Informal Discussion on:
The Recent Paper *Soft Hair on BHs*

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Organization

Five parts:

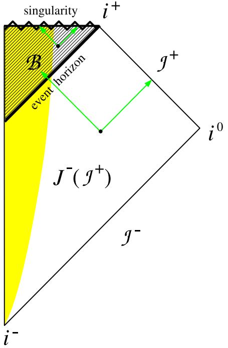
1) Sharing my understanding of the main ideas of the paper arXiv 1601.00921 by Hawking, Perry and Strominger

2) Discuss the conceptual issue of vacuum degeneracy that is central to the discussion. Although the paper seems to present this as new material, these are in fact old results from 1980s

3) Explain new ideas regarding horizons

4) Discuss what I see as limitations

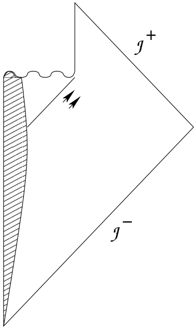
5) If time permits: Future prospects for us in LQG.



Much of the discussion in the paper uses *Maxwell theory* to illustrate vacuum degeneracy.

This is an **informal** discussion. Please stop and ask questions, suggest new ideas, directions, etc. I don't have to cover all five points in the hour we have.

1. Main ideas



A. The late time vacuum at $\mathcal{I}^+$ that results from the BH evaporation is not unique. There is an infinite vacuum degeneracy. This is related to the enlargement of the symmetry group from the Poincaré group to the BMS.

B. This degeneracy opens a door to restore correlations that appear to have been lost because for a long time the radiation is (approximately) thermal. Roughly: if two black holes with the same final (M, J) were produced by different incoming states, their final states will have radiation correlated with different final vacua and hence distinct.

C. Degeneracy also occurs on the horizon so even though the final BH produced by two different incoming states have same (M, J), the quantum states of their 'horizon geometry' would be different.

D. The argument is meant only to point a new direction to the resolution of the information loss puzzle. It does not address the issue of how the vacuum degeneracy at $\mathcal{I}^+$ leads to restoration of correlations to make the final state pure.

2. Vacuum Degeneracy

• Let us begin with the free Maxwell theory in Minkowski space. Using positive and negative frequency decomposition, we construct the 1-particle Hilbert space $\mathcal{H}$ and then the Fock representation. $\mathcal{H}$ consists of solutions F_{ab} to Maxwell's equations with finite norm $\|F\|$.

• Every solution F_{ab} is characterized by its initial data on $\mathcal{I}^+$ (or $\mathcal{I}^-$), which is: $\mathcal{E}_a = F_{\underleftarrow{ab}} n^b$. Therefore, the 1-particle Hilbert space is given by vector-fields $\mathcal{E}_a$ on $\mathcal{I}^+$ which have finite norm:

$$\|\mathcal{E}\|^2 = (1/\hbar) \int d^2S \int_0^\infty \frac{d\omega}{\omega} |\tilde{\mathcal{E}}(\omega, \theta, \phi)|^2.$$

• There are perfectly regular classical solutions (with finite energy-momentum and angular momentum) F_{ab} for which the norm is infinite. Key observation: At $\mathcal{I}^+$, this fact has a very simple, geometric interpretation in space-time terms, i.e., without having to do a Fourier decomposition.

• Suppose $\mathcal{E}(u, \theta, \phi)$ on $\mathcal{I}^+$ is in the Schwartz space. Then,

$$\|\mathcal{E}\|^2 < \infty \quad \text{iff} \quad Q_a(\theta, \phi) = \int_{-\infty}^{\infty} du\, \mathcal{E}_a(u, \theta, \phi) = 0.$$

Or, in the Newman-Penrose (NP) language, $Q(\theta, \phi) = \int_{-\infty}^{\infty} du\, \Phi_2^0(u, \theta, \phi) = 0.$

Maxwell theory (Continued)

- In terms of the Maxwell potential A_a adapted to t=const slices ($A_a \hat{t}^a = 0$) we have at $\mathcal{I}^+$:

  $\mathcal{E}_a = \partial_u A_a$ so that the condition becomes: $Q_a(\theta, \phi) = [A_a]_{u=-\infty}^{u=\infty} = 0$.

- If $Q(\theta, \phi) \neq 0$ then the 1-particle norm of the solution is infrared divergent.

- Take an $\mathring{\mathcal{E}}_a$ on $\mathcal{I}^+$ which is in Schwartz space which is infrared divergent. Then on the algebra of observables,

  $\hat{\mathcal{E}}_a \to \hat{\mathcal{E}}_a + \mathring{\mathcal{E}}_a \hat{I}$ on $\mathcal{I}^+$, or $\hat{F}_{ab} \to \hat{F}_{ab} + \mathring{F}_{ab} \hat{I}$ in space-time

  is not unitarily implementable. Therefore, the automorphism provides a new representation of the canonical commutation relations which is unitarily inequivalent to the Fock representation. In this representation, the vacuum expectation value of $\hat{\mathcal{E}}_a$ at $\mathcal{I}^+$ is $\mathring{E}_a$.

Maxwell Theory (continued)

• The new representation is labelled by the 'charge' $Q(\theta, \phi)$ and not by the individual $\mathring{\mathcal{E}}$ used in the automorphism.. If we choose a different $\mathring{\mathcal{E}}_a$ with the same infrared 'charge' $Q_(\theta, \phi)$, we obtain the same representation. For any given $Q_(\theta, \phi)$, we can choose a $\mathring{\mathcal{E}}_a$ with arbitrarily small energy. Hence these non-Fock representations are called infrared sectors.

• These non-Fock, infrared representations are essential to make the (order by order) S-matrix well-defined in QED. In presence of charges that scatter non-trivially, the outgoing photon state at $\mathcal{I}^+$ is in a non-Fock representation, labelled by $Q(\theta, \phi)$ which is determined by the difference between the asymptotic velocities of the charges in the distant past and those in the distant future.

These results on the Maxwell theory are not new. They were obtained in the early 80s. A paper by AA and K.S. Narain, presented at the IUMP conference in 1981 and sumarized in AA: Asymptotic Quantization Book, pages 78-83.

Vacuum Degeneracy: Einstein gravity

- The Maxwell work was done at the same time as the Asymptotic Quantization program was developed for Einstein gravity using $\mathcal{I}^+$.

- At $\mathcal{I}^+$ of any asymptotically flat space-time, the intrinsic metric and the null normal are *universal*. What differs from one space-time to another is the intrinsic connection D on $\mathcal{I}^+$. More precisely, the equivalence class $\{D\}$ of connections under conformal transformations since, unlike Maxwell theory, GR is not conformally invariant.

- $\{D\}$ encodes the two radiative modes of the full non-linear gravitational field. The (conformally invariant part of) curvature of $\{D\}$ is the Bondi News tensor N_{ab}. If $N_{ab} = 0$, $\{D\}$ is said to define a vacuum configuration (in analogy with the terminology in the Yang-Mills theory).

- Analogy with Maxwell/YM: $A_a \leftrightarrow \{D\}, \quad \text{and} \quad \mathcal{E}_a \leftrightarrow N_{ab}.$

Einstein Gravity (continued)

b Natural question: How many vacua $\{\mathring{D}\}$ are there? These are the equivalence classes of connections with trivial curvature (i.e. zero news).

- Detailed analysis shows that each $\{\mathring{D}\}$ is left invariant precisely by a Poincaré subgroup of the BMS group. So there are as many vacua as there are Poincaré subgroups. So:

There is a 1-1 correspondence between a classical vacuum $\{\mathring{D}\}$ and the quotient $\mathcal{S}/\mathcal{T}$ of the infinite-dimensional group of BMS supertranslations by its 4-d subgroup of translations.

That is the degeneracy of classical vacua is precisely why the Poincaré group is enlarged to the BMS group!

Einstein Gravity: Quantum considerations

• The phase space Γ_R of radiative modes of the full non-linear gravitational field can be conveniently coordinatized by the $\{D\}$ at $\mathcal{I}^+$. It has a natural symplectic structure, induced by that on the covariant phase space of general relativity. Using the translation subgroup of the MBS group, one can introduce on Γ_R unique complex structure that is compatible with the symplectic structure (i.e., unique $\pm$-frequency decomposition).

• We then obtain a unique Fock representation for the asymptotic quantum states of full non-linear GR. One can work out the mass and spin using any Poincaré subgroup of the Poincaré group. We obtain $m = 0$ and helicity $s = 2$. A precise relation between the non-linear and linear theory.

• As in the Maxwell theory we have non-Fock representations labelled by the charge: $Q_{ab}(\theta, \phi) = \int_{-\infty}^{\infty} N_{ab}(u, \theta, \phi)\, du$ or, in the NP notation, $Q(\theta, \phi) = \int_{-\infty}^{\infty} \dot{\sigma}(u, \theta, \phi)\, du = [\sigma]_{u=-\infty}^{u=\infty}(\theta, \phi)$.

Einstein Gravity: Quantum considerations (continued)

- As in the Maxwell theory, these are the infrared sectors that are distinguished from the Fock representation because they carry soft gravitons. The main idea is that In the BH evaporation process, the state in the distant future would be full of soft gravitons (relative to the distant past) and therefore not unique.

These ideas were spelled out in the 1980s:

Radiative modes and Classical Vacua:

A. Ashtekar, Radiative degrees of freedom of the gravitational field in exact general relativity, J. Math. Phys. 22, 2885-2895 (1981).
A. Ashtekar and M. Streubel, Symplectic geometry of radiative modes and conserved quantities at null infinity, Proc. R. Soc. (London) A376, 585-607 (1981).
A. Ashtekar and A. Magnon, On the symplectic structure of general relativity, Comm.Math. Phys. 86, 55-68 (1982).

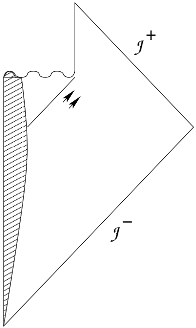
Quantization:

A. Ashtekar, Asymptotic quantization of the gravitational field, Phys. Rev. Lett. 46, 573-577 (1981);
A. Ashtekar, Quantization of the radiative modes of the gravitational field, In: Quantum Gravity 2; Edited by C. J. Isham, R. Penrose, and D. W. Sciama (Oxford University Press, Oxford, 1981).

Summary:

A. Ashtekar, Asymptotic Quantization (Bibliopolis, Naples (1987)).

1. Main ideas



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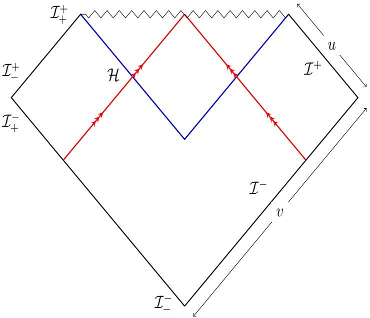
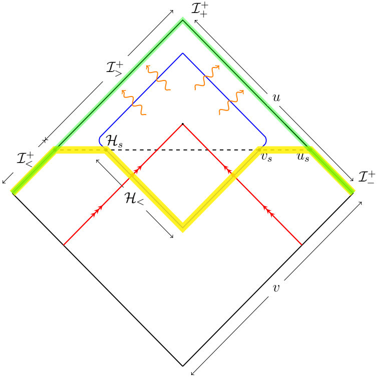
3. Horizon considerations

Figures from the HPS paper



Classical Collapse



'semi-classical' space-time

4. List of Comments/Issues

Limitations (based on my understanding):

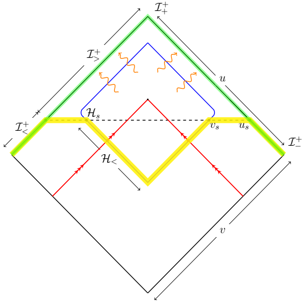
- Semi-classical versus full quantum space-time: The second Penrose diagram (Fig 2) in HPS is labelled "semi-classical evaporating black" hole. Yet it has no singularity. This is not the case even in CGHS case. There is no reason to believe that semi-classical effects will resolve the singularity.

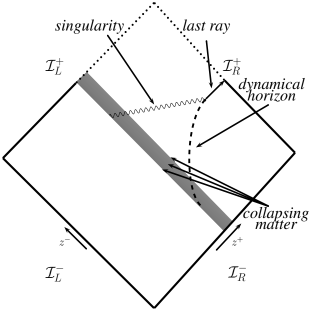
- Purity restored in the semi-classical regime? This seems to be a theme beyond HPS. In the semi-classical space-time, there will be a singularity and so there is no reason to believe that the state at $\mathcal{I}^+$ will be pure.

- No indication of how purity is restored. (However, this is a limitation explicitly acknowledged in the paper.)

- There is reference to S-matrix with an implicit assumption that $\mathcal{I}^+$ can be mapped unambiguously to $\mathcal{I}^-$. In presence of mass, there is Singularity at i^0 which would makes this identification extremely non-trivial if not impossible.

List of Comments/Issues (continued)





- Reference to a horizon in the HPS Fig 2 (shown in rthe left panel) is mysterious. The Penrose diagram has no singularity or an event horizon since the past of $\mathcal{I}^+$ is the entire space-time. What is referred to as $H_<$ is not a dynamical horizon either since there are no marginally trapped surfaces at least in the Minkowski portion of this space-time.

- In any case, what forms and evaporates is a dynamical horizon which is space-like in the classical region and becomes time-like once the inflow of classical matter goes to zero i.e. is dominated by quantum radiation. (Right panel.) So it is far from clear how horizon "soft hair" arise.

4. Future prospects for us in LQG

- Paradigm: Singularity resolution makes quantum space-time larger. Correlations can be restored by the part of the state arriving at $\mathcal{I}^+$ through the quantum region that replaces the singularity. What forms and evaporates is a dynamical horizon. No event horizons. $\mathcal{I}^+$ is complete.

- Information about the incoming state is encoded in the portion of the horizon that expands soon after the classical matter flow begins till the horizon reaches (approximate) isolation. The idea of 'screens' by the AEI group (Jaramillo, Rezzolla, et al) tells us that this information is encoded in the classical outgoing radiation.

- Long evaporation phase when the outgoing radiation is thermal: Need to understand this phase better through semi-classical calculations in 4-d.

- How is purity restored? Long wave length modes that slowly leak out. This will create an 'infrared vacuum' in the final stage. Understand space-time geometry and restoration of correlations. (partial understanding from CGHS).

Future prospects (continued)

- Two related but distinct issues:

i) Is the final state pure? Here we want to focus on how correlations that seem to be lost during the long Hawking phase are restored on $\mathcal{I}^+$. Current view is that it is the very long wave length infrared radiation that is correlated with the (nearly) thermal radiation during the Hawking phase. and,

ii) Is the evolution from $\mathcal{I}^-$ to $\mathcal{I}^+$ unitary? Here the early radiation in the classical theory that is correlated with the space-like dynamical horizon settles down to become (nearly) isolated will correspond to coherent state at early times that will carry information about the incoming state that led to the collapse. The long thermal phase in which the dynamical horizon is time-like and shrinks will essentially know only about the mass and angular momentum of the (nearly) isolated horizon during the phase when the quantum radiation is negligible. This would be correlated to the final state near i^+ which will likely have infrared radiation (soft quanta). Because of the soft quanta, the state may be in a displaced Fock representation at $\mathcal{I}^+$.