Unimodular quantum gravity as a solution to the cosmological constant problem

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1. The basic idea of unimodular gravity
2. Henneaux and Teitelboim, Plebanski
3. Hamiltonian formulation
4. The path integral quantum theory is unimodular
5. Hamiltonian quantization and the problem of time
6. Why is the cosmo constant is so small?

arXiv:0904.4841+ to come

The cosmological constant problem**s**

1. Why the cosmo constant is not enormous.
2. Why it has the particular value it has
3. Coincidence problems.

The cosmological constant problems

1. Why the cosmo constant is not enormous.
2. Why it has the particular value it has
3. Coincidence problems.

Many people have worked on unimodular gravity and commented that it might have something to do with the cosmological constant problems

Einstein	1919
Zee	1985
Sorkin	
Unruh	
Weinberg	
Ng and van Dam	
Henneaux and Teitelboim	1991
Bombelli, Couch, Torrence	1991

The basic idea:

The basic idea of unimodular gravity:

$$S^{uni} = \int_{\mathcal{M}} \epsilon_0 \left(-\frac{1}{8\pi G} \bar{g}^{ab} R_{ab} + \mathcal{L}^{matter}(\bar{g}_{ab}, \psi) \right)$$

det(g) has been constrained to be equal to a fixed volume element:

$$\sqrt{-g} = \epsilon_0$$

The diffeomorphism group is reduced to volume preserving diffeo's:

$$\partial_a(\epsilon_0 v^a) = 0$$

The eq's of motion are just the tracefree part of Einstein

$$R_{ab} - \frac{1}{4}\bar{g}_{ab} R = 4\pi G \left(T_{ab} - \frac{1}{4}\bar{g}_{ab} T \right)$$

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This has a decoupling symmetry:

$$\boxed{T_{ab} \rightarrow T'_{ab} = T_{ab} + g_{ab}C}$$

This means that contributions to the energy-momentum tensor proportional to the metric don't couple to gravity!

$$R_{ab} - \frac{1}{4}\bar{g}_{ab}R = 4\pi G\left(T_{ab} - \frac{1}{4}\bar{g}_{ab}T\right)$$

The divergence of this yields

$$\partial_a\left(R + 4\pi G T\right) = 0$$

which implies that there is a constant $\boxed{-4\Lambda = R + 4\pi G T}$

so one gets the Einstein equations with an arbitrary Λ

$$G_{ab} - \Lambda g_{ab} = 4\pi G T_{ab}$$

The decoupling symmetry is still present:

$$\boxed{T_{ab} \rightarrow T'_{ab} = T_{ab} + g_{ab}C}$$

now implies also

$$\boxed{\Lambda \rightarrow \Lambda - 4\pi G C}$$

Unimodular gravity is not a new theory, it is a reformulation of GR.

Why isn't this the solution to the first cosmological constant problem? Or, why isn't the fact that Λ *is not Planck scale evidence that this is the right formulation of GR for quantum physics?*

Weinberg discussed this in his 1989 review and said:

> *"In my view, the key question in deciding whether this is a plausible classical theory of gravitation is whether it can be obtained as the classical limit of any physically satisfactory [quantum] theory of gravitation."*

We will study this problem and see that the answer is YES.

The main result will be that the full quantum effective action

$$S^Q[<\bar{g}_{ab}>,<\Psi>] = S^0[<\bar{g}_{ab}>,<\Psi>] + \hbar\Delta S[<\bar{g}_{ab}>,<\Psi>]$$

is a functional of the unimodular metric, $\det(g_{ab})=1$

$$S^0 = S^{uni} = \int_{\mathcal{M}} \epsilon_0 \left(-\frac{1}{8\pi G}\bar{g}^{ab}R_{ab}[\bar{g}] + \mathcal{L}^{matter}(\bar{g},\psi)\right)$$

The full quantum equations of motion:

$$R_{ab} - \frac{1}{4}\bar{g}_{ab}R = 4\pi G\left(T_{ab} - \frac{1}{4}\bar{g}_{ab}T\right) + \hbar\left(W_{ab} - \frac{1}{4}\bar{g}_{ab}W\right)$$

$$W_{ab} \equiv \frac{\delta\Delta S}{\delta g_{ab}}$$

Hence the spacetime geometry is insensitive to any terms in T_{ab} or W_{ab} proportional to a constant times the metric.

Unimodular gravity in the Plebanski formalism

a la Henneaux-Teitelboim

Bombelli, Couch, Torrence

The Henneaux-Teitelboim reformulation of unimodular gravity

They introduce a new field, which is a three form a_{bcd}

$$\tilde{a}^a = \frac{1}{3!}\epsilon^{abcd}a_{bcd}$$

There is then a four form

$$b_{abcd} = da_{abcd} \qquad \tilde{b} = \frac{1}{4!}\epsilon^{abcd}b_{abcd} = \partial_a \tilde{a}^a$$

$$\boxed{S^{HT} = \int_{\mathcal{M}} \sqrt{-g}\left(-\frac{1}{8\pi G}(\bar{g}^{ab}R_{ab} + \phi) + \mathcal{L}^{matter}\right) + \frac{1}{8\pi G}\phi\tilde{b}}$$

ϕ is also a new field

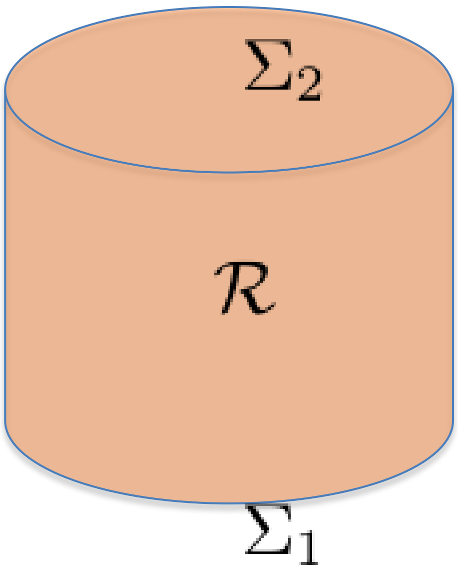
$$\frac{\delta S^{HT}}{\delta \phi} : \tilde{b} = da = \sqrt{-g}$$

The a_{bcd} field measures a global time:

$$\tilde{b} = da = \sqrt{-g}$$

implies that:

$$\boxed{\int_{\Sigma_2} a - \int_{\Sigma_1} a = Vol = \int_{\mathcal{R}} \sqrt{-g}}$$



We can put unimodular gravity in the Plebanski formalism:

Action:

$$\boxed{\begin{gathered} S^{HT} = \int_{\mathcal{M}} \left(B^i \wedge F_i - \Phi_{ij} B^i \wedge B^j - \phi B_i \wedge B^i + \mathcal{L}^{matter} \right) + \frac{1}{8\pi G} \phi \tilde{b} \\ \Phi_{ii} = 0 \end{gathered}}$$

Equations of motion:

$$F_i = \phi B_i + \Phi_{ij} B^j$$

$$B^i \wedge B^j - \frac{1}{3} \delta^{ij} B^i \wedge B_k = 0$$

$$\partial_a \phi = 0$$

$$\tilde{b} = B_i \wedge B^i$$

The canonical route to quantum gauge theories:

•Start with the classical action

•Work out Hamiltonian formulation

Gauge symmetries imply constraints

•Gauge fix to get deterministic dynamics in phase space.

•Construct fully gauge fixed path integral in phase space

“Faddeev-Poppov”

•Work backwards to configuration space path integral

•Construct quantum effective action for averaged fields.

Question: is the resulting quantum effective action unimodular?

If so, the decoupling symmetry is present quantum mechanically!

The answer is **YES.**

We eliminate B^i by its eom and plug back in:

$$\Phi'_{ij} = \Phi_{ij} + \frac{1}{3}\delta_{ij}\phi$$

$$\boxed{S^{HT2} = \int_{\mathcal{M}} \left(F^i \wedge F^j (\Phi')^{-1}_{ij} + \mathcal{L}^{matter}\right) + \frac{1}{8\pi G}\phi\tilde{b}}$$

Canonical decomposition:

$$\Phi'_{ij} = \Phi_{ij} + \frac{1}{3}\delta_{ij}\phi$$

$$S^{HT2} = \int_{\mathcal{M}} \left(F^i \wedge F^j (\Phi')^{-1}_{ij} + \mathcal{L}^{matter} \right) + \frac{1}{8\pi G}\phi\tilde{b}$$

Momenta:

$$\left.\begin{aligned} \tilde{E}^a_i &= \epsilon^{abc} F_{bc} (\Phi')^{-1}_{ij} \\ P^{ij}_{\Phi} &= \Pi^0_i = \pi_\phi = \pi_c = 0 \\ \mathcal{E} &= \pi_0 - \phi = 0 \end{aligned}\right\} \text{primary constraints}$$

Secondary constraints:

$$\mathcal{D}_a = \tilde{E}^b_i F^i_{ab} = 0$$

$$\mathcal{G}^i = \mathcal{D}_a \tilde{E}^{ai} = 0$$

$$\mathcal{H} = \epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j F_{abk} - \phi det(\tilde{E}^a_i) = 0$$

$$\mathcal{G}_c = \partial\phi = 0$$

Canonical decomposition:

$$\Phi'_{ij} = \Phi_{ij} + \frac{1}{3}\delta_{ij}\phi$$

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Momenta:

$$\tilde{E}^a_i = \epsilon^{abc} F_{bc} (\Phi')^{-1}_{ij}$$

$$P^{ij}_{\Phi} = \Pi^0_i = \cancel{\pi_\phi} = \pi_c = 0$$

$$\cancel{\mathcal{E} = \pi_0 + \phi = 0}$$

primary constraints

Secondary constraints:

Eliminate ϕ and π_ϕ

$$\mathcal{D}_a = \tilde{E}^b_i F^i_{ab} = 0$$

$$\mathcal{G}^i = \mathcal{D}_a \tilde{E}^{ai} = 0$$

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primary constraints

Secondary constraints:

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$$\mathcal{G}^i = \mathcal{D}_a \tilde{E}^{ai} = 0 \qquad \mathcal{G}_c = \partial\pi_0 = 0$$

$$\mathcal{H} = \epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j F_{abk} - \pi_0 det(\tilde{E}^a_i) = 0$$

There is a non-vanishing Hamiltonian

$$\boxed{H = \int_{\Sigma} (\partial_a \tilde{a}^a) \left(\frac{\epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j F_{abk}}{det(\tilde{E}^a_i)} \right)}$$

Canonical pairs: $(A_a^i, \tilde{E}_i^a), (\tilde{a}^a, \pi_a), (\tilde{a}^0, \pi_0)$

Constraints:

$$\mathcal{H} = \epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b F_{abk} - \pi_0 det(\tilde{E}_i^a) = 0$$

$$\mathcal{D}_a = \tilde{E}_i^b F_{ab}^i = 0 \qquad \pi_c = 0$$

$$\mathcal{G}^i = \mathcal{D}_a \tilde{E}^{ai} = 0 \qquad G_c = \partial_c \pi_0 = 0$$

Hamiltonian:

$$H = \int_\Sigma (\partial_a \tilde{a}^a) \left(\frac{\epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b F_{abk}}{det(\tilde{E}_i^a)} \right)$$

Fully constrained momentum space path integral:

$$Z = \int dA_a^i d\tilde{E}_i^a d\tilde{a}^0 d\tilde{a}^a d\pi_a d\pi_o \delta(\mathcal{H})\delta(\mathcal{D}_a)\delta(\mathcal{G}^i)\delta(\mathcal{G}_c)\delta(\pi_c)\delta(\text{ gauge fixing}) Det_{FP}$$
$$\times exp\ \imath \int dt \int_\Sigma \left(\tilde{E}_i^a \dot{A}_a^i + \pi_0 \dot{\tilde{a}}^0 + \pi_c \dot{\tilde{a}}^c - (\partial_a \tilde{a}^a) \left(\frac{\epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b F_{abk}}{det(\tilde{E}_i^a)} \right) \right)$$

The main result: the constrained momentum space path integral

$$Z = \int dA^i_a d\tilde{E}^a_i d\tilde{a}^0 d\tilde{a}^a d\pi_a d\pi_o \delta(\mathcal{H})\delta(\mathcal{D}_a)\delta(\mathcal{G}^i)\delta(\mathcal{G}_c)\delta(\pi_c)\delta(\text{ gauge fixing}) Det_{FP}$$

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becomes the unimodular configuration space path integral:

$$Z = \int dA^i_\mu de^\mu \delta\left(det(e) - \epsilon_0\right) \delta(\text{ gauge fixing}) Det'_{FP}$$

$$\times exp\ \imath \int dt \int_\Sigma \left(e^\mu \wedge e^\nu \wedge F^+_{\mu\nu} \right)$$

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or, if you prefer the Plebanski formalism:

$$Z = \int dA^i dB^i d\Phi_{ij} \delta\left(B^i \wedge B_i - \epsilon_0\right) \delta(\Phi_{ii}) \delta(\text{ gauge fixing}) Det'_{FP}$$
$$\times exp\, \imath \int dt \int_\Sigma \left(B^i \wedge F_i - \Phi_{ij} B^i \wedge B^j\right)$$

.

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$$Z = \int dA_a^i d\tilde{E}_i^a d\tilde{a}^0 d\tilde{a}^a d\pi_a d\pi_o \delta(\mathcal{H})\delta(\mathcal{D}_a)\delta(\mathcal{G}^i)\delta(\mathcal{G}_c)\delta(\pi_c)\delta(\text{ gauge fixing})Det_{FP}$$
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$$\times exp\, \imath \int dt \int_\Sigma \left(B^i \wedge F_i - \Phi_{ij} B^i \wedge B^j\right)$$

So Weinberg's challenge is met: *the semi-classical limit is unimodular gravity.* So if we define the quantum effective action, it is a function of the determinant-fixed metric. Hence the quantum effective equations of motion have the decoupling symmetry.

$$T_{ab} \to T'_{ab} = T_{ab} + g_{ab} C$$

$$Z = \int dA^i_\mu de^\mu \delta\left(det(e) - \epsilon_0\right) \delta(\text{ gauge fixing}) Det'_{FP}$$
$$\times exp\ \imath \int dt \int_\Sigma \left(e^\mu \wedge e^\nu \wedge F^+_{\mu\nu}\right)$$

To make this more precise we define the quantum effective action

Expand around flat spacetime, pick coordinates $\epsilon_0 = 1$

$$g_{ab} = [exp(h_{..})]_{ab} \quad \delta^{ab} h_{ab} = 0 \quad \mathcal{F}_b = \partial^a h_{ab} = 0$$

Introduce external current:

$$Z[J^{ab}] = e^{W[J]^{ab}]} = \int dh_{ab} d\Psi \delta(\text{ gauge fixing}) Det_{FP} e^{\imath(S^{uni} + \int_{\mathcal{M}} h_{ab} J^{ab})}$$

Define expectation value:

$$< h_{ab} >= \frac{\delta W}{\delta J^{ab}}|_{J=0} \qquad \delta^{ab} < h_{ab} >= 0$$

$$< \bar{g}_{ab} >= exp < h_{ab} >$$ **is unimodular**

In perturbation theory

$$\mathcal{S}^{eff}[< \bar{g}_{ab} >] = S^{uni}(< \bar{g}_{ab} >, \phi, \psi) + \hbar \Delta S(< \bar{g}_{ab} >, \phi, \psi)$$

The canonical quantization of unimodular gravity.

1) The connection representation.
2) The spin network representation.
3) Infrared regularization and finite temperature

See also Bombelli, Couch, Torrence 1991

Key results of loop quantum gravity:

- The Hilbert space of spatially diffeomorphism invariant states, $\boldsymbol{H^{diff}}$ is precisely defined.
- The volume operator is precisely defined on $\boldsymbol{H^{diff}}$.
- The hamiltonian constraint can be precisely defined on $\boldsymbol{H^{diff}}$
  - These and other operators are uv finite

Key open issues:

- The inner product on physical states, ie solutions also to the Hamiltonian constraint
- The issue of physical observables.
- The issue of time and evolution.

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- The inner product on physical states, ie solutions also to the Hamiltonian constraint
- The issue of physical observables.
- The issue of time and evolution.

Might unimodular gravity's global time provide a new approach to these? *Some first thoughts....*

The connection representation:

Rename $\tilde{a}^0 \longrightarrow \tilde{T} \qquad \pi_0 \longrightarrow \pi$

Accumulated four volume: $\tau = \int_\Sigma \tilde{T}$

Canonical pairs:

$$\{\tilde{T}(x), \pi(y)\} = \delta^3(x,y)$$

$$\{A_a^i(x), \tilde{E}_j^b(y)\} = \delta^3(x,y)\delta_a^b\delta_j^i$$

Wavefunctionals: $\Psi[A, \tilde{T}]$

The hamiltonian constraint:

$$\imath\hbar\frac{\partial}{\partial\tilde{T}}\hat{det}(e)\Psi(A,\tilde{T}) = \hat{\tilde{h}}\Psi(A,\tilde{T})$$

conventional hamiltonian constraint

The full set of quantum constraints:

$$\tilde{h} = \frac{1}{\sqrt{q}} \epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j F_{abk}$$

$$\imath\hbar \frac{\partial}{\partial \tilde{T}} \hat{det}(e) \Psi(A, \tilde{T}) = \hat{\tilde{h}} \Psi(A, \tilde{T})$$

$$\imath\hbar \partial_c \frac{\partial}{\partial \tilde{T}} \Psi(A, \tilde{T}) = 0$$

plus SU(2) gauge and spatial diffeomorphism constraints.

Physical observables are correlations between A and T.

To solve these as usual we go to the spin network representation:

$$\Psi(A, \tilde{T}) \rightarrow \Psi(\Gamma, \tilde{T})$$

spin network basis

Solving the Hamiltonian constraint, starting with a graph Γ

Partition space into regions, R_i, each containing one vertex (w volume)

$$\int_{\mathcal{R}_i} \tilde{T} = \tau_i \qquad \sum_i \tau_i = \tau$$

This defines a partition of the elapsed four volume time.

Associate each $\tau_\imath$ to the vertex v_i $\quad \Psi(\Gamma, \tau_i)$

Each region has a volume operator and hamiltonian operator

$$V_i = \int_{\mathcal{R}_i} \sqrt{q} \qquad h_i = \int_{\mathcal{R}_i} \tilde{h}$$

Acting on a single node $\quad \hat{V}_i \Psi(\gamma, \tau_i) = \hat{w}_i \Psi(\gamma, \tau_i)$

The Hamiltonian constraint is now:

$$\boxed{\imath\hbar \frac{\partial}{\partial \tau_i} \hat{w}_i \Psi(\Gamma, \tau_i) = \hat{h}_i \Psi(\Gamma, \tau_i)}$$

$$\imath\hbar\frac{\partial}{\partial\tau_i}\hat{w}_i\Psi(\Gamma,\tau_i)=\hat{h}_i\Psi(\Gamma,\tau_i)$$

There is one remaining constraint:

$$\imath\hbar\partial_c\frac{\partial}{\partial\tilde{T}}\Psi(\Gamma,\tilde{T})=0$$

$$\sum_i\tau_i=\tau$$

$$\int_{\mathcal{R}_i}\tilde{T}=\tau_i$$

which implies:

$$\frac{\partial\Psi(\gamma,\tau_i)}{\partial\tau_i}=\frac{\partial\Psi(\gamma,\tau_j)}{\partial\tau_i}$$

which is solved by

$$\Psi(\gamma,\{\tau_i\})=\Psi(\gamma,\tau)$$

so there is simultaneous evolution in a single time:

.

$$\boxed{\imath\hbar\frac{\partial}{\partial\tau}\hat{w}_i\Psi(\Gamma,\tau)=\hat{h}_i\Psi(\Gamma,\tau)}$$

STEP1: Compactify the N time coordinates:

$$0 \leq \tau_i \leq 2\pi\beta$$

STEP 2: Work in **H**$^{\mathbf{diff}}$, the hilbert space of gauge and spatially diffeomorphism invariant states times $[L^2 (s^1)]^N$

STEP 3: Fourier transform to discrete E's

$$E_i^n = \frac{\pi n}{\beta}$$

$$\Psi(\Gamma, \{E_i\}) = \oint \prod_i d\tau_i e^{-\imath E_i \tau_i} \hat{h}_i \Psi(\Gamma, \tau_i)$$

which solve time independent Schrodinger equations

$$\hat{\tilde{h}}_i \Psi(\Gamma, \{E_i\}) = E_i \hat{w}_i \Psi(\Gamma, \{E_i\})$$

STEP 4: For each set of discrete E's solutions to this define a subspace of **H**$^{\mathbf{diff}}$

$$H^{diff}_{\{E_i\}}$$

STEP 5: Solve the remaining constraint, which is now in the form

$$(E_i - E_j)\Psi(\Gamma, \{E_i\}) = 0$$

The solutions of this live in a subspace of H^{diff} defined by

$$\boxed{H^{phys} = \sum_n H^{diff}_{\{E_1=\Lambda_n, E_2=\Lambda_n, \dots\}}}$$

$$\Lambda_n = \frac{G\pi n}{\beta}$$

Why is the cosmological constant so small?

Could this be a quantum effect?

We can rework the partition function into a form conjectured by Ng & van Dam

$$Z = \int d\Lambda \int \prod_{x^a} dg_{ab} d\Psi \delta(\text{ gauge fixing}) Det_{FP}$$
$$\times exp\ \imath \int_{\mathcal{M}} \sqrt{-g}\,(R + 2\Lambda + \mathcal{L}^{matter})$$

In the semi-classical approximation:

$$Z \approx \int d\Lambda \sum_{g_{ab},\Psi} exp\ \imath \int_{\mathcal{M}} \sqrt{-g} \left(-\frac{\Lambda}{4\pi G} + (\mathcal{L}^{matter} - \frac{T}{2}) \right)$$

This is dominated by histories for which

$$\frac{\Lambda}{4\pi G} \approx \frac{\int_{\mathcal{M}} \sqrt{-g}(\mathcal{L}^{matter} - T)}{Vol} = < (\mathcal{L}^{matter} - \frac{T}{2}) >$$

What is the meaning of:

$$\frac{\Lambda}{4\pi G} \approx \frac{\int_{\mathcal{M}} \sqrt{-g}(\mathcal{L}^{matter} - T)}{Vol} =< (\mathcal{L}^{matter} - \frac{T}{2}) >$$

For perfect fluids $\mathcal{L}^{matter} = P$

So we find, roughly, neglecting P:

$$\boxed{\frac{\Lambda}{2\pi G} \approx \frac{\int_{\mathcal{M}} \sqrt{-g}\rho}{Vol}}$$

Conclusions:

- Unimodular gravity can be quantized via path integrals and the resulting quantum theory is also unimodular.

Thus, the quantum equations of motion have the decoupling symmetry

$$T_{ab} \rightarrow T'_{ab} = T_{ab} + g_{ab}C$$

Hence the first cosmological constant problem is solved.

- There is a physical time coordinate, which is elapsed four volume. The hamiltonian quantization can be carried out in LQG and this time *might* be used to give a new approach to the physical inner product and physical observables.

- The second, why so small, problem and third, coincidence problem are also addressed at least at a hand-waving, semiclassical level, a la Ng and van Dam.

Details of the calculation of the path integral

Begin with the completely gauge fixed, constrained path integral:

$$Z = \int dA^i_a d\tilde{E}^a_i d\tilde{a}^0 d\tilde{a}^a d\pi_a d\pi_o \delta(\mathcal{H})\delta(\mathcal{D}_a)\delta(\mathcal{G}^i)\delta(\mathcal{G}_c)\delta(\pi_c)\delta(\text{ gauge fixing})Det_{FP}$$
$$\times exp\ \imath \int dt \int_\Sigma \left(\tilde{E}^a_i \dot{A}^i_a + \pi_0 \dot{\tilde{a}}^0 + \pi_c \dot{\tilde{a}}^c - (\partial_a \tilde{a}^a)\left(\frac{\epsilon^{ijk}\tilde{E}^a_i \tilde{E}^b_j F_{abk}}{det(\tilde{E}^a_i)}\right)\right)$$

Integrate over $d\pi_0$, eliminating $\delta(H)$:

$$\pi_0 \longrightarrow \left(\frac{\epsilon^{ijk}\tilde{E}^a_i \tilde{E}^b_j F_{abk}}{det(\tilde{E}^a_i)}\right)$$

$$Z = \int dA^i_a d\tilde{E}^a_i d\tilde{a}^0 d\tilde{a}^a d\pi_c \delta(\mathcal{D}_a)\delta(\mathcal{G}^i)\delta(\mathcal{S}_c)\delta(\pi_c)\delta(\text{ gauge fixing})Det_{FP}$$
$$\times exp\ \imath \int dt \int_\Sigma \left(\tilde{E}^a_i \dot{A}^i_a + \left(\frac{\epsilon^{ijk}\tilde{E}^a_i \tilde{E}^b_j F_{abk}}{det(\tilde{E}^a_i)}\right)(\dot{\tilde{a}}^0 + \partial_c \tilde{a}^c) + \pi_c \dot{\tilde{a}}^c\right)$$

where G_c has become S_c

$$S_c \equiv \partial_c \left(\frac{\epsilon^{ijk}\tilde{E}^a_i \tilde{E}^b_j F_{abk}}{det(\tilde{E}^a_i)}\right) = 0$$

•Exponentiate S_c with a new vector density w^a, integrate $d\pi_c$

$$Z = \int dA_a^i d\tilde{E}_i^a d\tilde{a}^0 d\tilde{a}^a d\tilde{w}^c \delta(\mathcal{D}_a)\delta(\mathcal{G}^i)\delta(\text{ gauge fixing}) Det_{FP}$$

$$\times exp\ \imath \int dt \int_\Sigma \left(\tilde{E}_i^a \dot{A}_a^i + \left(\frac{\epsilon^{ijk}\tilde{E}_i^a \tilde{E}_j^b F_{abk}}{det(\tilde{E}_i^a)} \right) (\dot{\tilde{a}}^0 + \partial_c \tilde{a}^c + \partial_c \tilde{w}^c) \right)$$

•Shift $a^a \rightarrow a^a - w^a$, then do integral over dw^a

•Exponentiate Gauss, diffeo constraints with A_0 and N^a

$$Z = \int dA_a^i dA_0^i d\tilde{E}_i^a dN^a d\tilde{a}^0 d\tilde{a}^c \delta(\text{ gauge fixing}) Det_{FP}$$

$$\times exp\ \imath \int dt \int_\Sigma \left(\tilde{E}_i^a \dot{A}_a^i + \left(\frac{\epsilon^{ijk}\tilde{E}_i^a \tilde{E}_j^b F_{abk}}{det(\tilde{E}_i^a)} \right) (\dot{\tilde{a}}^0 + \partial_c \tilde{a}^c) + N^a \mathcal{D}_a + A_0^i \mathcal{G}^i \right)$$

•Introduce the lapse, N, with a definition of unity:

$$1 = \int dN \delta \left(N - \frac{\dot{\tilde{a}}^0 + \partial_c \tilde{a}^c}{Det(\tilde{E}^{ai})} \right)$$

This gives us:

$$
\begin{aligned}
Z &= \int dA^i_a dA^i_0 d\tilde{E}^a_i dN^a dN d\tilde{a}^0 d\tilde{a}^c \delta\left(N - \frac{\dot{\tilde{a}}^0 + \partial_c \tilde{a}^c}{Det(\tilde{E}^{ai})}\right) \delta(\text{ gauge fixing}) Det_{FP} \\
&\times exp\, \imath \int dt \int_\Sigma \left(\tilde{E}^a_i \dot{A}^i_a + N\epsilon^{ijk}\tilde{E}^a_i \tilde{E}^b_j F_{abk} + N^a \mathcal{D}_a + A^i_0 \mathcal{G}^i\right)
\end{aligned}
$$

Change variables from E,N, N^a to e^μ:

$$
\begin{aligned}
Z &= \int dA^i_a dA^i_0 de^\mu d\tilde{a}^0 d\tilde{a}^c \delta\left(det(e) - \dot{\tilde{a}}^0 + \partial_c \tilde{a}^c\right) \delta(\text{ gauge fixing}) Det_{FP} \\
&\times exp\, \imath \int dt \int_\Sigma \left(e^\mu \wedge e^\nu \wedge F^+_{\mu\nu}\right)
\end{aligned}
$$

Now we specify the gauge fixing functions in the delta functions:

$$
\tilde{f}_0 = \tilde{a}^0 - t(\epsilon_0 - \partial_c \tilde{a}^c) = 0, \qquad \tilde{f}^c = \tilde{a}^c = 0
$$

After which we can integrate over the a^0 and a^a.

This yields finally:

$$\boxed{\begin{aligned} Z &= \int dA^i_\mu de^\mu \delta\left(det(e) - \epsilon_0\right) \delta(\text{ gauge fixing}) Det'_{FP} \\ &\quad \times exp\ \imath \int dt \int_\Sigma \left(e^\mu \wedge e^\nu \wedge F^+_{\mu\nu}\right) \end{aligned}}$$