SPIN-FOAMS from LOOP QUANTUM GRAVITY

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AIM:

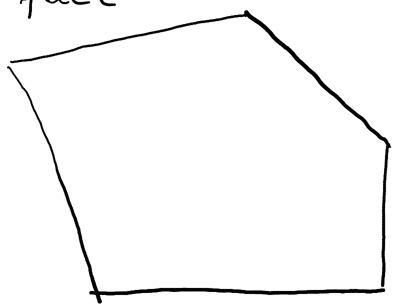
To introduce spin-foams for all the graphs used in LQG, not only for the graphs given by triangulations. In this way we will go back to the fundamental LQG

PLAN

1. GENERALIZATION OF SPIN FOAM TO SPIN-NETWORKS OF LQG
2. HOW IT FITS THE STRUCTURE OF SPIN FOAM MODELS
3. HOW IT FITS THE BC MODEL
4. HOW IT FITS THE E-P-R-L/F-K

SF INGREDIENTS: LINEAR 2-CELL COMPLEX

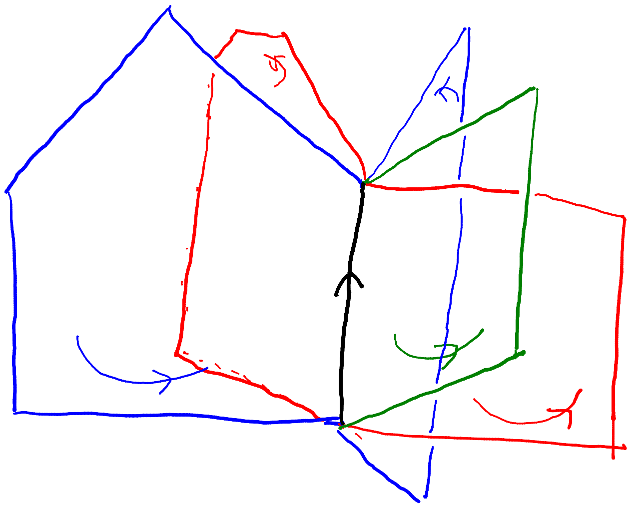
face



edge

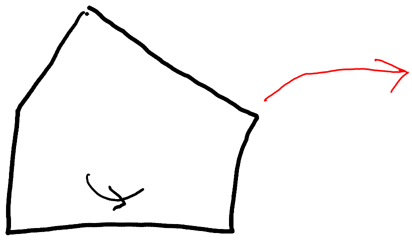
vertex

glued along the edges:

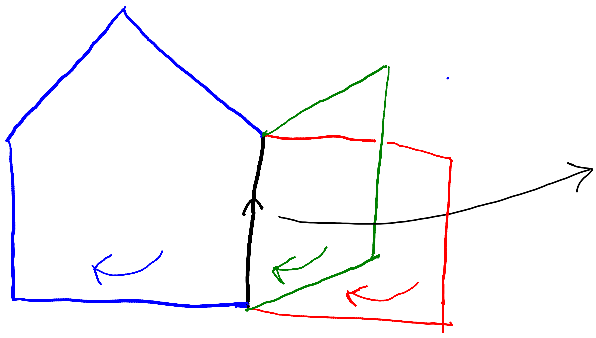


SF INGREDIENTS: LABELLINGS

given group G



$\rho \in \mathrm{IrR}(G)$



$\iota \in \mathrm{Inv}(\rho \overset{*}{\otimes} \rho \overset{*}{\otimes} \rho)$

BOUNDARY SPIN-NETWORKS

ASSUMPTION ABOUT THE SPIN FOAM BOUNDARIES ∂:

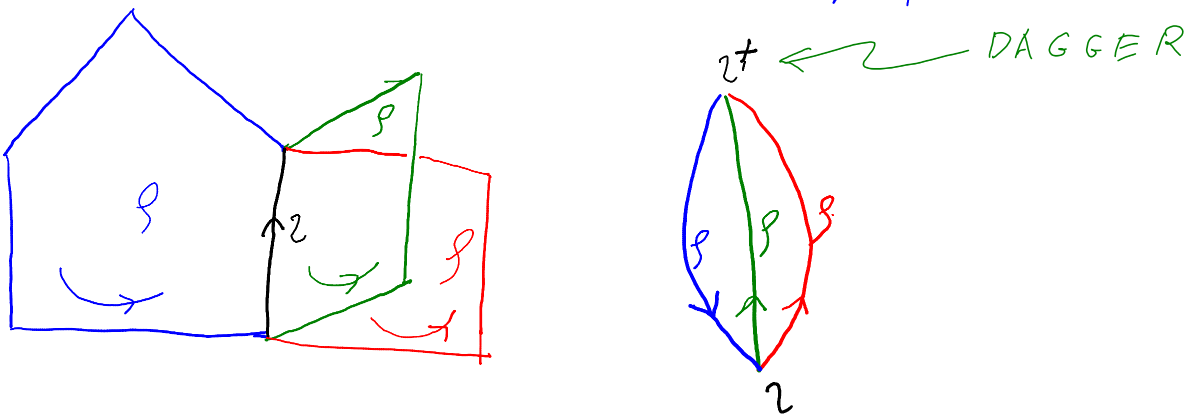
- ∂ is a graph (a 1-cell sub-complex)
- a neighbourhood of ∂ in it's SF is:

$$\partial \times [0, \epsilon) \subset \text{Spin-foam}$$

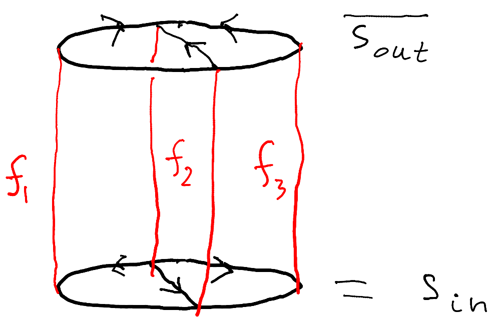
Given a spin-foam, on the boundary ∂ there is induced a spin-network.

EXAMPLE 1:

the boundary spin-network:



EXAMPLE 2. Spin-foam history of a spin-network

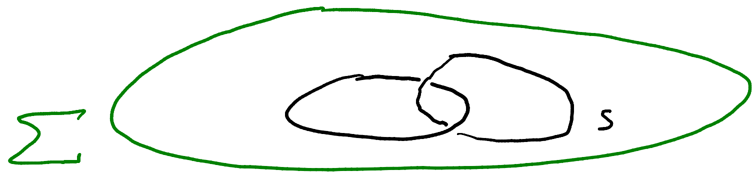


EMBEDDED SPIN-FOAMS

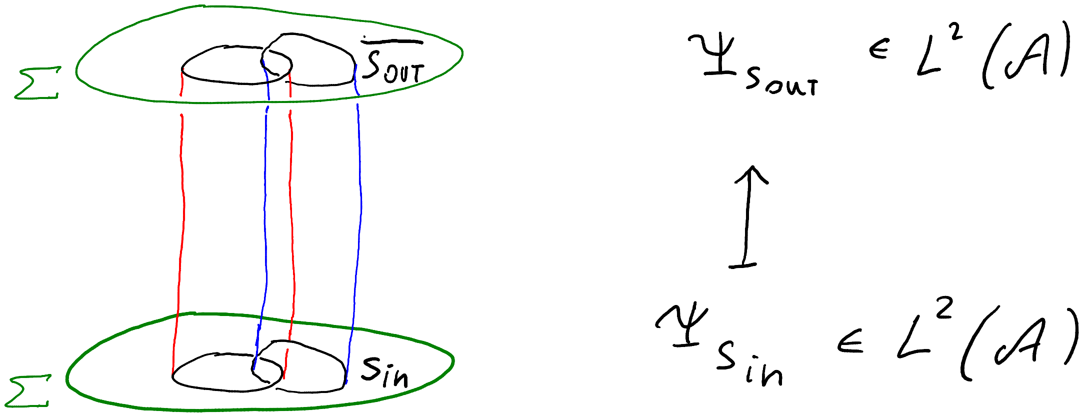
Σ - 3 manifold of the canonical 3+1 gravity

$\mathcal{H} = L^2(\mathcal{A})$ – the Hilbert space of functions of connections on Σ

$\mathcal{H} \ni \Psi_s$ – a state labelled by a spin-network s EMBEDDED in Σ



A spin-foam history of embedded spin-network



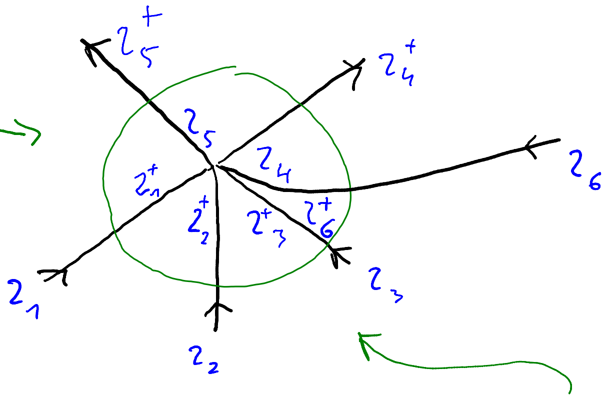
THESE SPIN-FOAM HISTORIES ARE EMBEDDED IN

$$\Sigma \times \mathbb{R}$$

THE NATURAL SPIN-FOAM AMPLITUDE

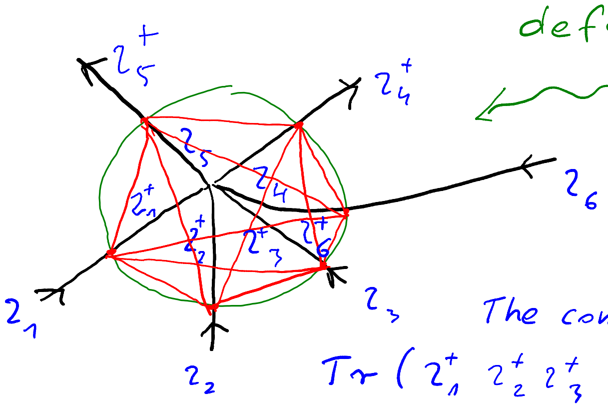
SF edge

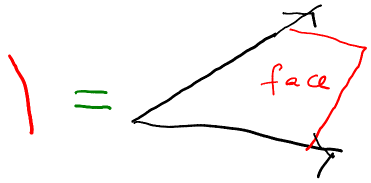
SF vertex



there is a natural way to contract the tensors ι at each vertex

Each spin-foam vertex defines a spin-network





The contraction

$$Tr(\iota_1^+ \, \iota_2^+ \, \iota_3^+ \, \iota_4 \, \iota_5 \, \iota_6^+)$$

coincides with the Penrose's evaluation of the spin-network. It gives the vertex amplitude

$$A(v)$$

A generalization of the "n-j symbols."

The total spin-foam amplitude is:

$$A(SF) = \prod_{v \notin \partial} A(v) \cdot \prod_f A(f)$$

$= \dim \rho_f$

CONCLUSION: The amplitude used in the simplecial spin-foam models is naturally generalized to the spin-foams proposed above.

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THE SCHEME OF THE SF MODELS OF ++++ GR

- $G = SO(4),\ SU(2)\times SU(2),$
- The spin-foams: either embedded or unembedded either all the linear 2-cell complexes or only simplecial complexes
- The natural amplitude defined above
- Constraints on the labellings guessed from the simplicity conditions

$$B^{IJ}_{S} = \int_{S} *(e^{I}\wedge e^{J})$$

- and imposed on the $SU(2)\times SU(2)$ spin-networks

- Examples: Barrett-Crane, Engle-Pereira-Rovelli-Livine

THE BARRETT-CRANE LABELLING GENERALIZED TO ARBITRARY GRAPHS

THE TOOLS:

THE AREA OPERATORS (S - 2-surfaces)

$$\hat{A}^+_S = \int_S \sqrt{(e^I \wedge e^J)^+ \wedge (e_I \wedge e_J)^+}$$

$$\hat{A}^-_S = \int_S \sqrt{(e^I \wedge e^J)^- \wedge (e_I \wedge e_J)^-}$$

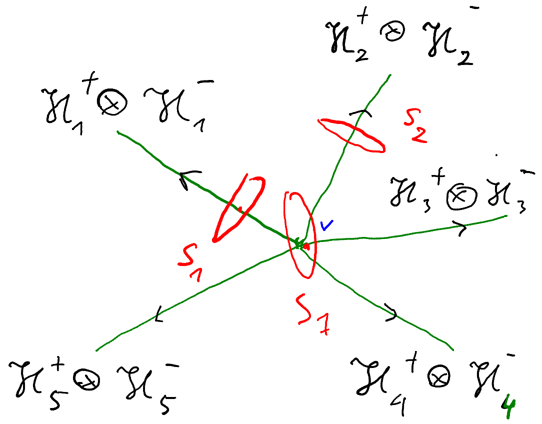
IN THE HILBERT SPACE ASSOCIATED TO A GRAPH:

$$\mathrm{Inv}(\mathcal{H}^+_1 \otimes \ldots \mathcal{H}^+_N) \otimes \mathrm{Inv}(\mathcal{H}^-_1 \otimes \ldots \mathcal{H}^-_N)$$

THE BC CONSTRAINTS:

$$\hat{A}^+_S \,\imath = \hat{A}^-_S \,\imath$$

FOR ARBITRARY $S = S_1, \ldots$



NATURAL SOLUTION TO THE BC CONSTRAINT

IN THE ABSTRACT INDEX NOTATION:

$$\mathrm{Inv}\left(\mathcal{H}_1^+ \otimes \ldots\right) \otimes \mathrm{Inv}\left(\mathcal{H}_1^- \otimes \ldots\right) \ni \imath^{M_1^+ \ldots M_1^- \ldots}$$

THE CONDITION READS:

$$A_{S\,K_1^+\ldots}{}^{M_1^+\ldots}\,\imath^{K_1^+\ldots\,M_1^-\ldots} = A_{S\,K_1^-\ldots}{}^{M_1^-}\,\imath^{M_1^+\ldots K_1^-\ldots}$$

$$\Rightarrow \quad \mathcal{H}_1^+ = \mathcal{H}_1^-, \ \ldots$$

$$\Rightarrow \quad \imath = \text{" id with raised index "}$$

THE "id" IS THE PROJECTION:

$$P : \mathcal{H}_1^- \otimes \ldots \otimes \mathcal{H}_N^- \to \mathrm{Inv}\left(\mathcal{H}_1^+ \otimes \ldots \otimes \mathcal{H}_1^+\right)$$

$$P = P^{M_1^+\ldots}{}_{M_1^-\ldots}$$

IN EACH REPRESENTATION OF SU(2) WE HAVE A NATURAL BI-LINEAR FORM ϵ

$$\imath = P \circ \epsilon^{-1}$$

$$\boxed{\imath^{M_1^+\ldots M_1^-\ldots} = P^{M_1^+\ldots}{}_{K_1^-\ldots}\,\epsilon^{K_1^- M_1^-}\ldots}$$

THE EPRL / FK MODEL

$$S = \frac{1}{2\varkappa}\int *e^I \wedge e^J \wedge F_{IJ} + \frac{1}{\gamma} e^I \wedge e^J \wedge F_{IJ}$$

The simplicity constraints:

$$\mathcal{H}_1^+ = \mathcal{H}_{j_1^+}, \quad \mathcal{H}_1^- = \mathcal{H}_{j_1^-}, \quad \ldots$$

$$j_1^+ = k_1 2|1+\gamma|, \quad j_1^- = k_1 2|1-\gamma|, \quad \ldots$$

$$k_I = \frac{1}{2}, 1, \ldots$$

$$\iota^{M_1^+ \ldots M_1^- \ldots} = P^{M_1^+ \ldots}_{A_1^- \ldots} c^{A_1^- M_1^-}_{m_1 \ldots} \iota_{SU(2)}^{m_1 \ldots}$$

$$\epsilon^{A_1^- M_1^-} \leftarrow \begin{matrix} \gamma \to \infty \\ k \to 0 \end{matrix}$$

Barrett-Crane