Generic singularities

by
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- Past singularities = ”cosmological singularities.”
- Singularity → variable scale
- Singularity theorems: One dynamical input – the Raychaudhuri equation for the expansion; expansion = variable scale.
- Blow up of expansion – factor it out!

Some desirable features when factoring out the expansion:

- Preservation of causal structure.
- Adaption to self-similar solutions, i.e., use of scale-invariance.

The scale-invariant conformal approach:

Conformal regularization of Einstein's equations

$$g = \Psi^2 G$$

$$G = \Psi^{-2} g$$

where Ψ has dimension length (or, equivalently, time);

g = is the physical metric;

G is a dimensionless conformal metric.

Consider the temporal development along a timelike congruence toward a singularity.

$H\Psi$ bounded and >0 toward the singularity, where

$H = \frac{1}{3}\theta$ is the `Hubble´ variable; we assume H>0.

The conformal orthonormal frame approach (advantage: weighted spatial derivative operators)

For simplicity: Conformal Hubble-normalization

$$\mathbf{g} = H^{-2}\,\mathbf{G} = H^{-2}\,\eta_{ab}\,\boldsymbol{\Omega}^a\,\boldsymbol{\Omega}^b \qquad \langle\,\boldsymbol{\Omega}^a,\,\boldsymbol{\partial}_b\,\rangle = \delta^a{}_b$$

$$\boldsymbol{\partial}_0 = H^{-1}\,\boldsymbol{e}_0 = \mathcal{M}^{-1}\partial_t$$

$$\boldsymbol{\partial}_\alpha = H^{-1}\,\boldsymbol{e}_\alpha = \mathcal{M}_\alpha\,\mathcal{M}\boldsymbol{\partial}_0 + E_\alpha{}^i\partial_i$$

$${}^3G^{ij} = \delta^{\alpha\beta}E_\alpha{}^iE_\beta{}^j$$

Decoupling of the equations for H because of dimensional reasons (kinematically defines q and r_α):

$$\boldsymbol{\partial}_0 H = -\,(q+1)\,H \qquad\qquad \boldsymbol{\partial}_\alpha H = -\,r_\alpha\,H$$

Gauge variables: $(\mathcal{M}, \mathcal{M}_\alpha, W^\alpha, \dot{U}^\alpha, R^\alpha)$

Conformally Hubble-normalized state space variables:

$$\mathbf{X} = (E_\alpha{}^i) \oplus \mathbf{S}$$

$$\mathbf{S}_{\rm vac} = (\Sigma_{\alpha\beta}, A_\alpha, N_{\alpha\beta})$$

$$\mathbf{S} = \mathbf{S}_{\rm vac} \oplus (H\text{-normalized matter variables})$$

$$\{\Omega,\ P,\ Q^\alpha,\ \Pi_{\alpha\beta}\} = \{\rho,\ p,\ q^\alpha,\ \pi_{\alpha\beta}\}/(3H^2)$$

$$\boldsymbol{\partial}_0\, E_\alpha{}^i = F_\alpha{}^\beta\, E_\beta{}^i$$

$$(E_\alpha{}^i, \mathcal{M}_\alpha, W_\alpha, \dot{U}_\alpha, r_\alpha) = 0$$

Invariant set: The silent boundary.

The *equations* (=ODE) on the silent boundary are *identical to those of the spatially homogeneous models* (the equations for the spatial frame variables decouple in the SH case). Here the state space is an infinite dimensional set of copies – one at each spatial point.

Asymptotic silence gauge condition:

$$0 < \mathcal{M} < \infty, \qquad \mathcal{M}_\alpha \to 0$$

<u>Asymptotic silence condition</u>: $\quad E_\alpha{}^i \to 0$

which is equivalent to $\quad {}^3G^{ij} \to 0$

These conditions imply conditions on

$q = -\mathcal{H}$ and $q\delta_\alpha{}^\beta - \Sigma_\alpha{}^\beta = -\Theta_\alpha{}^\beta$

namely, the conformal expansion has to be negative in all directions almost always in the vicinity of an asymptotically silent singularity; note that this is a purely kinematical result.

The strategy here is thus opposite to that of the usual conformal treatment of asymptotic flatness: there one `contracts and brings in' the `nice' region out to and including infinity – here we `blow up and push out' the `nasty' region in the vicinity of the singularity!

Asymptotic silence and locality

Asymptotic locality gauge condition:

$$(\mathcal{M}_\alpha, W_\alpha, \dot{U}_\alpha, r_\alpha) \to 0, \qquad 0 < \mathcal{M} < \infty$$

Suggests useful gauges:

- Inverse mean curvature flow foliation (separable volume gauge).
- Constant mean curvature foliation.

Asymptotic silence and locality condition:

$$(E_\alpha{}^i, \mathcal{M}_\alpha, W_\alpha, \dot{U}_\alpha, r_\alpha) \to 0, \qquad 0 < \mathcal{M} < \infty,$$

$$\boldsymbol{\partial}_\alpha(\text{coordinate scalars appearing in the field equations}) \to 0$$

(Previous considerations are purely kinematical.)

Dynamical issues:

• According to Einstein's field equations, how large is the class of models that admit asymptotically silent and local singularities?

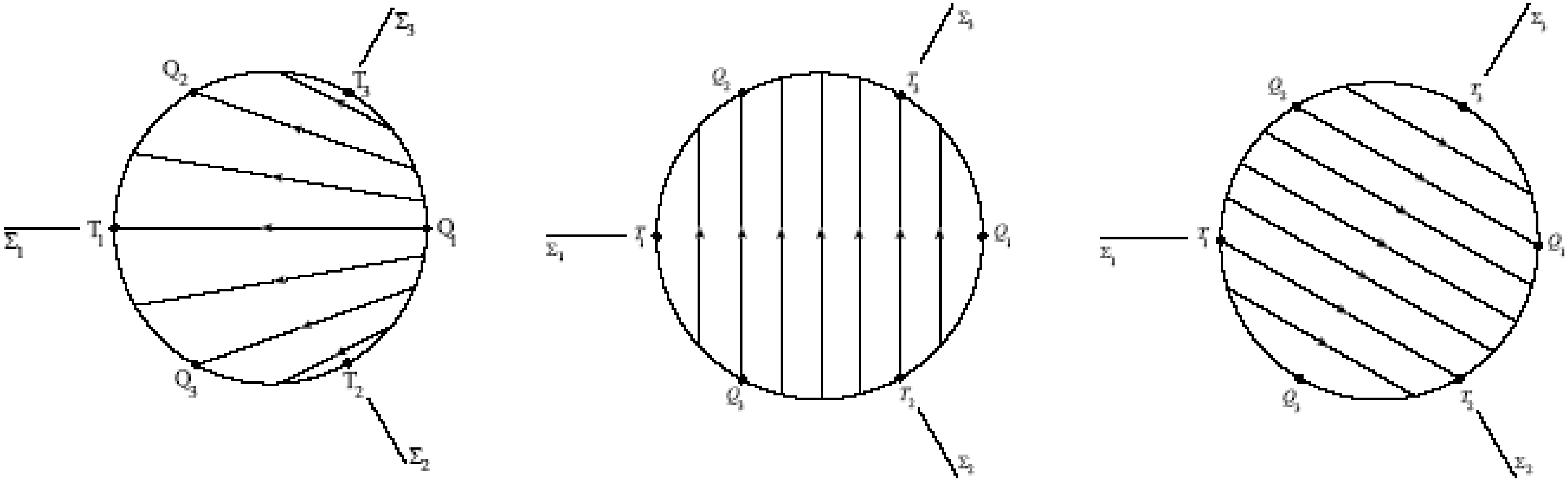
• In general not all of a part of a singularity that is asymptotically silent is asymptotically local. Hence, according to Einstein's field equations, how many of the timelines that end at a singularity in such models have asymptotically silent and local dynamics determined by the dynamics on the \`local part´ of the silent boundary?

Generic spacelike singularities

Strategy :

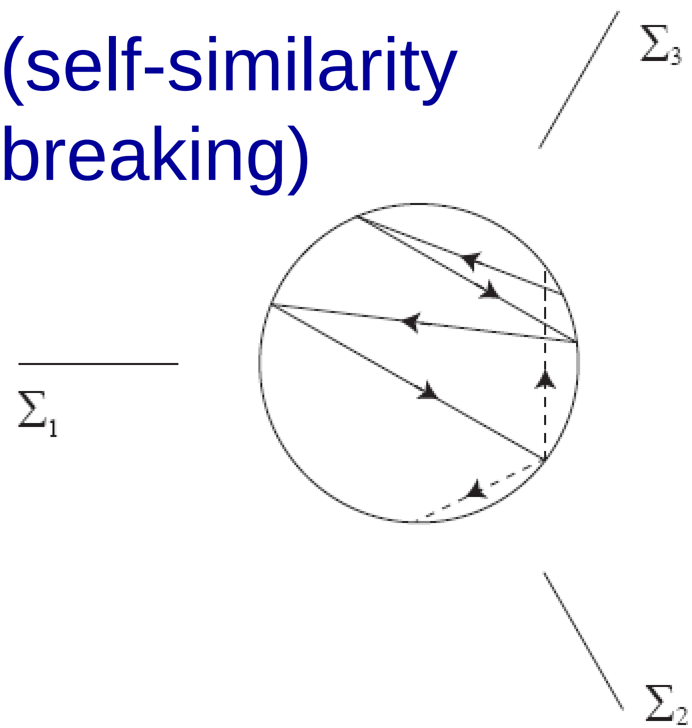
- Identification of the attractor on the silent boundary.

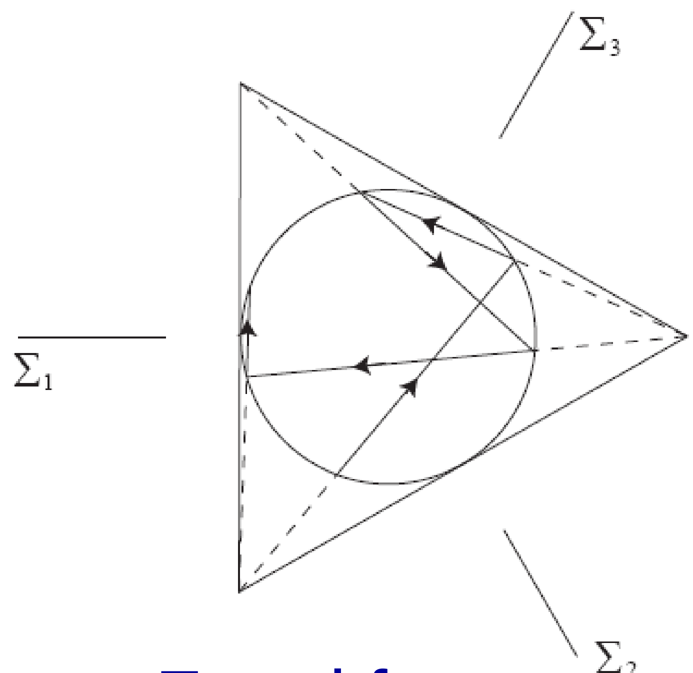
- Perturbation of the attractor to establish if it is stable or not.



(a) $\mathcal{T}_{N_1}$ (b) $\mathcal{T}_{R_1}$ (c) $\mathcal{T}_{R_3}$

(self-similarity breaking)

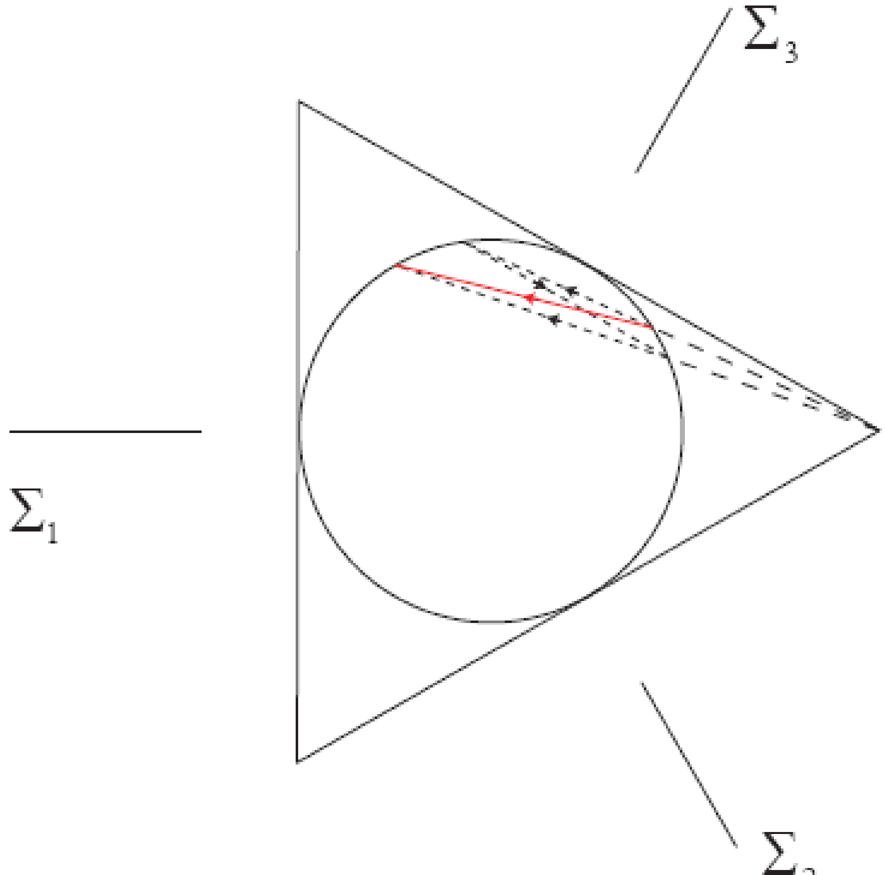




Fermi frame

Non-generic asymptotically non-local behavior *at* special timelines.

Conjecture: Infinitely recurring spike transitions. Happens on *partially* silent boundaries.



Recent developments

(Overlapping) tools: Heuristic methods and insights; analytic results concerning special models; numerics; specific solutions as examples.

- Momentum and angular momentum matters for fluids with soft equations of state.
- Recurring non-local (i.e. non-BKL) spike formation.
- Bianchi type IX is misleading!
- No proofs about BKL, not even for type IX. Statistical cumulative effects; fragility of generic behaviour – connection with weak null singularities?
- The role of the scale and spatial diffeomorphism groups – hierarchical structures, conserved quantities and monotone functions & quantization?