Isolated Horizons: Ideas, framework and applications

Jonathan Engle
Centre de Physique Theorique, Marseille

(Abhay Fest, 2009)

(Workers in IH/DH: Ashtekar, Baez, Beetle, Booth, Corichi, Dryer, JE, Fairhurst, Krasnov, Krishnan, Lewandowski, Pawlowski, Schnetter, Schoemaker, Sudarsky, Wisniewski, Van Den Broeck ...
Many more recent people: Ansorg, Chatterjee, Ghosh, Jaramillo, Liko, Limousin, Xiaoning Wu, ...)

Goals

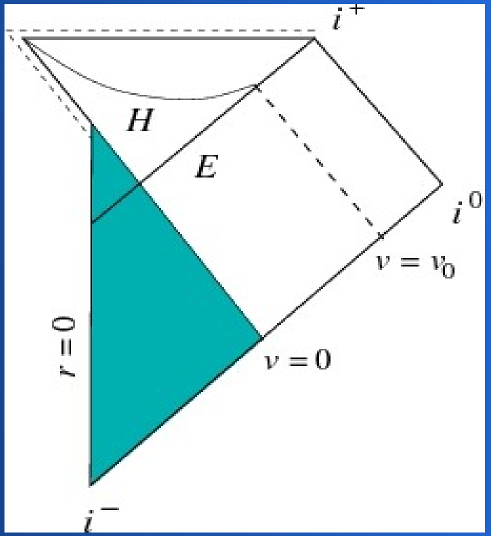
- To summarize *Isolated Horizons* and their results
  - Motivations and definitions
  - Covariant Hamiltonian framework: mass, angular momentum, multipoles, laws of BH mechanics
  - Mention Dynamical Horizons: Energy momentum flux and area balance law
- To summarize their applications
  - Classical (Numerical Relativity)
  - Quantum (Quantum Horizons and entropy)

Motivation

- Most commonly used horizons: Event H. and Killing H.
- Many applications
  - Event H. only `hard' way to define BH's generally
  - Laws of BH mechanics (e.g.:)

$$\delta M = \frac{\kappa}{8\pi G}\delta a + \Omega \delta J$$

- However: Event H. “too global"
- Furthermore: In 1st law, M & J defined at ∞, a at horizon, and k & Ω mixed.
- Killing Horizons assume symmetry beyond just at horizon – not available in realistic cases



- Isolated Horizons: Horizon is stationary but time dependence outside.
- Dynamical Horizons: Horizon itself is time dependent.

  (can be related to Hayward's trapping horizons)

Definition: Isolated Horizon

More precisely, an IH Δ is a null, $S^2 x R$ hypersurface s.t.

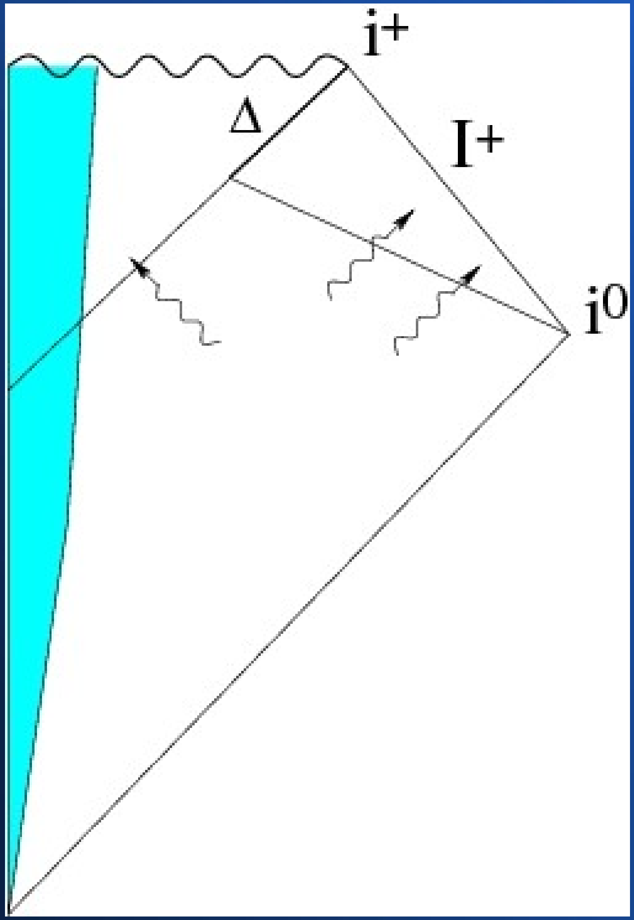
i. $-T_{ab}\, \ell^b$ is future causal; where ℓ^a is null normal

ii. $\Theta_{(\ell)} =_\Delta 0$ (Expansion-free) Ensures ∇_a induces D_a on Δ

iii. D_a commutes with $\mathcal{L}_\ell$ acting on vt fields on Δ

- Let q_{ab} := pull back of g_{ab}. Def'n implies $\mathcal{L}_\ell q_{ab} =_\Delta 0$.
- Generically ℓ^a unique upto constant rescalings: reminiscent of Killing horizons
- Killing horizons are IH, but infinitely many more ex.s of IH (local ex. Thm of Lewandowski)

Settling to IH at late times



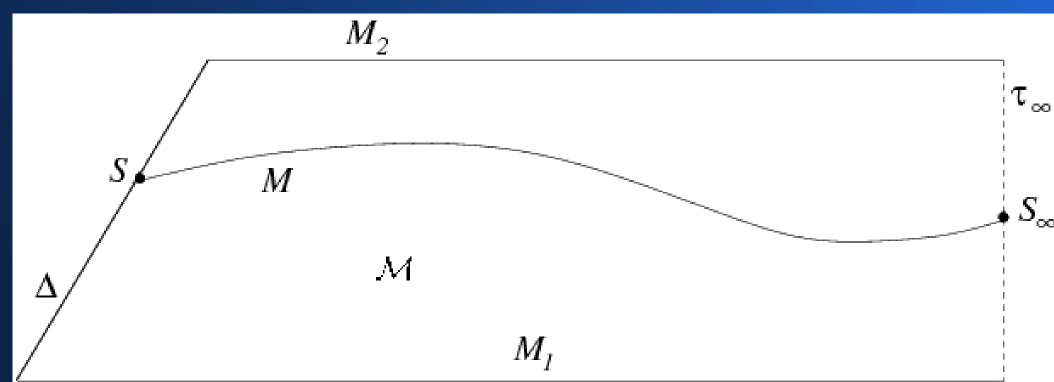
Laws of BH Mechanics

- 0^{th} law:

  $\ell^a D_a \ell^b = \kappa_{(\ell)} \ell^b \quad (\tilde{\ell} = c\ell \Rightarrow \kappa_{(\tilde{\ell})} = c\kappa_{(\ell)})$

  surface gravity $\kappa_{(\ell)}$ is const. on Δ .

- 1^{st} law: *use Hamiltonian methods* – covariant phase space



Angular Momentum

- Fix φ^a on space-time s.t. It is asymptotic rotation at infinity, and s.t. $\mathcal{L}_\varphi q_{ab} = 0$, $\mathcal{L}_\ell \varphi^a = 0$ at the horizon.
- Generator of this rotation is

$J^\varphi = J^\varphi_{ADM} - J^\varphi_\Delta$, where

$$J_\Delta = \frac{-1}{8\pi G}\oint \omega_a \varphi^a d^2V = \frac{-1}{4\pi G}\oint f\, \Im[\Psi_2] d^2V$$

$$D_a l^b = \omega_a l^b \qquad \varphi^a \epsilon_{ab} = \partial_b f$$

- J^φ is ang. mtm. of radiation in the bulk (is zero if φ is global KVF)
- J^φ_Δ is ang. mtm. of horizon; is defined locally – no ref. to infinity.

Fix φ at the horizon and restrict to space-times compatible with it there

Energy

- Fix t^a on space-time s.t. It is asymptotic TT at infinity,

  and s.t. $t^a = c_{(t)}\ell^a - \Omega_{(t)}\varphi^a$ at Δ, for some $c_{(t)}$ and $\Omega_{(t)}$ const on Δ.

  $\Omega_{(t)}$: angular velocity, $c_{(t)}$: determines surf. grav. $k_{(t)} = c_{(t)}k_{(\ell)}$.

- Evolution along t^a is Hamiltonian, i.e., *is generated by* ***some*** $E^{(t)} = E^{(t)}_{ADM} - E^{(t)}_{\Delta}$, iff

  a) $\kappa_{(t)}$ and $\Omega_{(t)}$ are functions only of a_Δ and J^φ_Δ

  b) $$\frac{\partial \kappa_{(t)}}{\partial J_\Delta^{(\varphi)}} = 8\pi G \frac{\partial \Omega_{(t)}}{\partial a_\Delta}$$

  But these are suff. to imply First Law – *for* $E^{(t)}_\Delta$*!*

  $$\delta E_\Delta^{(t)} = \frac{\kappa_{(t)}}{8\pi G}\delta a_\Delta + \Omega_{(t)}\delta J_\Delta^{(\varphi)}$$

  Furthermore $E^{(t)}_\Delta$ *is a local fn at Δ –* has interp of horizon energy corr. to $t^a|_\Delta$

Canonical choice of energy

- A priori, can construct infinite number of possible t^a and $E^{(t)}{}_{\Delta}$: each choice of $\kappa_{(t)}(a_\Delta, J^{(\varphi)}{}_\Delta)$ determines $\Omega(a_\Delta, J^{(\varphi)}{}_\Delta)$ determines t^a and $E^{(t)}{}_\Delta$. All of them satisfy first law.
- Is there canonical choice of t^a and hence $E^{(t)}{}_\Delta$? YES! *Stipulate that t^a reduce to stationary KVF on stationary space-times.*

Implies $$\kappa_o = \kappa_{kerr}(a_\Delta, J_\Delta) = \frac{R_\Delta^4 - 4G^2 J_\Delta^2}{2R_\Delta^3 (R_\Delta^4 + 4G^2 J_\Delta^2)}$$ $(a_\Delta = 4\pi R_\Delta^2)$

Leading to $$E_\Delta^{(t)} = \frac{\sqrt{R_\Delta^4 + 4G^2 J_\Delta^2}}{2GR_\Delta} \equiv M_\Delta$$ Satisfies "canonical" first law

Local expression, surprisingly simple, but derived – not postulated.

Symmetry Groups

- `horizon geometry' (q_{ab}, D_a)
  - Pull-back of metric to Δ
  - Derivative operator induced on Δ
- Type I: (q_{ab}, D_a) is spherically symmetric
- Type II: (q_{ab}, D_a) is axially symmetric
- Type III: (q_{ab}, D_a) has only ℓ symmetry

Chern-Simons symplectic structure

Was not said, but in the can. framework, in order for the symplectic structure `Ω' to be conserved, it is nec. to include a boundary term in the sympl. str.

In Type I case, the boundary sympl. str. takes Chern-Simons form in terms of Ashtekar-Barbero connection $A^i_a = \Gamma^i_a + \gamma K^i_a$:

$$\Omega_S(\delta_1, \delta_2) = \frac{-a_\Delta}{8\pi^2 G\gamma} \int_S Tr\, \delta_1 A \wedge \delta_2 A$$

(Also happens in Type II case, but more subtle …)

Is important for quantization of IH (next talk).

Further developments

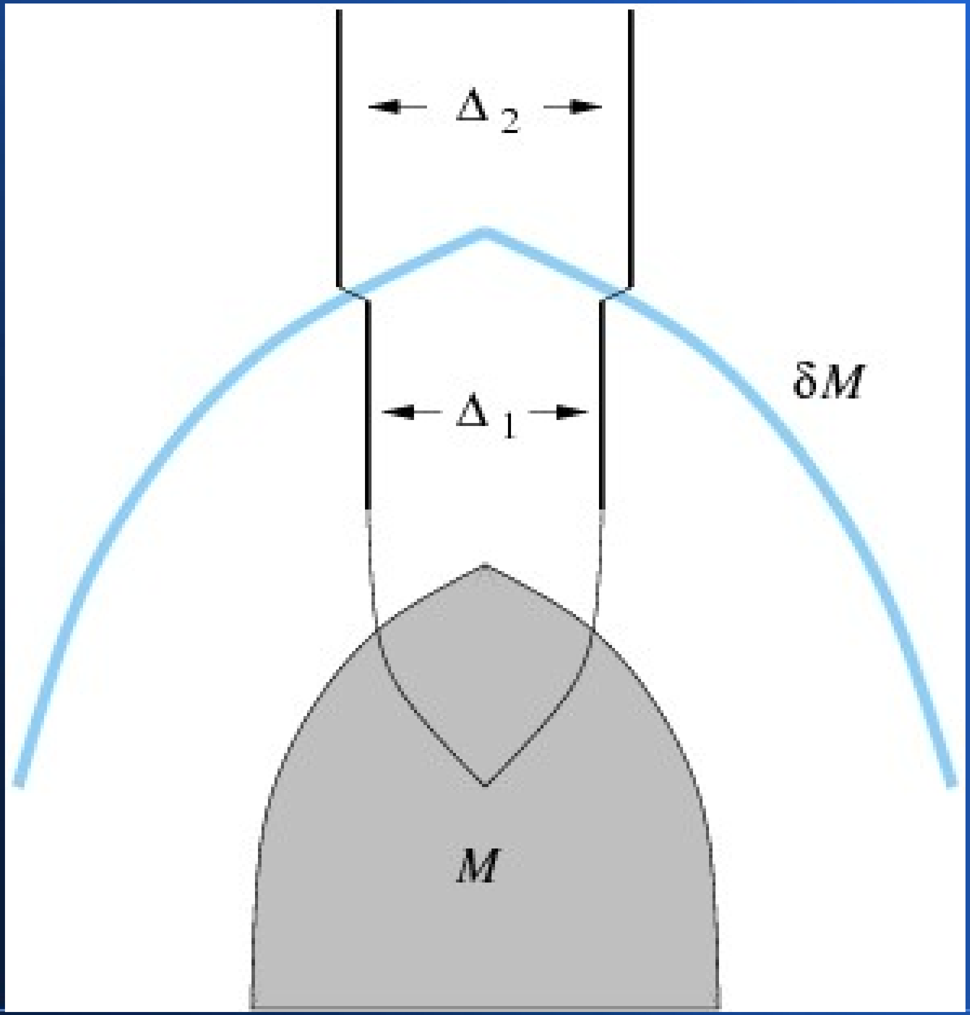
- Laws of BH mech. extend to Einstein-Maxwell and Einstein-YM cases
- Mass and angular momentum multipoles M_n , J_n in type II case
- Natural foliation of Δ, cov.-defined coordinates and tetrad in a neighbd of Δ
- Dynamical Horizons
  - Local expr. for energy-mtm flux across horizon
  - Area balance law and integral version of 1st law
  - 2nd law: Area always increases
- Higher dim. IH, and more recently: supersymm IH (Booth, Liko)...

Applications

- Numerical Relativity
  - Each continuous piece of apparent horizon world tube is a Dynamical Horizon
  - Many IH constructions can still be used (horizon angular mtm, horizon mass, multipoles)
- Quantum Isolated Horizons and Black hole entropy
  - See next talk
  - (new paper out on the non-gauge-fixed SU(2) calculation: JE, Noui, Perez, arxiv:0905.3168)

Summary

- IH and DH give quasilocal description of black holes allowing radiation arbitrarily close to horizon. IH: horizon is in equilibrium. (can be related to Hayward's trapping horizons)
- Can construct Hamiltonian framework for IH, define Ang. Mtm., Mass, Multipoles. 1^{st} law is satisfied with all quantities quasilocally defined.
- Is used in interpreting numerical simulations: simple and well-motivated expressions
- Is used in BH entropy calculations in LQG


Δ 2
Δ 1
δM
M