alternatively, one could $f_+ = 0$ and consider the collapse of f_-.)

But why is there a black hole? Fields f_+, Θ, Φ are all smooth on the entire manifold M_o. Recall, however, that the physical metric is given by $g^{ab} = \Omega\eta^{ab} \equiv \Theta^{-1}\Phi\,\eta^{ab}$. On the entire manifold M_o, Θ is smooth and nowhere vanishing. However, it is easy to verify that Φ vanishes along a space-like line (see Fig 1.5b). On this line g^{ab} becomes degenerate and its scalar curvature diverges. Thus there is a space-like singularity; the physical space-time manifold M on which g_{ab} is well defined is only a part of the fiducial Minkowski manifold M_o (see Fig 1.5b). Is it hidden behind an event horizon? To ask this question, we should first verify that (M, g_{ab}) admits a *complete* [53] future null infinity $\mathcal{I}^+$ and the past of $\mathcal{I}^+$ does not contain the singularity. In 2 space-times dimensions, past as well as future null infinity has two pieces, one to the right and the other to the left and they are joined only by points $i^\pm$ at time-like infinity. In the solutions under consideration $\mathcal{I}^+_{\rm R}$ is complete but $\mathcal{I}^+_{\rm L}$ is not. Therefore strictly we can meaningfully ask if there is a black hole only with respect to $\mathcal{I}^+_{\rm R}$ and the answer is in the affirmative. Fortunately to analyze the Hawking radiation and information loss, we can focus just on $\mathcal{I}^+_{\rm R}$. Before going on to these issues, it is interesting to note that there is a black hole in spite of the fact that the solution (f, θ, Φ) is perfectly regular. This is because the physical meaning of the solution has to be analyzed using the physical geometry determined by g.

Solution (1.10) represents a black hole formed by the gravitational collapse of f_+. In the spirit of Hawking's original derivation, let us study the dynamics of a *test* quantum field $\hat{f}_-$ on this black hole geometry. Now $\mathcal{I}^-$ of every physical metric g coincides with the past null infinity $\mathcal{I}^{o-}$ of Minkowski space (M_o, η) and $g = \eta$ in a neighborhood of $\mathcal{I}^{o-}_{\rm L}$. So we can begin with the vacuum state $|0\rangle_-$ at $\mathcal{I}^{o-}_{\rm L}$ and ask for its dynamical content. In the Heisenberg picture, the operators evolve and state remain fixed. The issue then is that of interpretation of the fixed state $|0\rangle_-$ in the geometry given by g in a neighborhood of $\mathcal{I}^+_{\rm R}$. Now, two important factors of the geometry come into play. First, although the physical metric g is asymptotically flat, *it does not agree with* η *even at* $\mathcal{I}^+_{\rm R}$. More precisely, the affine parameter y^- at $\mathcal{I}^+_{\rm R}$ is a non-trivial function of z^-, reflecting the fact that the asymptotic time translation of g does not coincide with any of the asymptotic time translations of η. Therefore there is a mixing of positive and negative frequency modes. Since $|0\rangle_-$ is defined using z^-, it is populated with particles defined at $\mathcal{I}^+_{\rm R}$ by g. Second, $\mathcal{I}^+_{\rm R}$ is a proper subset of $\mathcal{I}^{o+}_{\rm R}$. Therefore, we have to trace over modes of $\hat{f}_-$ with support on $\mathcal{I}^{o+}_{\rm R} - \mathcal{I}^+_{\rm R}$. Therefore, as far as measurements of observables near $\mathcal{I}^+_{\rm R}$ are concerned, the state $|0\rangle_-$ is indistinguishable from a density matrix ρ on the Hilbert space $\mathcal{H}$ of $\hat{f}_-$ at $\mathcal{I}^+_{\rm R}$. Detailed calculation shows that at late times, ρ is precisely the thermal state at temperature $\hbar\kappa/2\pi$ [54]! Thus, in the CGHS model there is indeed Hawking radiation and therefore, by repeating the reasoning summarized in section 1.3.2 one can conclude that there must be information loss.

I will conclude this section by summarizing the similarities and differences in the 4 and 2 dimensional analyses. In both cases there is a formation of a black hole due to gravitational collapse and the test quantum field is distinct from the field that collapses. Thanks to asymptotic flatness at past null infinity, the vacuum state $|0\rangle_-$ of the test field is well-defined and the key issue is that of its physical interpretation in the physical geometry near future null infinity. Finally although κ was introduced as a constant in the CGHS theory, one can verify that it is in fact the surface gravity of the stationary black hole in the future of the support of f_+. In both cases the Hawking temperature is this given by $\hbar/2\pi$ times the surface gravity. However, there are also some important differences. First, whereas there is just one $\mathcal{I}^-$ and $\mathcal{I}^+$ in 4 dimensions in the CGHS case we have two copies of each and the clear-cut black hole interpretation holds only with respect to $\mathcal{I}^+_{\mathrm{R}}$. Second, while in 4 dimensions κ and hence the Hawking temperature is inversely proportional to the mass of the black hole, in the CGHS case it is a constant. Finally, at a technical level, even in the spherically symmetric reduction of the 4-dimensional theory, the equation satisfied by the scalar field f is much more complicated than the CGHS wave equation. Therefore, while analysis of the CGHS black hole does provide valuable insights for the 4 dimensional case, one cannot take directly over results.

1.3.4 Quantum geometry

Since the model is integrable classically, many steps in the passage to quantum theory are simplified [15]. Our basic fields will again be $\hat{f}, \hat{\Theta}, \hat{\Phi}$. The true degree of freedom is in the scalar field f and it satisfies just the wave equation on Minkowski space (M_o, η). Therefore, it is straightforward to construct the Fock space $\mathcal{F} = \mathcal{F}_+ \otimes \mathcal{F}_-$ and represent $\hat{f}_\pm$ as operator valued distributions on $\mathcal{F}$. Classically, we have explicit expressions (1.10) of fields Θ and Φ in terms of f on all of M_o. In quantum theory, because of trace anomaly the equations satisfied by $\hat{\Theta}, \hat{\Phi}$ are more complicated. Therefore explicit solutions are not available. However, these are hyperbolic equations on the fiducial Minkowski space and the boundary values *at* $\mathcal{I}^{o-}$ are given by the (unambiguous) operator versions of (1.10). Therefore, in principle, it should be possible to solve them. A conjecture based on approximate solutions is that $\hat{\Theta}$ would be an operator field and $\hat{\Phi}$ an operator valued distribution on $\mathcal{F}$.

At first, may seem surprising that there is no Hilbert space corresponding to geometry. However, already at the classical level the covariant phase space can be coordinatized completely by the scalar field f and geometric fields Θ, Φ are just functionals on this phase space. The situation in quantum theory is precisely what one would expect upon quantization. While the full quantum theory is still incomplete in the CGHS model, there is a simpler and interesting system in which this feature is realized in detail: cylindrical gravitational waves in 4-dimensional general relativity. This system is equivalent to 2+1 Einstein gravity coupled to an axi-symmetric scalar field. Again because there are no gravitational degrees of freedom in 2+1 dimensions, the true degree of freedom can be encoded in the scalar field which now satisfies a wave equation in a fiducial 2+1 dimensional Minkowski space. The regulated metric operator is represented as an operator valued distribution on the Fock space of the scalar field [55] and leads to interesting and unforeseen quantum effects [56].

Returning to the CGHS model, we can now ask: What is a quantum black hole? In the classical theory, black holes result if we specify a smooth profile f^o as initial data for f_+ on $\mathcal{I}_{\rm R}^{o-}$ and zero data for f_- on $\mathcal{I}_{\rm L}^{o-}$. In quantum theory, then, a candidate black hole would a quantum state $|\Psi\rangle$ which is peaked at this classical data on $\mathcal{I}^{o-}$: $|\Psi\rangle = |0\rangle_- \otimes |C_f^o\rangle_+$ where $|0\rangle_-$ is the vacuum state in $\mathcal{F}_-$ and $|C_f^o\rangle_+$ is the coherent state in $\mathcal{F}_+$ peaked at the classical profile f_o of f_+. One can show that if one solves the quantum equations for $\hat{\Theta}, \hat{\Phi}$ in a certain approximation (the 1st step in a certain bootstrapping), then the states Ψ do emerge as black holes: the expectation values of $\hat{g}^{ab}, \hat{\Phi}$ are precisely those of the classical black hole solutions. In particular, $\langle\hat{\Phi}\rangle$ vanishes along a space-like line which appears as the singularity in the classical theory. However, the true *quantum* geometry near this classical singularity is perfectly regular [15]: the *operator* $\hat{\Phi}$ does not vanish, only its expectation value does. Furthermore, one can also calculate fluctuations and show that they are small near infinity but huge near the classical singularity. Consequently, the expectation values are poor representations of the actual quantum geometry in a neighborhood of the classical singularity. The fact that the quantum metric $\hat{g}^{ab}$ is regular on M_o already in this approximation suggests that the singularity may be resolved in the quantum theory making the quantum space-time larger than the classical one. There is then a possibility that there may be no information loss.

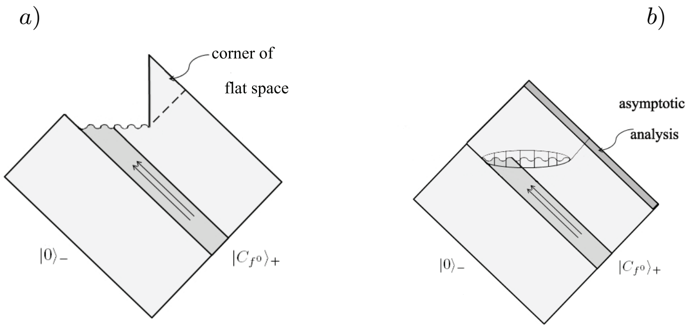

Fig. 1.6. *a*) The CGHS analog of the Penrose diagram 1.4b. This diagram has been used for a number of years to describe space-time geometry after inclusion of back reaction. Singularity is still part of the future boundary and so the information is lost. *b*) The space-time diagram suggested by the asymptotic analysis of mean field equations near $\mathcal{I}_{\rm R}^+$. In the quantum space-time $\mathcal{I}_{\rm R}^+$ is 'as long as' $\mathcal{I}_{\rm R}^{o+}$, whence $|0\rangle_-$ is a pure state also with respect to the physical metric g. It is however populated by particles and resembles thermal density matrix at an intermediate region of $\mathcal{I}_{\rm R}^+$.

This issue is probed using the mean field approximation (MFA) [15]. Here, one first takes the expectation value of the the quantum equations governing $\hat{\Theta}, \hat{\phi}$ in the state Ψ and, furthermore, replaces Φ, θ by their expectation values. Thus, for example, $\langle\hat{\Theta}\,\hat{\Phi}\rangle$ is replaced by $\langle\hat{\Theta}\rangle\,\langle\hat{\Phi}\rangle$ but $\langle: (\partial\hat{f})^2 :\rangle$ is kept as is.

This amounts to ignoring the quantum fluctuations in the geometric operators $\hat{\Theta}, \hat{\Phi}$ but not those in the matter field $\hat{f}$. This approximation can be justified in the limit in which there is a large number N of scalar fields $\hat{f}$ rather than just one and we restrict ourselves to regions in which "fluctuations in the geometry are less than N times the fluctuations in any one matter field". In this region, the mean field approximation provides a good representation of the geometry that includes back reaction of the Hawking radiation.

It turns out that the resulting equations on $\bar{\Theta} := \langle\hat{\Theta}\rangle$ and $\bar{\Phi} := \langle\hat{\Phi}\rangle$ were already obtained sometime ago using functional integral techniques and solved numerically [57]. By making appeal to the 4-dimensional theory whose symmetry reduction gives the CGHS models, one can introduce the notion of marginally trapped 'surfaces' and their 'area'. Simulations showed that marginally trapped surfaces do form due to infalling matter, the marginally trapped tube is first space-like —i.e., is a dynamical horizon— but, after the inflow of collapsing matter ends, becomes time-like due to the leakage of the Hawking radiation. Thus the scenario based on quasi-local horizons is realized. In the dynamical horizon phase, the horizon area $a_{\rm hor}$ increases and in the subsequent Hawking evaporation, it decreases to zero: It is again the MTT that evaporates. However, further evolution to the future moves one closer to what was the classical singularity. As I mentioned above, in this region the quantum fluctuations in geometry become huge and so the mean field approximation fails. The simulations cannot be continued further. However, since the area of the marginally trapped surface shrunk to zero, it was assumed —as is reasonable— that the Bondi mass at the corresponding retarded instant of time would be zero on $\mathcal{I}_{\rm R}^{+}$. Therefore, following what Hawking did in 4-dimensions, it became customary to attach by hand a 'corner of Minkowski space' to the numerically evolved space-time thereby arriving at a Penrose diagram of figure 1.6a. Note that in this diagram, the future boundary for the $\hat{f}_{-}$ modes consists not just of $\mathcal{I}_{\rm R}^{+}$ but also a piece of the singularity. As I argued in section 1.3.2, if this is an accurate depiction of the physical situation, one would conclude that $|0\rangle_{-}$ at $\mathcal{I}_{\rm L}^{o-}$ would evolve to a density matrix on $\mathcal{I}_{\rm R}^{+}$ and information would indeed be lost.

Note however that the key to the information loss issue lies in the geometry near future infinity and MFA should be valid there. Thus, rather than attaching a corner of flat space by hand at the end of the numerical simulation, we can use the mean field equations near $\mathcal{I}_{\rm R}^{+}$ and let them tell us what the structure of $\mathcal{I}_{\rm R}^{+}$ of the *physical* metric is.

To realize this idea, one has to make three assumptions: i) exact quantum equations can be solved and the expectation value $\bar{g}^{ab}$ of $\hat{g}^{ab}$ admits a smooth right null infinity $\mathcal{I}_{\rm R}^{+}$ which coincides with $\mathcal{I}_{\rm R}^{o+}$ *in the distant past* (i.e. near $i_{\rm R}^{o}$); ii) MFA holds in a neighborhood of $\mathcal{I}_{\rm R}^{+}$; and, iii) Flux of quantum radiation vanishes at some finite value of the affine parameter y^{-} of $\mathcal{I}_{\rm R}^{+}$ defined by the asymptotic time translation of $\bar{g}$. All three assumptions have been made routinely in the analysis of the information loss issue, although they are often only implicit. Indeed, one cannot even meaningfully ask if information is lost unless the first two hold. (The third assumption can be weakened to allow the flux to decay sufficiently fast in the future.) Then, a systematic analysis of the MFA equations shows [15] that *the right future null infinity* $\mathcal{I}^+_{\rm R}$ *of the physical metric* $\bar{g}$ *coincides with that of* η; $\mathcal{I}^+_{\rm R} = \mathcal{I}^{o+}_{\rm R}$ (see Fig 1.6). This implies that to interpret $|0\rangle_-$ at $\mathcal{I}^+_{\rm R}$ we no longer have to trace over any modes; in contrast to the situation encountered in the external field approximation discussed in section 1.3.3, all modes of $\hat{f}_-$ are now accessible to the asymptotically stationary observers of $\bar{g}$. The vacuum state $|0\rangle_-$ of η is pure also with respect to $\bar{g}$. But is it in the asymptotic Fock space of $\bar{g}$? Calculation of Bogoluibov coefficients shows [15] that the answer is in the affirmative. Thus, the interpretation of $|0\rangle_-$ with respect to $\bar{g}$ is that it is a pure state populated by pairs of particles at $\mathcal{I}^+_{\rm R}$. *There is neither information loss nor remnants.*

Let us summarize the discussion of CGHS black holes. A key simplification in this model is that the matter field satisfies just the wave equation on (M_o, η^{ab}). Therefore, given initial data on $\mathcal{I}^{o-}$, we already know the state everywhere both in the classical and the quantum theory. However, the state derives its physical interpretation from geometry which is a complicated functional of the matter field. We do not yet know the quantum geometry everywhere. But approximation methods suggest that $\hat{g}^{ab}$ is likely to be well-defined (and nowhere vanishing) everywhere on M_o. By making rather weak assumptions on the asymptotic behavior of its expectation value $\bar{g}^{ab}$, one can conclude that the right future null infinity $\mathcal{I}^+_{\rm R}$ of $\bar{g}^{ab}$ coincides with $\mathcal{I}^{o+}_{\rm R}$ of η^{ab} and the affine parameters y^- and z^- defined by the two metrics are such that the exact quantum state $|0\rangle_-$ is a pure state in the asymptotic Fock space of $\bar{g}^{ab}$. The S-matrix is unitary and there is no information loss. Thus the asymptotic analysis leads us to a Penrose diagram of Fig 1.6b which is significantly different from Fig 1.6a, based on Hawking's original proposal [43]. In particular, the quantum space-time does not end at a future singularity and is larger than that in 1.6a. The singularity is replaced by a genuinely quantum region in which quantum fluctuations are large and the notion of a smooth metric tensor field is completely inadequate. However, in contrast to the situation in quantum cosmology of section 1.2, a full solution to the quantum equations is still lacking.

1.4 Discussion

In section 1.2 we saw that many of the long standing questions regarding the big bang have been answered in detail in the FRW cosmologies with a massless scalar field and the results are physically appealing. Main departures from the WDW theory occur due to *quantum geometry effects* of LQG. There is no fine tuning of initial conditions, nor a boundary condition at the singularity, postulated from outside. Also, there is no violation of energy conditions. Indeed, quantum corrections to the matter Hamiltonian do not play any role in the resolution of singularities of these models. The standard singularity theorems are evaded because the geometrical side of the classical Einstein's equations is modified by the quantum geometry corrections of LQC. While the detailed results presented in section 1.2.5 are valid only for these simplest models, partial results have been obtained also in more complicated models indicating that the singularity resolution is rather robust.

In this respect there is a curious similarity with the very discovery of physical singularities in general relativity. They were first encountered in special examples. But the examples were also the physically most interesting ones —e.g., the big-bang and the Schwarzschild curvature singularities. At first it was thought that these space-times are singular because they are highly symmetric. It was widely believed that generic solutions of Einstein'e equations should be non-singular. As is well-known, this belief was shattered by the Penrose-Hawking singularity theorems. Some 40 years later we have come to see that the big bang and the Schwarzschild singularities are in fact resolved by quantum geometry effects. Is this an artifact of high symmetry? Or, are there robust *singularity resolution theorems* lurking just around the corner?

A qualitative picture that emerges is that the non-perturbative quantum geometry corrections are *'repulsive'*.[12] While they are negligible under normal conditions, they dominate when curvature approaches the Planck scale and can halt the collapse that would classically have lead to a singularity. In this respect, there is a curious similarity with the situation in the stellar collapse where a new repulsive force comes into play when the core approaches a critical density, halting further collapse and leading to stable white dwarfs and neutron stars. This force, with its origin in the Fermi-Dirac statistics, is *associated with the quantum nature of matter*. However, if the total mass of the star is larger than, say, 5 solar masses, classical gravity overwhelms this force. The suggestion from LQC is that a new repulsive force *associated with the quantum nature of geometry* comes into play and is strong enough to counter the classical, gravitational attraction, irrespective of how large the mass is. It is this force that prevents the formation of singularities. Since it is negligible until one enters the Planck regime, predictions of classical relativity on the formation of trapped surfaces, dynamical and isolated horizons would still hold. But assumptions of the standard singularity theorems would be violated. There would be no singularities, no abrupt end to space-time where physics stops. Non-perturbative, background independent quantum physics would continue.

One can also analyze the CGHS models using LQG [58]. However, I used the more familiar Fock spaces to illustrate the fact that the basic phenomenon

[12] We saw in section 1.2.4 that there is no connection operator in LQG. As a result the curvature operator has to be expressed in terms of holonomies and becomes non-local. The repulsive force can be traced back to this non-locality. Heuristically, the polymer excitations of geometry do not like to be packed too densely; if brought too close, they repel.

of singularity resolution by quantum geometry effects is more general. In the CGHS case the analysis is not as complete as in the cosmological models because the CGHS model has an infinite number of degrees of freedom. But results obtained using various approximations strongly suggest that, as in the cosmological case, quantum space-times are larger than what the classical theory suggests. However, nature of the quantum space-time is quite different in the two cases. In the cosmological case the state remains sharply peaked around a smooth geometry even near the bounce. The expression of the effective metric which provides an excellent approximation to the exact quantum state does have an explicit dependence on $\hbar$ due to quantum corrections. However, it is smooth everywhere. In the CGHS model on the other hand quantum fluctuations in the metric become large in the Planck regime whence one cannot approximate the quantum geometry by *any* smooth geometry. Rather, there is a genuine quantum bridge joining the smooth metric in the distant past to that in the distant future.

At first one might think that, since quantum gravity effects concern only a tiny region, whatever quantum effects there may be, their influence on the global properties of space-time should be negligible whence they would have almost no bearing on the issue of the Beginning and the End. However, as we saw, once the singularity is resolved, vast new regions appear on the 'other side' ushering in new possibilities that were totally unforeseen in the realm of Minkowski and Einstein. Which of them are realized generically? Is there a manageable classification? If, as in the CGHS case, there are domains in which geometry is truly quantum, classical causality would be rendered inadequate to understand the global structure of space-time. Is there a well-defined but genuinely quantum notion of causality which reduces to the familiar one on quantum states which are sharply peaked on a classical geometry? Or, do we just abandon the idea that space-time geometry dictates causality and formulate physics primarily in relational terms? There is a plethora of such exciting challenges. Their scope is vast, they force us to introduce novel concepts and they lead us to unforeseen territories. These are just the type of omens that foretell the arrival of a major paradigm shift to take us beyond the space-time continuum of Minkowski and Einstein.

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References

1. P. Hajicek, Origin of black hole radiation, Phys. Rev. **D36**, 1065 (1987)
2. A. Ashtekar and J. Lewandowski, Background independent quantum gravity: A status report, Class. Quant. Grav. **21**, R53-R152 (2004)
3. C. Rovelli,*Quantum Gravity.* (Cambridge University Press, Cambridge (2004))
4. T. Thiemann, *Introduction to Modern Canonical Quantum General Relativity.* (Cambridge University Press, Cambridge, (2007))
5. A. Ashtekar, J. Lewandowski, D. Marolf, J. Mourão and T. Thiemann, Quantization of diffeomorphism invariant theories of connections with local degrees of freedom. Jour. Math. Phys. **36**, 6456–6493 (1995)
6. R. Rovelli and L. Smolin, Discreteness of area and volume in quantum gravity, Nucl. Phys. **B442**593-622 (1995); Erratum **B456**, 753 (1996)
7. A. Ashtekar and J. Lewandowski, Quantum theory of geometry I: Area operators, Class. Quantum Grav. **14**, A55-81 (1997)
8. A. Ashtekar and J. Lewandowski, Quantum theory of geometry II: Volume operators, Adv. Theo. & Math. Phys. **1**, 388-429 (1997)
9. A. Perez, Spin foam models for quantum Class. Quant. Grav. **20**, R43-R104 (2003)
10. M. Bojowald, Absence of singularity in loop quantum cosmology, Phys. Rev. Lett. **86**, 5227-5230 (2001), `arXiv:gr-qc/0102069`.
11. M. Bojowald, Loop quantum cosmology. Liv. Rev. Rel. **8**, 11 (2005).
12. A. Ashtekar, An introduction to loop quantum gravity through cosmology, Nuovo Cimento, **112B**, 1-20 (2007), `arXiv:gr-qc/0702030`
13. A. Ashtekar and M. Bojowald, Quantum geometry and the Schwarzschild singularity. Class. Quantum Grav. **23**, 391–411 (2006)
    L. Modesto, Black hole interior from loop quantum gravity, `gr-qc/0611043`
14. C. G. Boehmer and K. Vandersloot, Loop Quantum Dynamics of the Schwarzschild Interior, Phys. Rev. **D76** 104030
15. A. Ashtekar, V. Taveras and M. Varadarajan, Information is not lost in 2-dimensional black hole evaporation, `arXiv:0801.1811`
16. J. B. Hartle and S. W. Hawking, Wave Function Of The Universe. Phys. Rev. D **28**, 2960 (1983)
17. M. Gasperini and G. Veneziano, The pre-big bang scenario in string cosmology, Phys. Rept. **373**, 1 (2003) `arXiv:hep-th/0207130`.
18. J. Khoury, B. A. Ovrut, P. J. Steinhardt, N. Turok, The Ekpyrotic Universe: Colliding Branes and the Origin of the Hot Big Bang, Phys.Rev. D64 (2001) 123522 `hep-th/0103239`.
19. J. Khoury, B. . Ovrut, N. Seiberg, P. J. Steinhardt, N. Turok, From Big Crunch to Big Bang, Phys.Rev. D65 (2002) 086007 `hep-th/0108187`.
20. A. Ashtekar and J. Stachel, eds *Conceptual problems of Quantum Gravity* (Birkhäuser, Boston, 1988)
21. A. Komar, Quantization program for general relativity. In: *Relativity*, Carmeli, M., Fickler, S. I., Witten, L. (eds.) (Plenum, New York (1970);
    K. Kuchař, Canonical methods of quantization. In: *Quantum Gravity 2, A Second Oxford Symposium*, Isham, C. J., Penrose, R., Sciama, D. W. (eds.) (Clarendon Press, Oxford (1981))
22. B. S. DeWitt (1967): Quantum theory of gravity I. The canonical theory. *Phys. Rev.* **160**, 1113–1148 (1967).

23. C. W. Misner, Mixmaster universe. Phys. Rev. Lett. **22**, 1071-1074 (1969); Minisuperspace. In: *Magic without Magic: John Archibald Wheeler; a collection of essays in honor of his sixtieth birthday.* (W. H. Freeman, San Francisco (1972)
24. A. Ashtekar, T. Pawlowski and P. Singh, Quantum nature of the big bang: An analytical and numerical investigation I, Phys. Rev. **D73**, 124038 (2006)
25. A. Ashtekar, T. Pawlowski and P. Singh, Loop quantum cosmology in the pre-inflationary epoch (in preparation)
26. C. Rovelli, Quantum mechanics without time: A model, Phys. Rev. **D42**, 2638 (1990)
27. A. Ashtekar, T. Pawlowski and P. Singh, Quantum nature of the big bang: Improved dynamics, Phys. Rev. **D74**, 084003 (2006)
28. C. Kiefer, Wavepsckets in in minisuperspace, Phys. Rev. **D38**, 1761-1772 (1988)
29. D. Marolf, Refined algebraic quantization: Systems with a single constraint. `arXive:gr-qc/9508015`;
Quantum observables and recollapsing dynamics. Class. Quant. Grav. **12**, 1199–1220 (1994);
A. Ashtekar, L. Bombelli and A. Corichi, Semiclassical states for constrained systems, Phys. Rev. **D72**, 025008 (2005)
30. A. Kamenshchik, C. Kiefer and B. Sandhofer, Quantum cosmology with big break singularity, Phys. Rev. **D76**, 064032 (2007)
31. A. Ashtekar, T. Pawlowski, P. Singh and K. Vandersloot, Loop quantum cosmology of k=1 FRW models. Phys. Rev. **D75**, 0240035 (2006);
L. Szulc, W. Kaminski, J. Lewandowski, Closed FRW model in loop quantum cosmology. Class. Quant. Grav. **24**, 2621–2635 (2006)
32. J. Lewandowski, A. Okolow, H. Sahlmann and T. Thiemann, Uniqueness of diffeomorphism invariant states on holonomy flux algebras, Comm. Math. Phys. **267**, 703-733 (2006);
C. Fleishchack, Representations of the Weyl algebra in quantum geometry. `arXiv:math-ph/0407006`.
33. A. Ashtekar, M. Bojowald and J. Lewandowski, Mathematical structure of loop quantum cosmology. Adv. Theo. Math. Phys. **7**, 233–268 (2003)
34. A. Ashtekar, T. Pawlowski and P. Singh, Quantum nature of the big bang, Phys. Rev. Lett. **96**, 141301 (2006), `arXiv:gr-qc/0602086`.
35. J. Willis On the low energy ramifications and a mathematical extension of loop quantum gravity. Ph.D. Dissertation, The Pennsylvaina State University (2004);
A. Ashtekar, M. Bojowald and J. Willis, Corrections to Friedmann equations induced by quantum geometry, IGPG preprint (2004);
V. Taveras, LQC corrections to the Friedmann equations for a universe with a free scalar field, IGC preprint (2007)
36. M. Bojowald, Dynamical coherent states and physical solutions of quantum cosmological bounces, Phys. Rev. D 75, 123512 (2007)
37. A. Ashtekar, A. Corichi and P. Singh, Robustness of predictions of loop quantum cosmology, Phys. Rev. **D77**, 024046 (2008)
38. D. Green and W. Unruh (2004): Difficulties with recollapsing models in closed isotropic loop quantum cosmology. Phys. Rev. **D70**, 103502; `arXiv:gr-qc/04-0074`.
39. A. Ashtekar and T. A. Schilling, Geometrical formulation of quantum mechanics. In: *On Einstein's Path: Essays in Honor of Engelbert Schücking*, Harvey, A. (ed.) (Springer, New York (1999)), 23–65, `arXiv:gr-qc/9706069`.

40. R. Penrose, Zero rest mass fields including gravitation, *Proc. R. Soc. (London)* **284** 159-203 (1965)
41. S. W. Hawking The event horizon in *Black holes*, DeWitt B S and DeWitt C M (eds) (North-Holland, Amsterdam (1972))
42. S. W. Hawking, Gravitational radiation from colliding black holes *Phys. Rev. Lett.* **26**, 1344-1346 (1971)
43. S. W. Hawking, particle creation by black holes, Commun. Math. Phys. **43**, 199-220 (1975)
44. A. Ashtekar, C. Beetle, O. Dreyer, S. Fairhurst, B. Krishnan, J. Lewandowski, and J. Wisniewski, Generic Isolated Horizons and their Applications, Phys. Rev. Lett. 85, 3564-3567 (2000)
45. A. Ashtekar and B. Krishnan, Dynamical horizons: energy, angular momentum, fluxes and balance laws *Phys. Rev. Lett.* **89** 261101-261104 (2002)
46. Ashtekar A and Krishnan B Dynamical horizons and their properties, *Phys. Rev.* **D68** 104030-104055 (2003)
47. Ashtekar A and Krishnan B (2004) Isolated and dynamical horizons and their applications *Living Rev. Rel.* **10** 1-78, `gr-qc/0407042`
48. A. Ashtekar and G. Galloway, Some uniqueness results for dynamical horizons, Adv. Theor. Math. Phys. **9** 1-30 (2005)
49. A. Ashtekar, J. Baez A. Corichi and K. Krasnov, Quantum Geometry and Black Hole Entropy, Phys. Rev. Lett. **80** 904-907 (1998);
A. Ashtekar, J. Baez and K. Krasnov, Quantum geometry of isolated horizons and black hole entropy, Adv. Theor. Math. Phys. **4** 1-94 (2000);
A. Ashtekar, J. Engle, C. Van Den Broeck, Quantum horizons and black hole entropy: Inclusion of distortion and rotation, Class. Quant. Grav. **22**, L27-L33 (2005)
50. A. Ashtekar and M. Bojowald, Black hole evaporation: A paradigm, Class. Quant. Grav. **22**,(2005) 3349-3362 (2005)
51. C. G. Callen, S. B. Giddings, J. A. Harvey and A. Strominger, Evanescent black holes, Phys. Rev. **D45**, R1005-R1010 (1992)
52. S. B. Giddings, Quantum mechanics of black holes, `arXiv:hep-th/9412138`;
A. Strominger, Les Houches lectures on black holes, `arXiv:hep-th/9501071`
53. R. Geroch and G. Horowitz, Asymptotically simple does not imply asymptotically Minkowskian, Phys. Rev. Lett. **40**, 203-207 (1978); Erratum: Phys. Rev. Lett. 40, 483 (1978)
54. S. B. Giddings and W. M. Nelson, Quantum emission from two-dimensional black holes, Phys. Rev. **D46**, 2486-2496 (1992)
55. A. Ashtekar and M. Pierri, Probing quantum gravity through exactly soluble midi-superspaces I, J. Math. Phys. **37**, 6250-6270 (1996)
56. A. Ashtekar, Large quantum gravity effects: Unexpected limitations of the classical theory, Phys.Rev.Lett. **77**, 4864-4867 (1996)
57. D. A. Low, Semiclassical approach to black hole evaporation, Phys. Rev. **D47**, 2446-2453 (1993);
T. Pirqan and A. Strominger, Numerical analysis of black hole evaporation, Phys. Rev. **D48**, 4729-4734
58. A. Laddha, Polymer quantization of the CGHS model I, **24**, 4969-4988 (2007) Polymer quantization of the CGHS model II, Class. Quantum Grav. **24**, 4989-5009 (2007)