The Classical Physics of Our Quantum Universe

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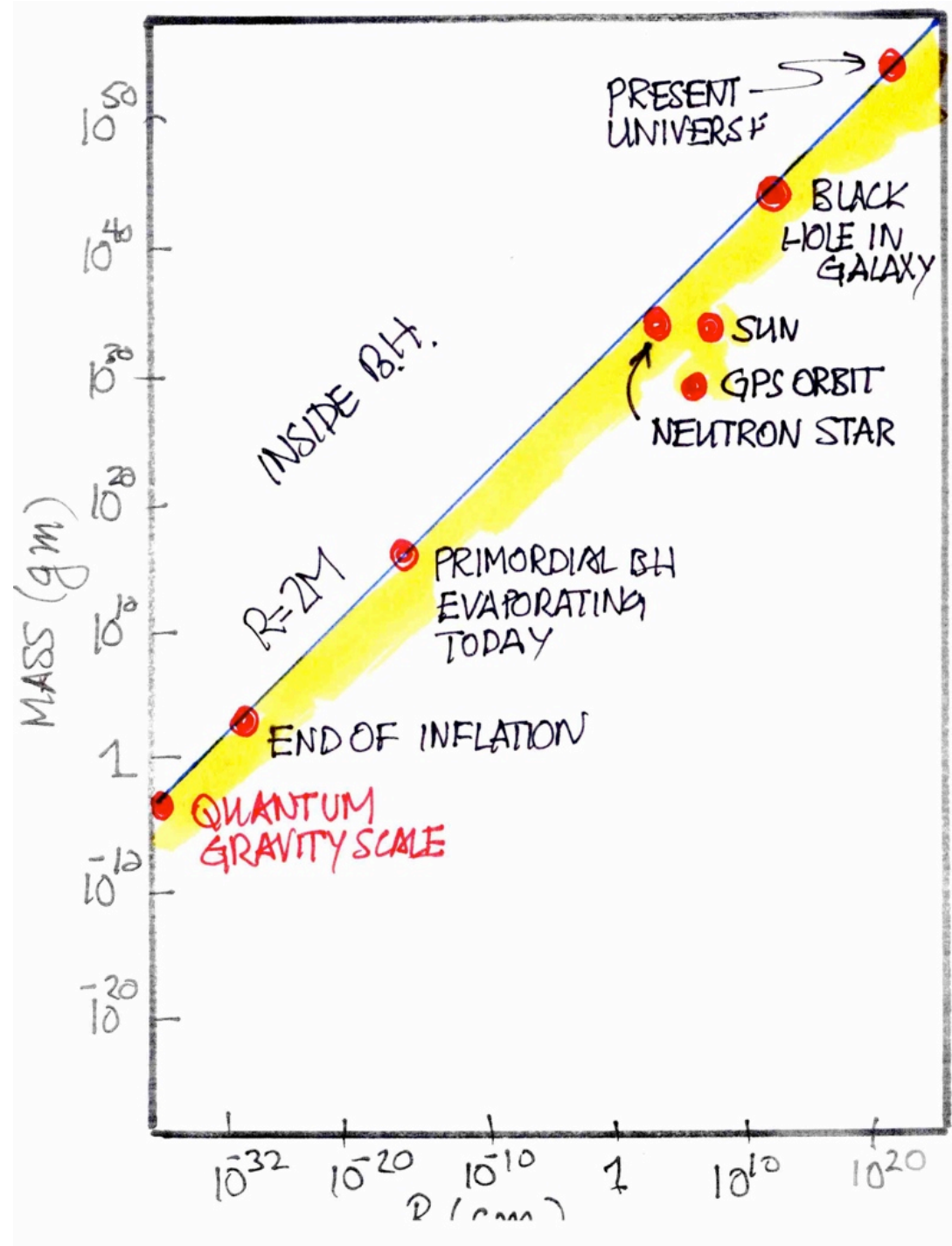
The Universe of GR in 1956

Einstein's obituary

``In the forty years that have elapsed [since the classic initial tests] these have remained the principal, and, with one exception the only connection between the general theory and experience.''
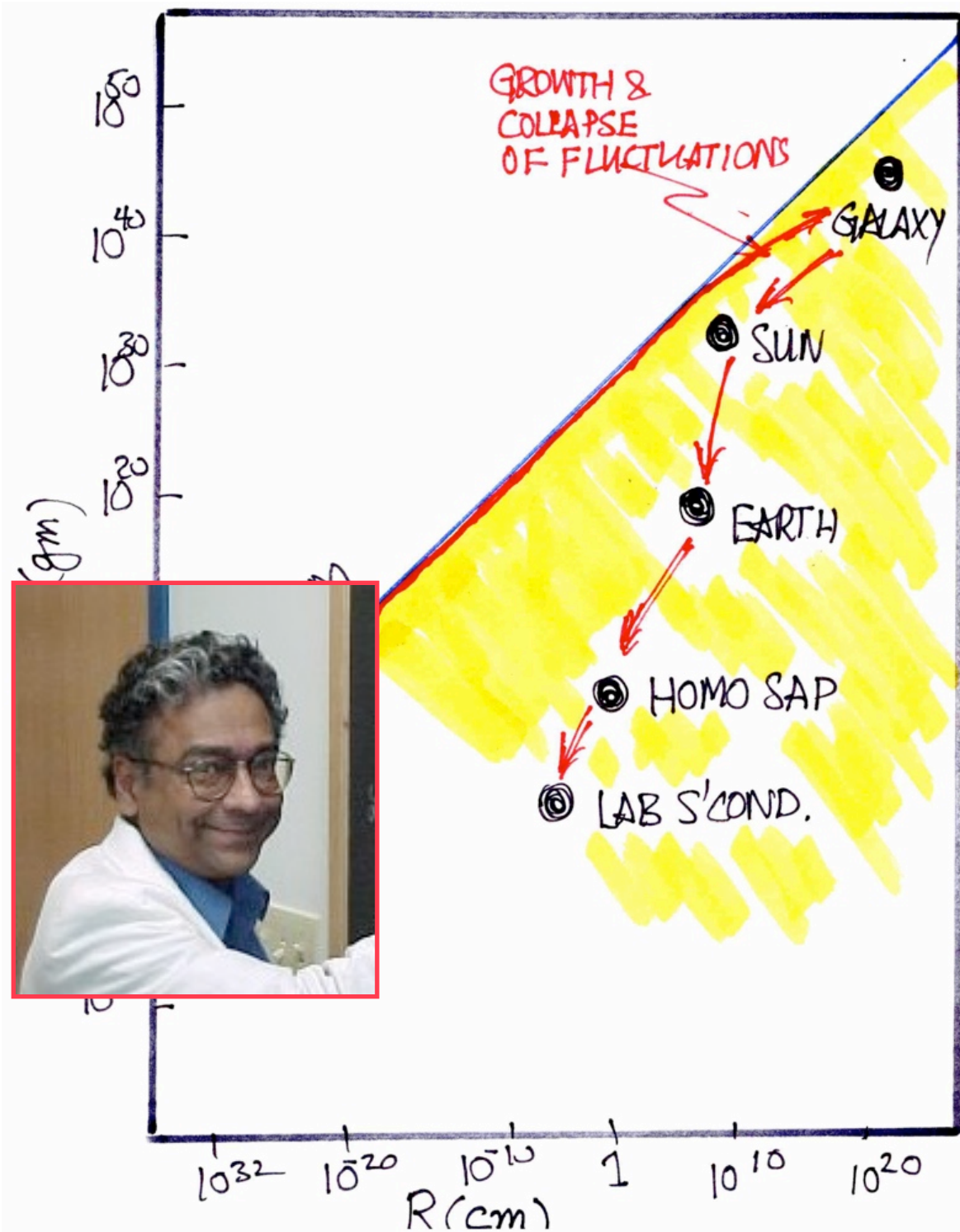

Einstein and Oppenheimer

The Universe of Classical GR Today

$$\frac{GM}{Rc^2} \sim 1$$

The Universe of Quantum General Relativity is much Larger

The origin of subsystems for which GR is unimportant today lies in the early quantum universe where gravity was central.

The Quasiclassical Realm - A feature of our Universe

The wide range of time, place and scale on which the laws of classical physics hold to an excellent approximation.

- Time --- from the Planck era forward.
- Place --- everywhere in the visible universe.
- Scale --- macroscopic to cosmological.

What is the origin of this quasiclassical realm in a quantum universe characterized fundamentally by indeterminacy and distibuted probabilities?

Classical spacetime is the key to the origin of the rest of the quasiclassical realm.

Origin of the Quasiclassical Realm

- Classical spacetime emerges from the quantum gravitational fog at the beginning.

- Local Lorentz symmetries imply conservation laws.

- Sets of histories defined by averages of densities of conserved quantities over suitably small volumes decohere.

- Approximate conservation implies these quasiclassical variables are predictable despite the noise from mechanisms of decoherence.

- Local equilibrium implies closed sets of equations of motion governing classical correlations in time.

Only Certain States Lead to Classical Predictions

- Classical orbits are not predictions of every state in the quantum mechanics of a particle.

- Classical spacetime is not a prediction of every state in quantum gravity.

Classical spacetime is the key to the origin of the quasiclassical realm.

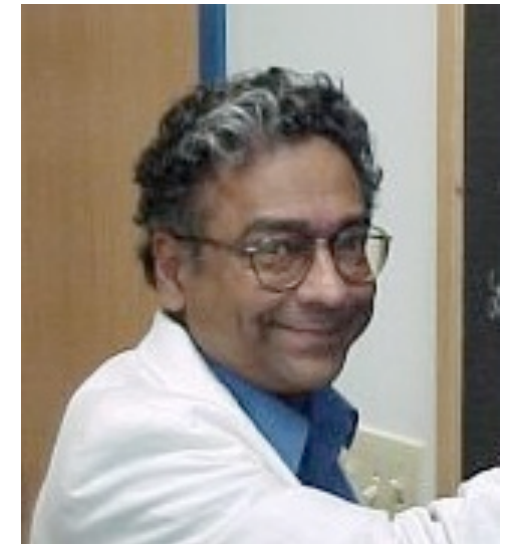The quantum state of the universe is the key to the origin of classical spacetime.

Ψ
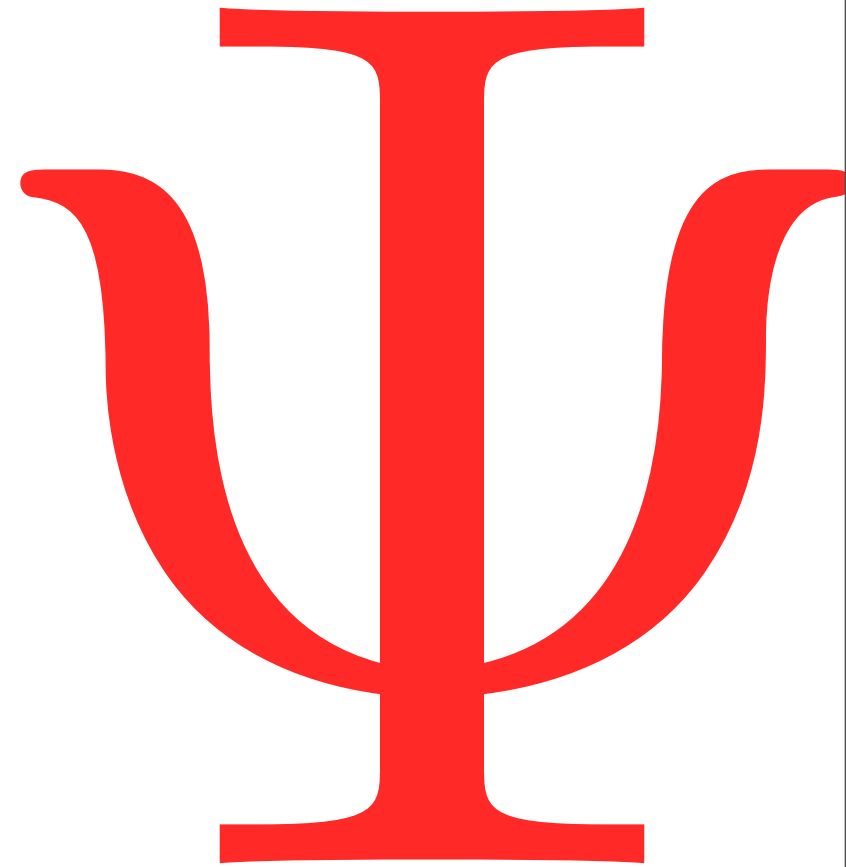
Loop Quantum Cosmology

Loop quantum gravity provides a fundamental framework for formulating a theory of the state and deriving its predictions for cosmology.
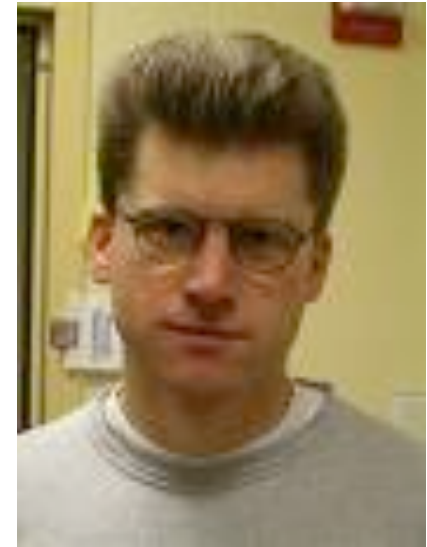
But at the moment we all seem forced to models.

The Classical Spacetimes of Our Universe

We seek a state that will not just predict some classical spacetime but which predicts classical spacetimes with a high probability for properties consistent with our cosmological observations.

- homogeneity and isotrop
- the amount of matter
- the amount of inflation
- a spectrum of density fluctuations the CMB and growth of large scale stru
- The thermodynamic arrow of time.

These are late time properties (compared to the Planck time) for which semiclassical physics may be adequate.

This Talk:

We analyze the classical spacetimes predicted by Hawking's no-boundary quantum state for a class of minisuperspace models.

$$\Psi = \int_{\mathcal{C}} \delta g \, \delta \phi \, \exp(-I[g, \phi])$$

Minisuperspace Models

Geometry: Homogeneous, isotropic, closed.

$$ds^2 = (3/\Lambda)\left[N^2(\lambda)d\lambda^2 + a^2(\lambda)d\Omega_3^2\right]$$

Matter: cosmological constant Λ plus homogeneous scalar field moving in a quadratic potential.

$$V(\Phi) = \frac{1}{2}m^2\Phi^2$$

Theory: Low-energy effective gravity.

$$I_C[g] = -\frac{m_p^2}{16\pi}\int_M d^4x(g)^{1/2}(R - 2\Lambda) + (\text{surface terms})$$

Classical Pred. in NRQM ---Key Points

Semiclassical form:

$$\Psi(q_0) = A(q_0)e^{iS(q_0)/\hbar}$$

- When S(q$_0$) varies rapidly and A(q$_0$) varies slowly, high probabilities are predicted for classical correlations in time of suitably coarse grained histories.

- For each q$_0$ there is a classical history with probability:

$$p_0 = \nabla S(q_0) \qquad p(\text{class.hist.}) = |A(q_0)|^2$$

NRQM -- Two kinds of histories

$$\Psi(q_0) = A(q_0)e^{iS(q_0)/\hbar}$$

- $S(q_0)$ might arise from a semiclassical approximation to a path integral for $\Psi(q_0)$ but it doesn't have to.

- If it does arise in this way, the histories for which probabilities are predicted are generally distinct from the histories in the path integral supplying the semiclassical approximation.

No-Boundary Wave Function (NBWF)

$$ds^2 = (3/\Lambda) \left[N^2(\lambda)d\lambda^2 + a^2(\lambda)d\Omega_3^2 \right]$$

$$\Psi(b, \chi) \equiv \int_{\mathcal{C}} \delta N \delta a \delta \phi \exp(-I[N(\lambda), a(\lambda), \phi(\lambda)]/\hbar)$$
.

The integral is over all $(a(\lambda), \phi(\lambda))$ which are regular on a disk and match the (b, χ) on its boundary. The complex contour is chosen so that the integral converges and the result is real.

Semiclassical Approx. for the NBWF

$$\Psi(b,\chi) \equiv \int_{\mathcal{C}} \delta N \delta a \delta \phi \exp(-I[N(\lambda), a(\lambda), \phi(\lambda)]/\hbar)$$

- In certain regions of superspace the steepest descents approximation may be ok.

- To leading order in ℏ the NBWF will then have the semiclassical form:

$$\Psi(b,\chi) \approx \exp\{[-I_R(b,\chi) + iS(b,\chi)]/\hbar\}.$$

- The next order will contribute a prefactor which we neglect. Our probabilities are therefore only relative.

Instantons and Fuzzy Instantons

In simple cases the extremal geometries may be real and involve Euclidean instantons, but in general they will be a complex --- fuzzy instantons.
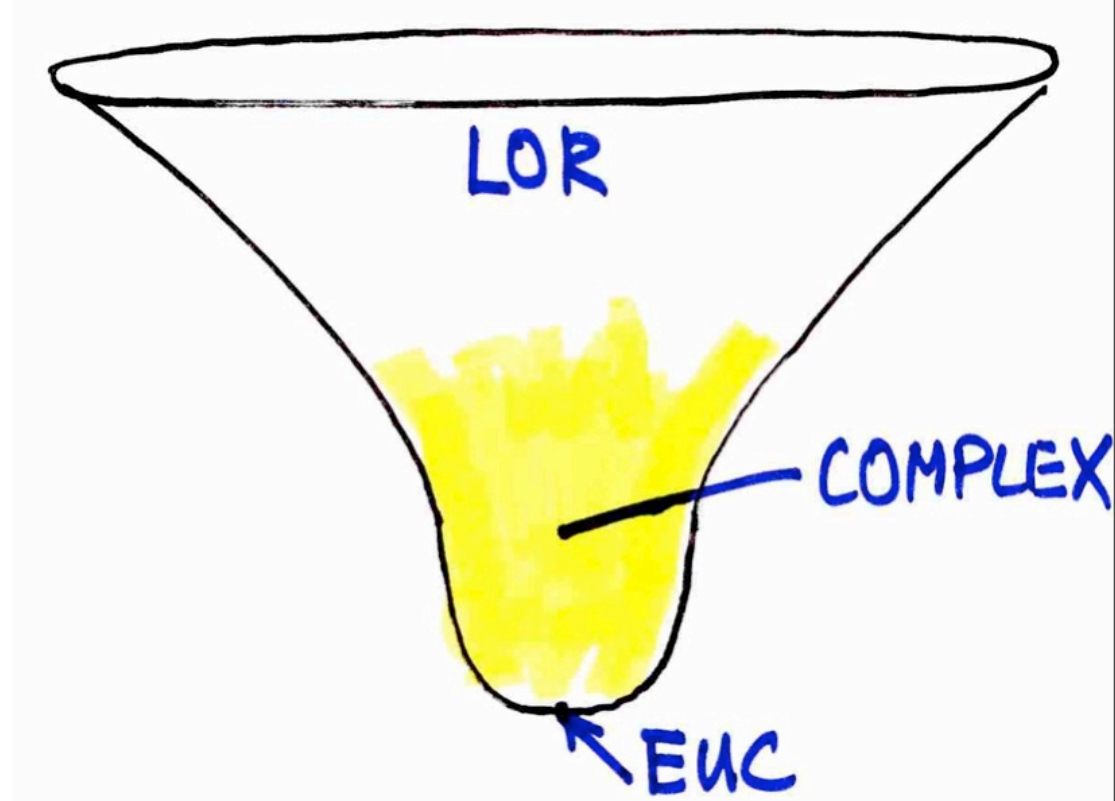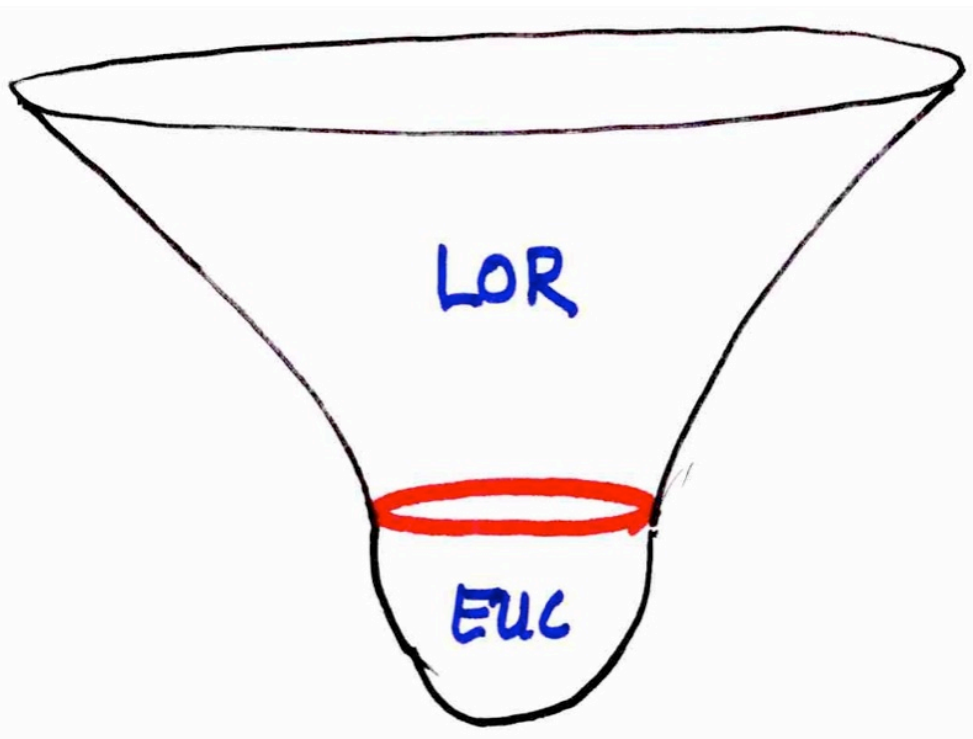
Classical Prediction in MSS and The Classicality Constraint

$$\Psi(b, \chi) \approx \exp\{[-I_R(b, \chi) + iS(b, \chi)]/\hbar\}$$

- Following the NRQM analogy this semiclassical form will predict classical Lorentian histories that are the integral curves of S, ie the solutions to:

$$p_A = \nabla_A S \qquad\qquad p(\text{class. hist.}) \propto \exp(-2I_R/\hbar)$$

- However, we can expect this only when S is rapidly varying and I_R is slowly varying, i.e.

$$|\nabla_A I_R| \ll |\nabla_A S|. \qquad\qquad |(\nabla I_R)^2| \ll |(\nabla S)^2|.$$

These consitute the classicality condition.

Hawking (1984), Grischuk &Rozhansky (1990), Halliwell(1990)

Class. Prediction --- Key Points

- The NBWF predicts probabilities for an ensemble of entire, 4d, classical histories.

- These real, Lorentzian, histories are not the same as the complex extrema that supply the semiclassical approximation to the integral defining the NBWF.
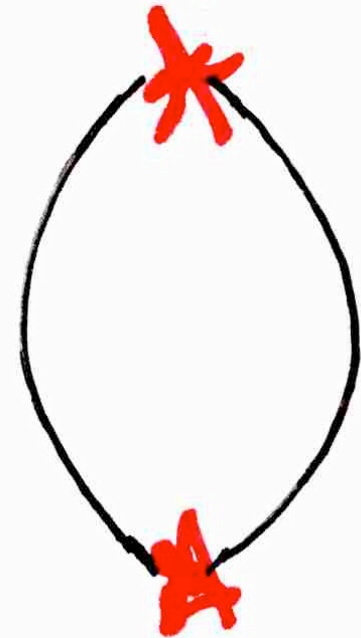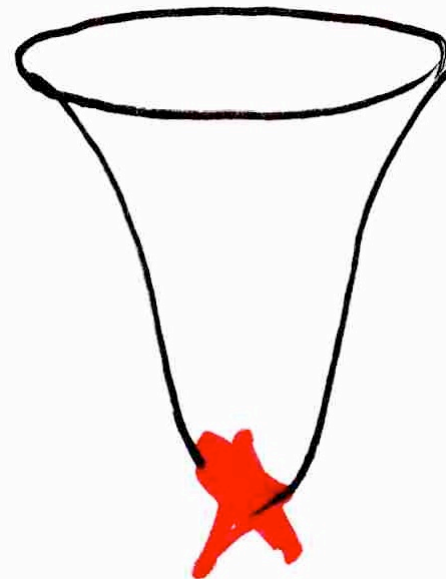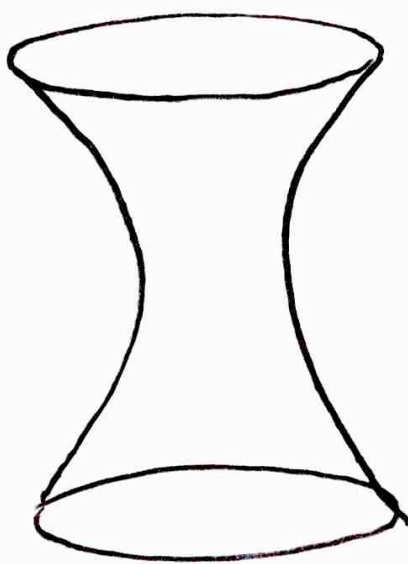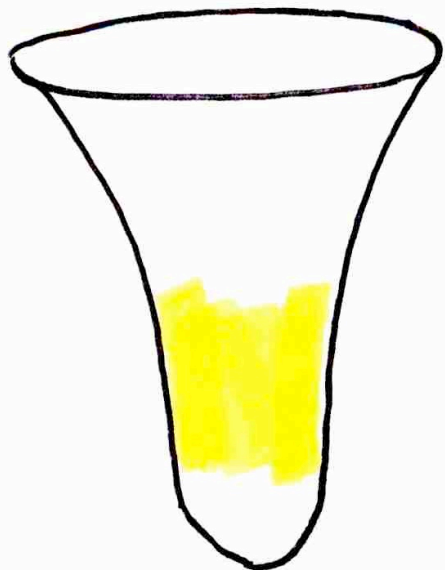
No-Boundary Measure on Classical Phase Space

The NBWF predicts an ensemble of classical histories that can be labeled by points in classical phase space. The NBWF gives a measure on classical phase space.
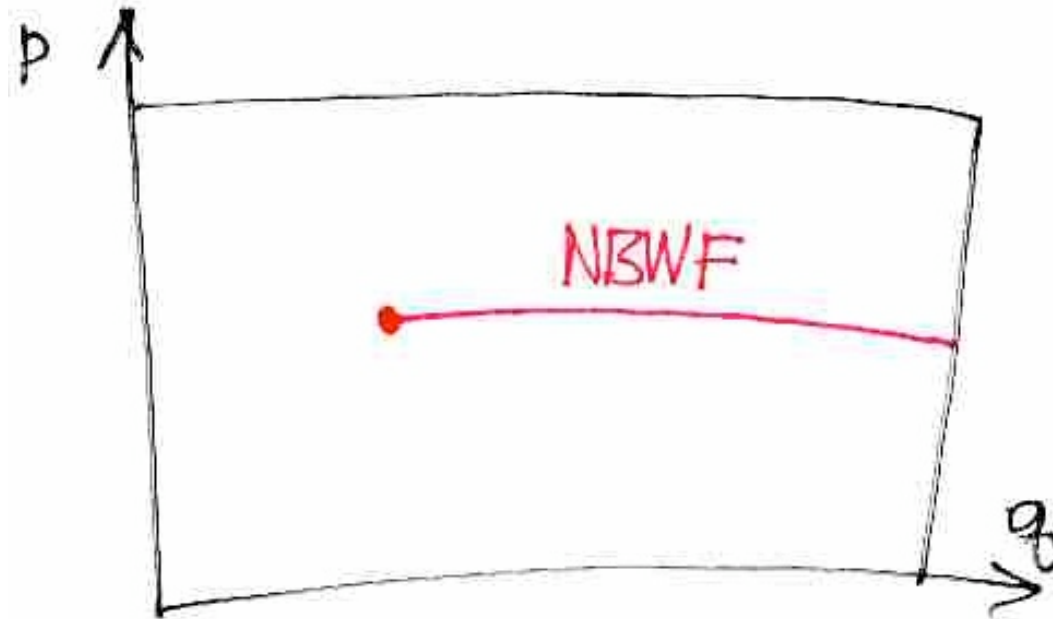

Gibbons
Turok '06

The NBWF predicts a one-parameter subset of the two-parameter family of classical histories, and the classicality constraint gives that subset a boundary.

Singularity Resolution

- The NBWF predicts probabilities for entire classical histories not their initial data.

- The NBWF therefore predicts probabilities for late time observables like CMB fluctuations whether or not the origin of the classical history is singular.

- The existence of singularities in the extrapolation of some classical approximation in quantum mechanics is not an obstacle to prediction but merely a limitation on the validity of the approximation as loop quantum cosmology has shown.
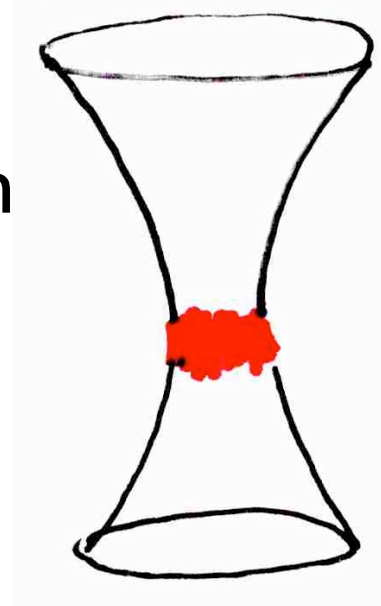
Equations and BC

$$\hbar = c = G = 1, \ \mu \equiv (3/\Lambda)^{1/2}m, \ \phi \equiv (4\pi/3)^{1/2}\Phi, \ H^2 \equiv \Lambda/3$$

Extr
Equa

You won't follow this.
I just wanted to show how much work we did.

$$\ddot{a} + 2a\dot{\phi}^2 + a(1 + \mu^2\phi^2) = 0 \ .$$

Regularity at
Sou

The only important point is that there is one classical history for each value of the field at the south pole $\phi_0 \equiv |\phi(0)|$.

Para
matching:

$$(\phi_0, \gamma, X, Y) \longleftrightarrow (b, \chi, 0, 0)$$

Finding Solutions

- For each ϕ_0 tune remaining parameters to find curves in (b, χ) for which I_R approaches a constant at large b.

- Those are the Lorentzian histories.

- Extrapolate backwards using the Lorentizan equations to find their behavior at earlier times -- bouncing or singular.

- The result is a one-parameter family of classical histories whose probabilities are

$$p(\phi_0) \propto \exp(-2I_R)$$

Gallery
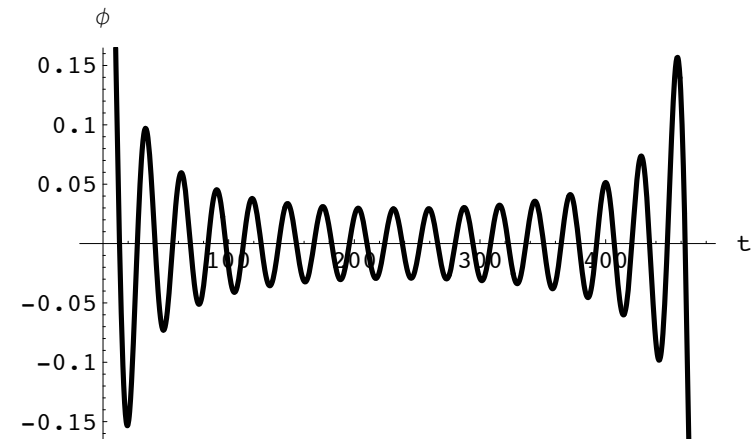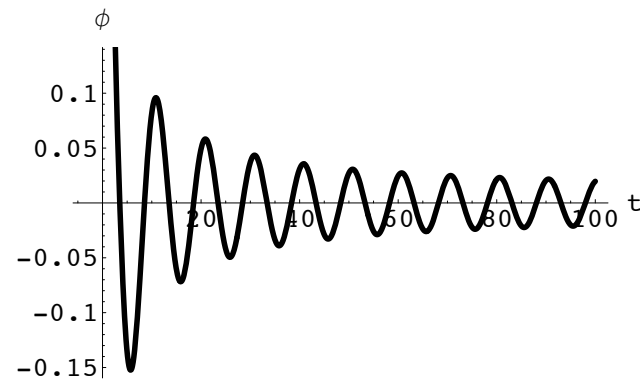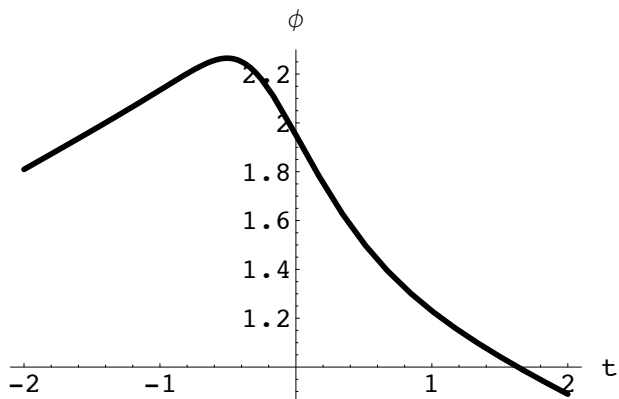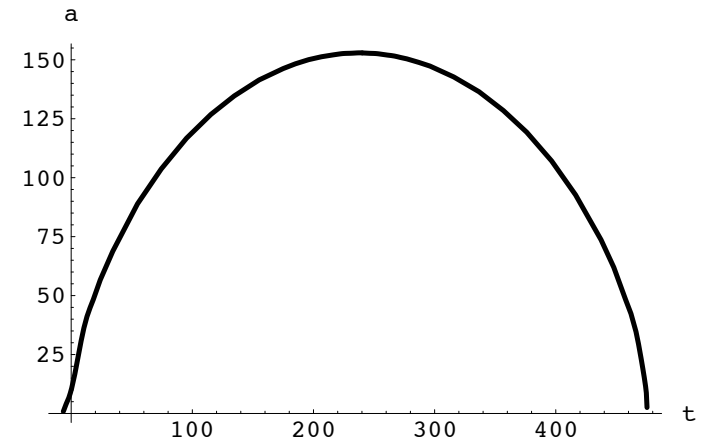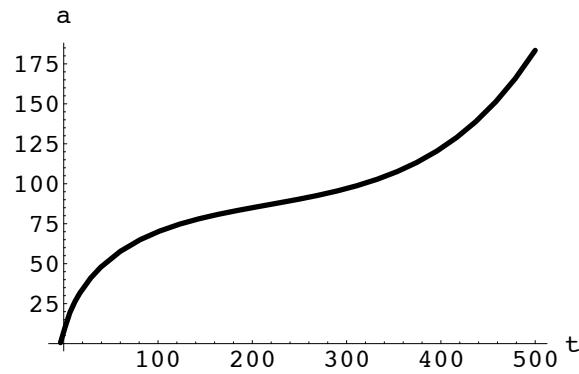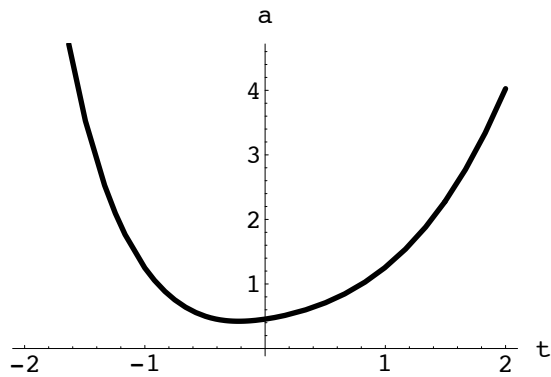
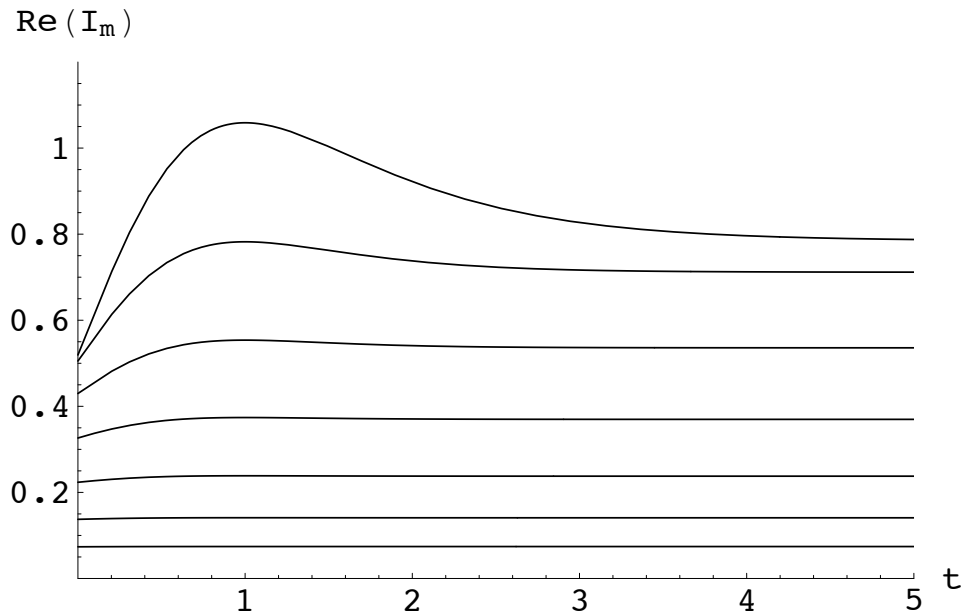

$$\mu = 1.65 \qquad \mu = 2.25 \qquad \mu = 63$$
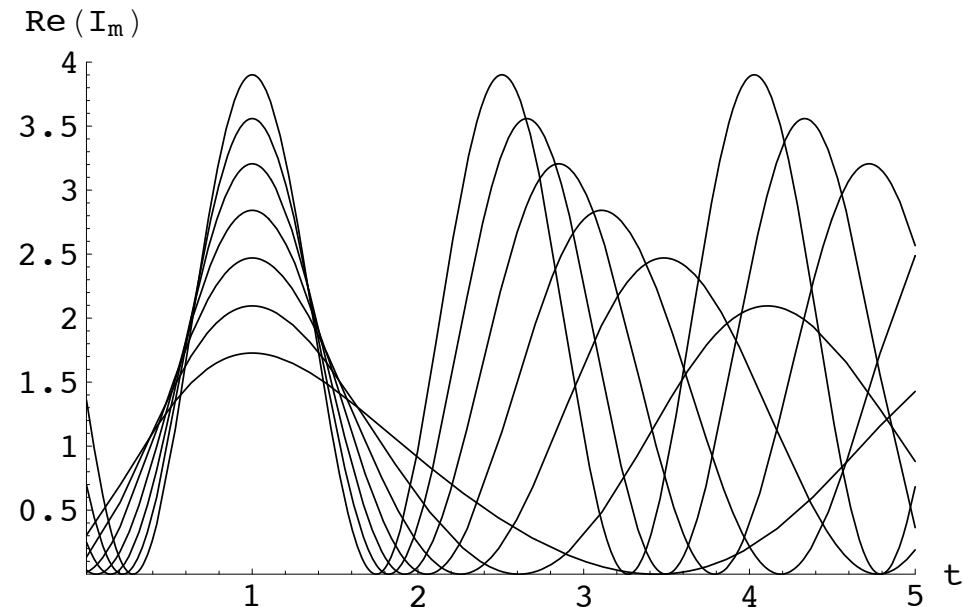
$$\mu \sim m/\Lambda^{1/2} \qquad \phi_0 = 1.32$$

Classicality Constraint ---Pert. Th.

Small field perts on deSitter space.

$\mu < 3/2$
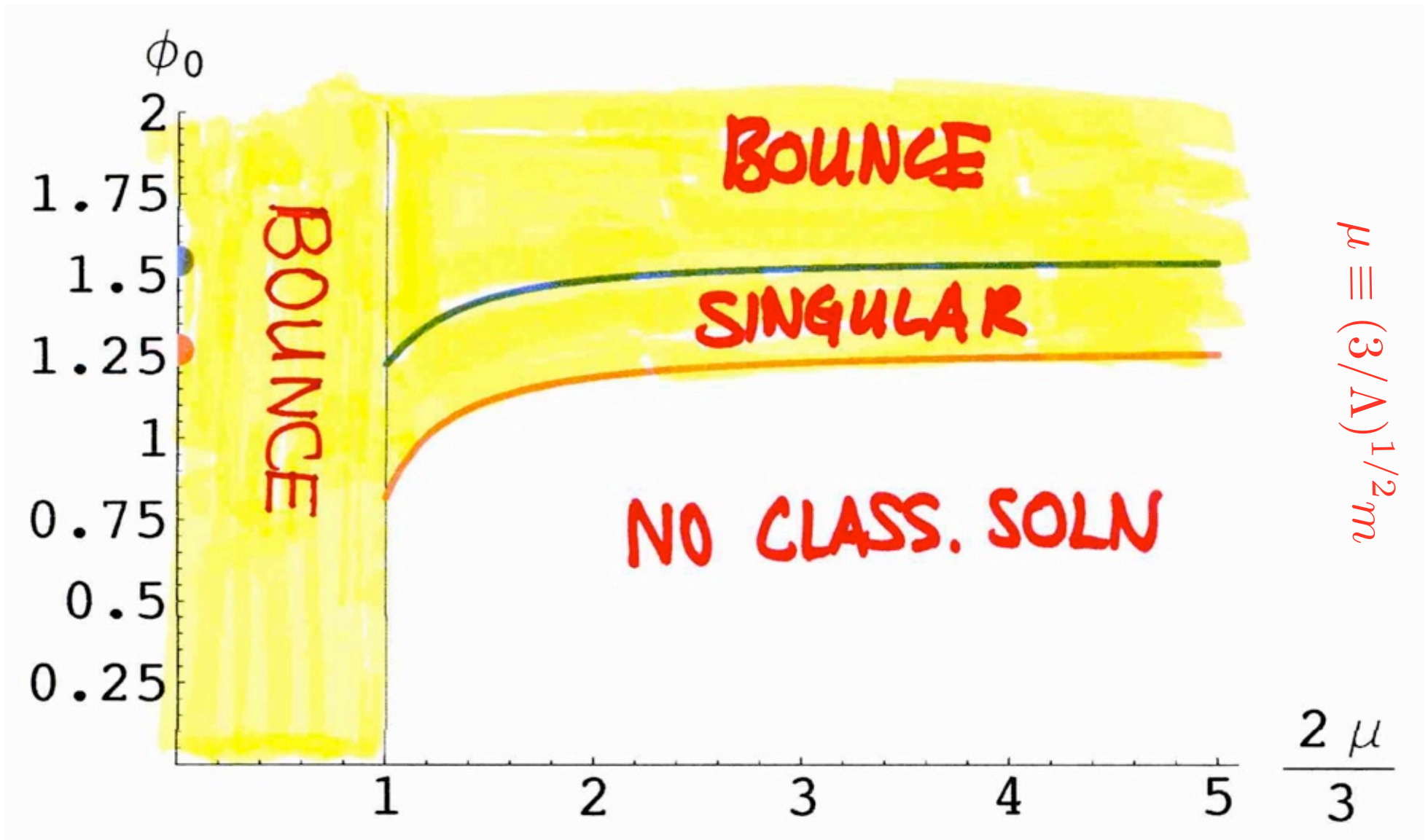
$\mu > 3/2$

Classical $\mu \equiv (3/\Lambda)^{1/2} m$ Not-classical
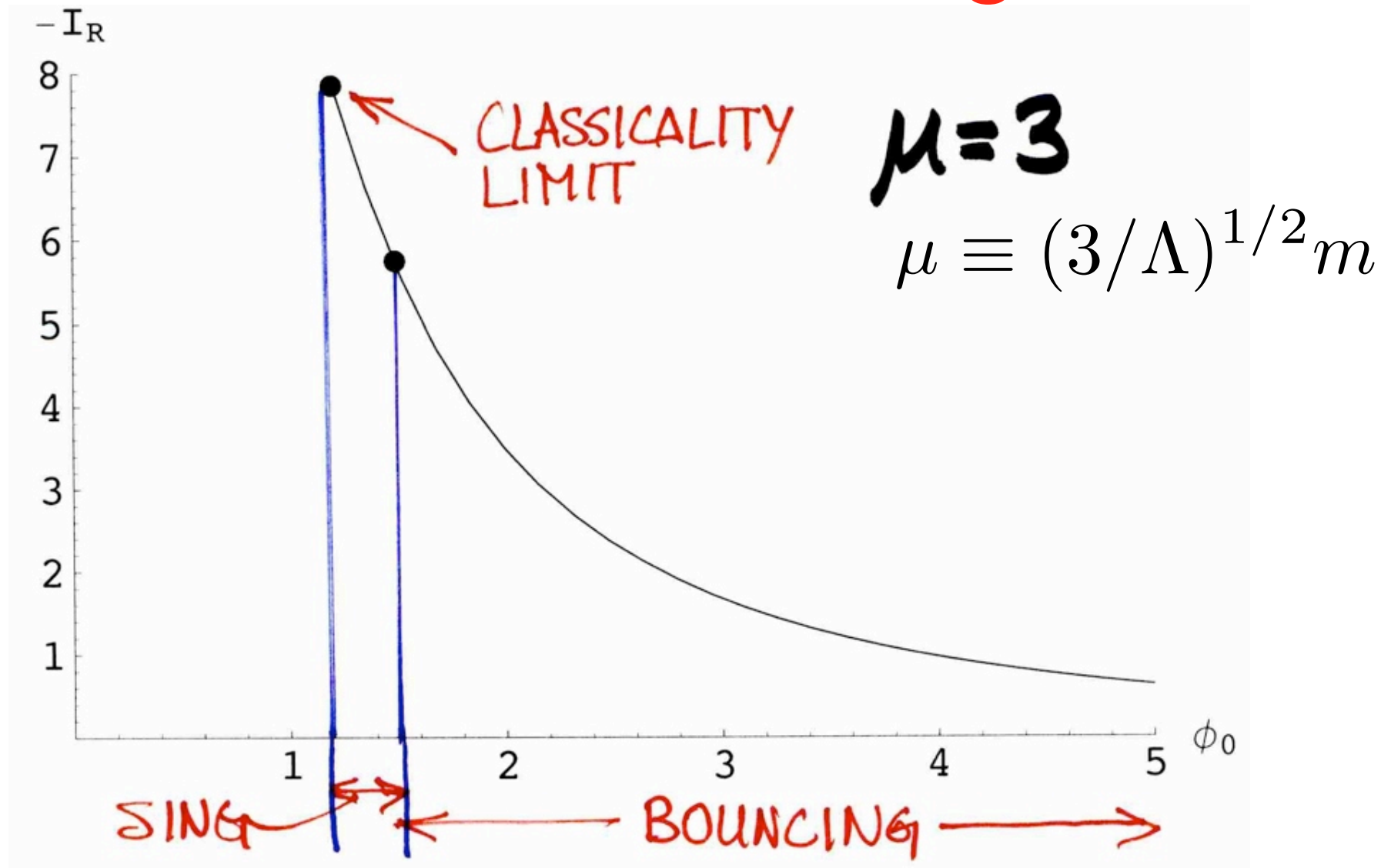
This is a simple consequence of two decaying modes for $\mu < 3/2$, and two oscillatory modes for $\mu > 3/2$.

Origins

No nearly empty models for μ >3/2, a minimum amount of matter is needed for classicality.
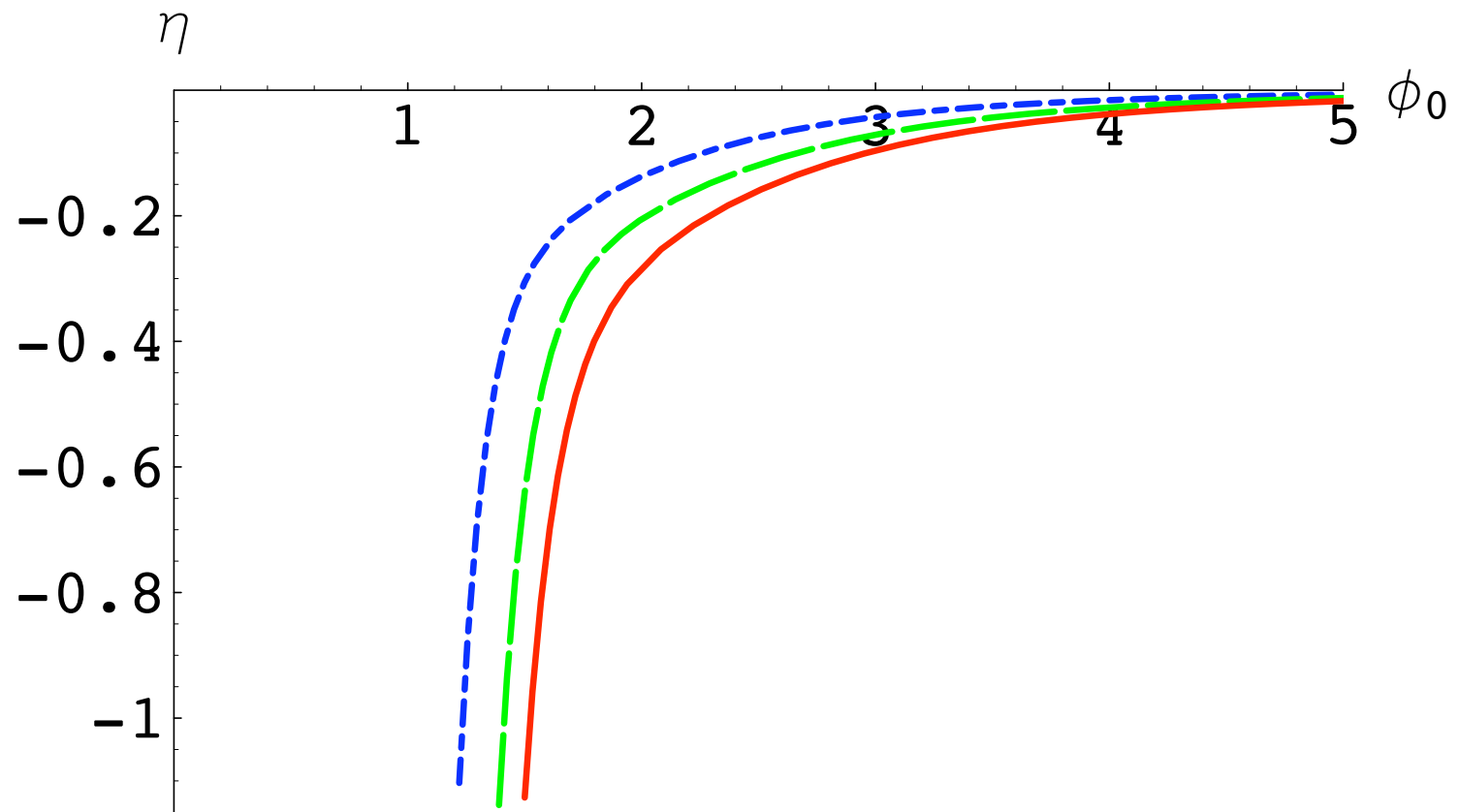
Probabilities and Origins

$$\mu = 3$$

$$\mu \equiv (3/\Lambda)^{1/2} m$$

There is a significant probability that the universe never reached the Planck scale in its entire evolution.

Time Asymmetry

$$\eta \equiv \frac{\dot{\phi}_b}{\phi_b}$$

- Individual histories are not time-symmetric, although the time asymmeties for bouncing universes are not large.

- The ensemble of classical histories is time-symmetic.

Arrows of Time

- The growth of fluctuations defines an arrow of time, order into disorder.

- NBWF fluctuations vanish at the South Pole of the fuzzy instanton.

- Fluctuations therefore increase away from the bounce on both sides.

- Time's arrow points in opposite directions on the opposite sides of the bounce.

- Events on one side will therefore have little effect on events on the other.

compare Caroll & Chen

Inflation

$\mu = 3$

$\mu = 3$
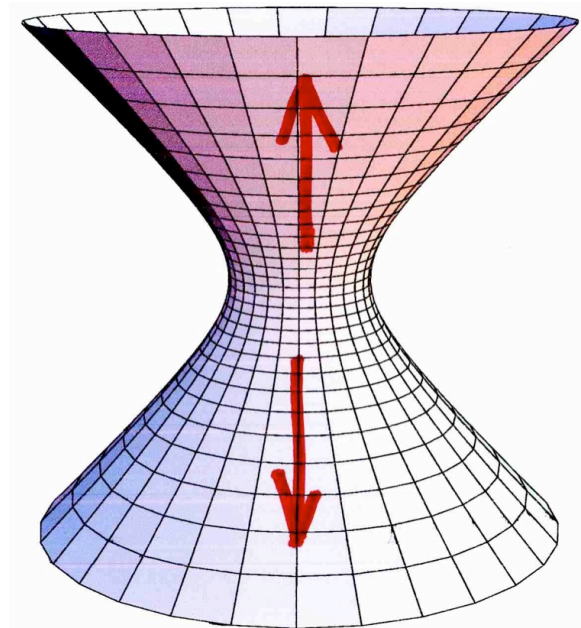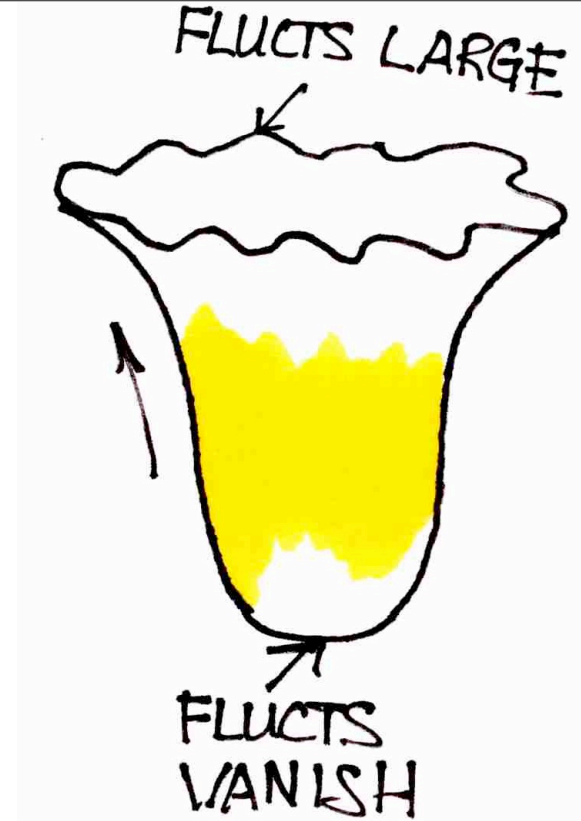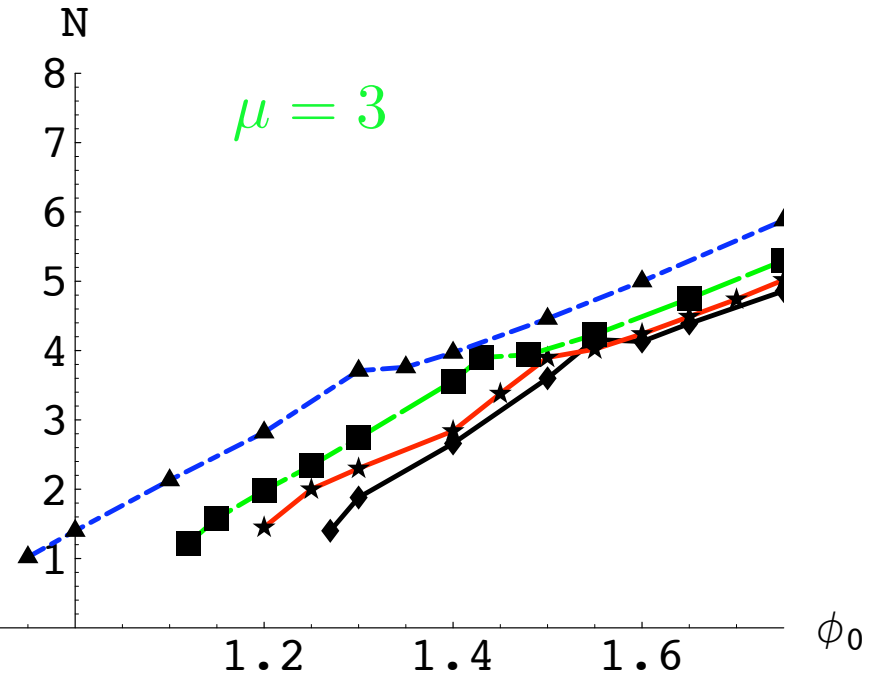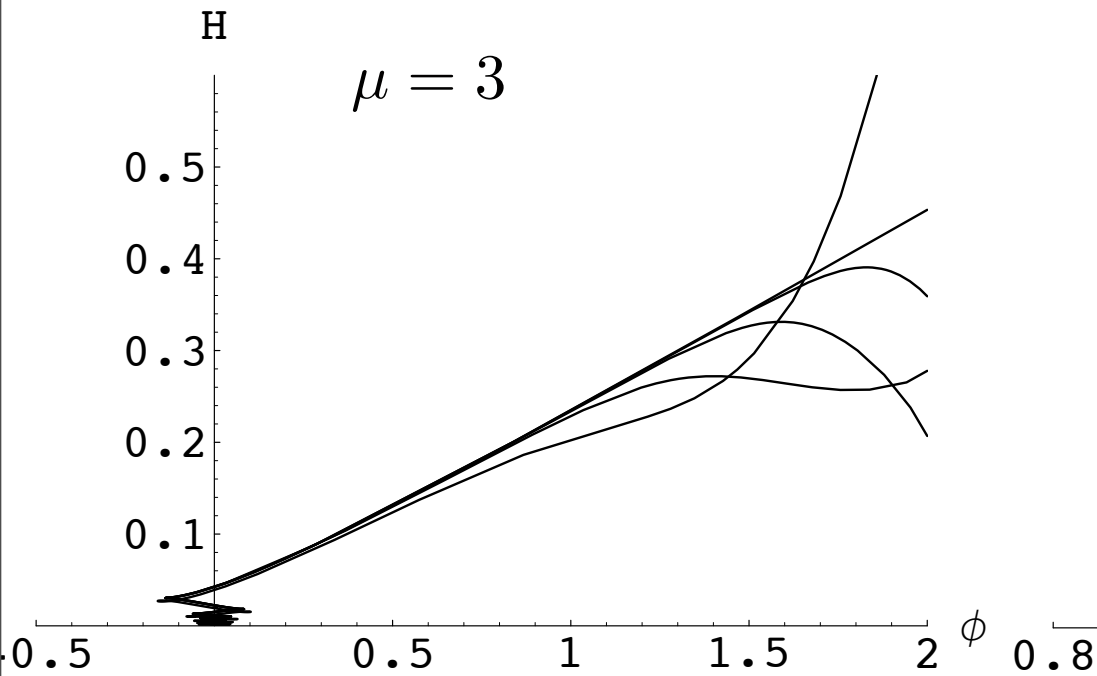
There is scalar field driven inflation for all histories allowed by the classicality constraint, but a small number of efolds N for the most probable of them.

$\mu = 3, \Lambda = .03$
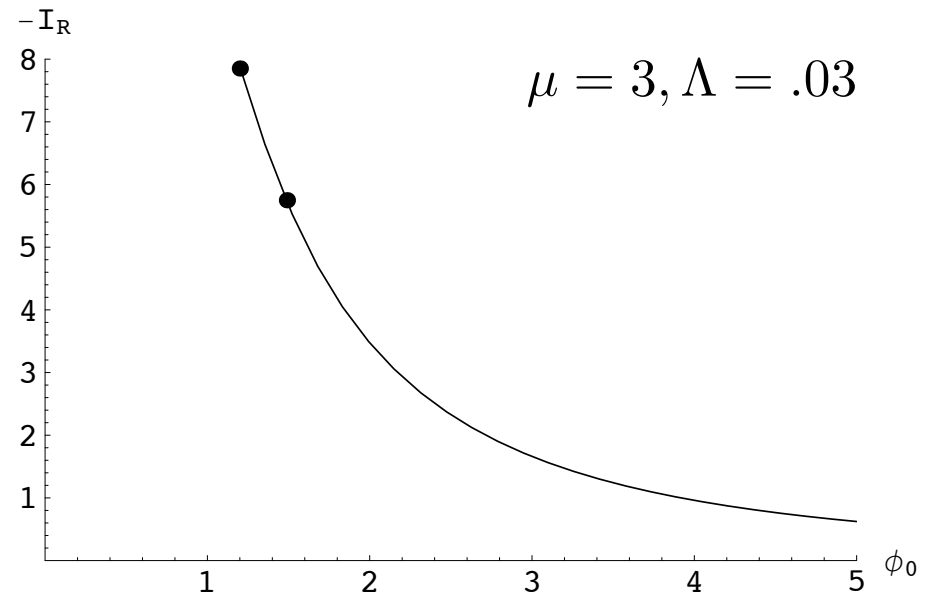
Probabilities for Our Observations

- The NBWF predicts probabilities for entire 4-d histories.

- We so not somehow observe 4-d histories from the outside.

- Rather, we are physical systems within the universe, living at some particular location in spacetime that is partially specified by our data D.

- Probabilities for observations are therefore conditioned on D.

- The probabilities for observations of the CMB for instance depend on when and where they are made.

Conditioning on Our Data

- The NBWF predicts probabilities for entire classical histories.

- Our observations are restricted to a part of a light cone extending over a Hubble volume and located somewhere in spacetime.

- To get the probabilities for our observations we must sum over the probabilities for the classical spacetimes that contain our data at least once, and then sum over the possible locations of our light cone in them.

Sum over location in homo/iso models

- Assume our data locate us on a surface of homogeneity, and approx. data on the past light cone by data in a Hubble vol. on that surface

- Assume we are rare. (If we are everywhere there is no sum).

- The sum multiplies the probability for each history ϕ_0 by

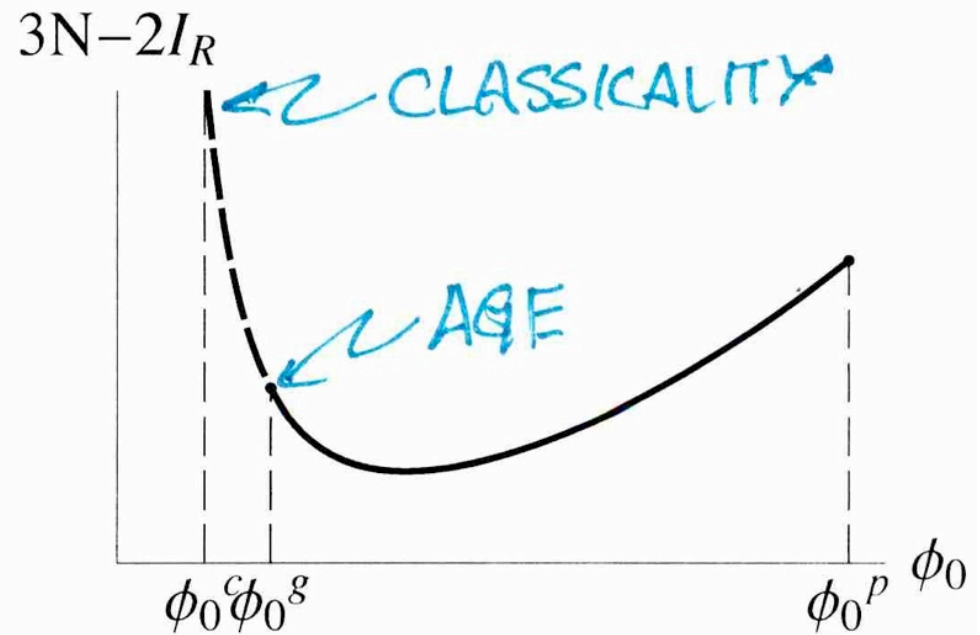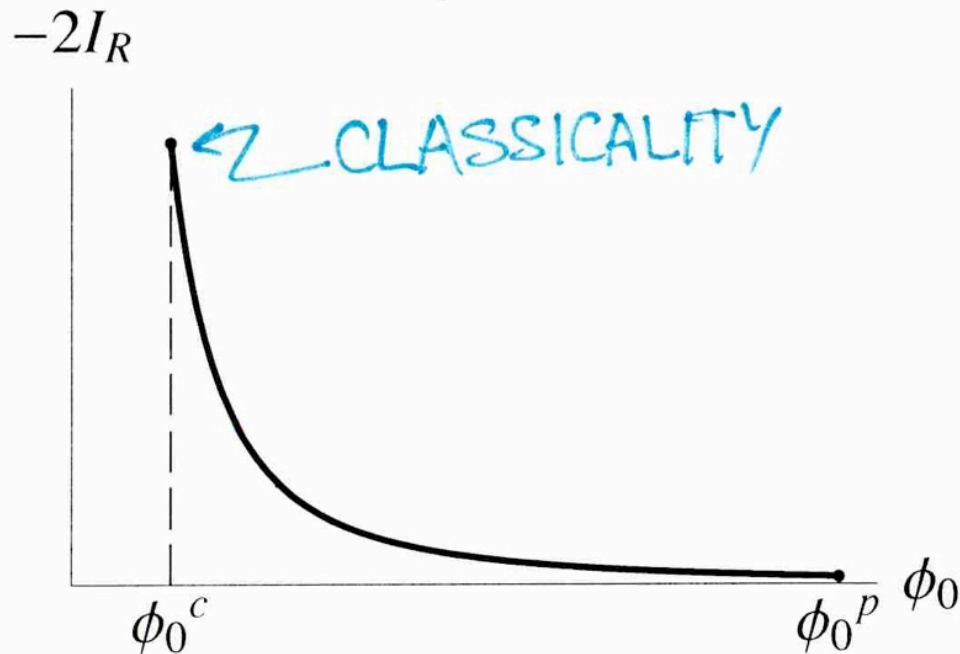$$N_h = V_{\mathrm{surf}}/V_{\mathrm{Hubble}} \approx \exp\left(3N\right)$$

N= # efoldings
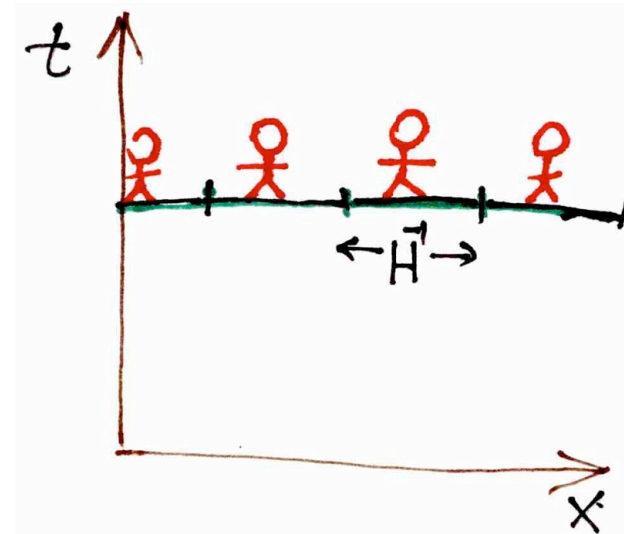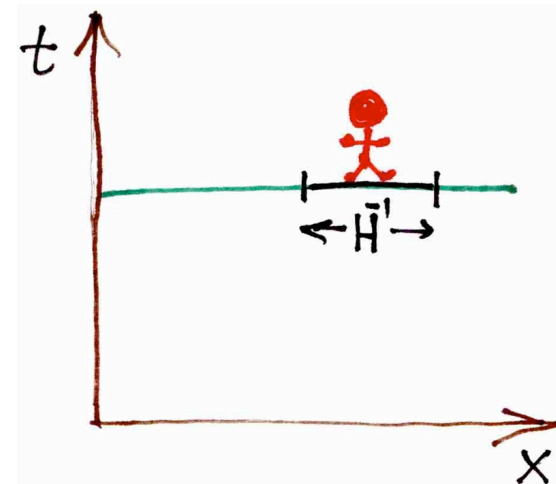
Volume Weighting favors Inflation

By itself, the NBWF + classicality favor low inflation, but we are are more likely to live in a universe that has undergone more inflation, because there are more places for us to be.

$$p(\phi_0 | H_0, \rho) \propto \exp(3N) p(\phi_0) \propto \exp(3N - 2I_R)$$

Replication and Regulation

- In an infinite universe volume weighting breaks down.
- In an infinite universe the probability is unity that we are replicated elsewhere. We are then not rare.
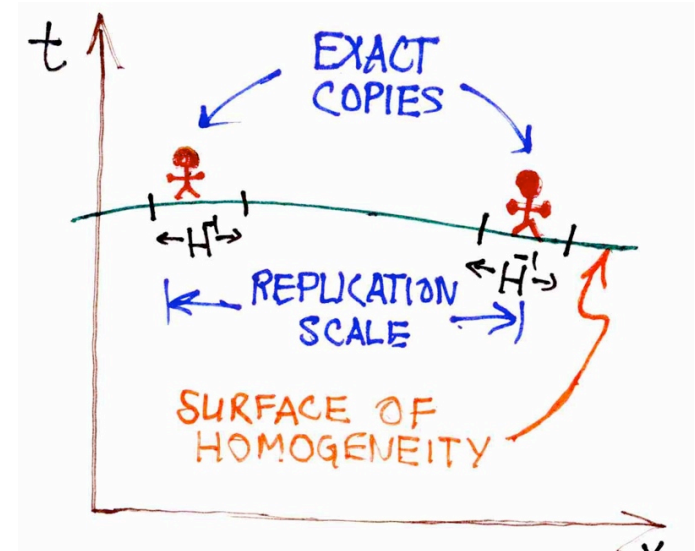
- We are quantum physical systems within the universe that have a probability p_E to exist in any Hubble volume.

- Rather than volume, probabilities should be weighted by the probability that the is at least one instance of us in the universe (all we know for certain).

$$1 - (1 - p_E)^{N_h}$$

Srednicki a.o. 07
Hertog a.o   09

- This is finite for infinite number of Hubble volumes N_h but reduces to volume weighting when p_E is small (rare).

Forthcoming Results on Inhomogeneous Fluctuations

- We calculated the NBWF probabilities for small fluctuations away from homogeneity and isotropy conditioned on at least one instance of our data.

- Fluctuations on observable scales are gaussian with small corrections arising from summing over our possible locations.

- On larger scales that left the horizon in the regime of eternal inflation the universe is predicted to be significantly inhomogenous.

The Main Points Again

Homogeneous, isotropic, scalar field in a quadratic potential, $\mu > 3/2$

- Classical spacetime is the key to the origin of the quasiclassical realm.

- Only special states in quantum gravity predict classical spacetime.

- The NBWF predicts probabilities for a restricted set of entire classical histories that may bounce or be singular in the past. All of them inflate.

- The classicality constraint requires a minimum amount of scalar field (no big empty U's).

- Probabilities of the past conditioned on limited present data favor many efolds of inflation.

Happy Birthday Abhay!

Keep on Quantizing!