Three-Charge Supertubes in a Rotating Black Hole Background

http://arXiv.org/abs/hep-th/0612085

Eastern Gravity Meeting

The Pennsylvania State University

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May 12, 2008

Abstract

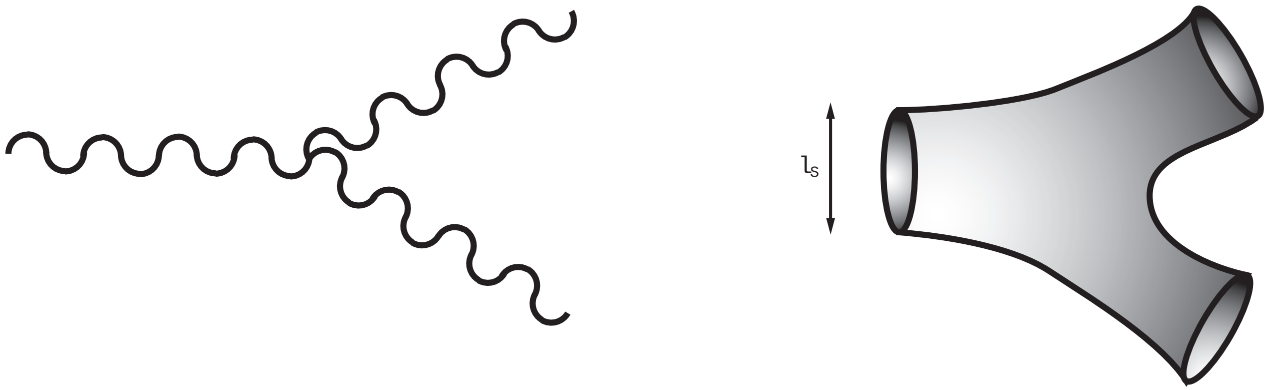
- String theory has provided a precise quantum description of certain supersymmetric black holes, one of which is the Breckenride-Myers-Peet-Vafa (BMPV) black hole.

- We use a string theoretical object called a **supertube** to probe the black hole.

- We determine under what circumstances a supertube with three charges can merge with the black hole and find evidence that the merger can cause fragmentation of the black hole.

- Thus we extend and generalize the investigations have been performed with the two-charge supertube.

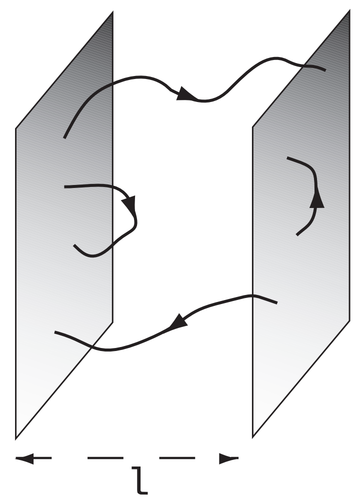
String Theory



[Sagnotti and Sevrin [hep-ex/0209011]]

- String theory describes elementary particles, including the graviton, as quantum states of one dimensional strings instead of zero-dimensional point particles.

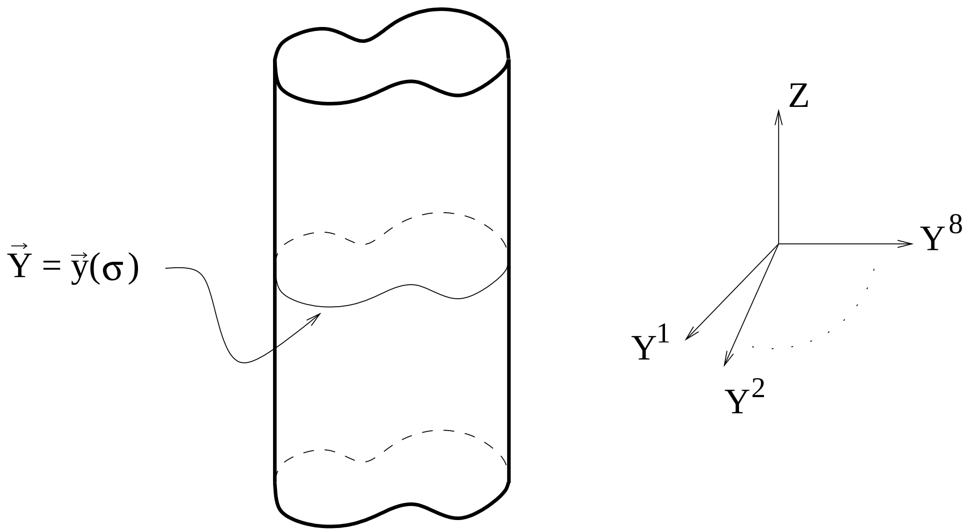
D-branes



[Sagnotti and Sevrin, hep-ex/0209011]

- In string theory there are both open strings and closed strings (loops of string). Open strings end on objects called **D-branes**.

Supertubes



[Mateos, Ng and Townsend, hep-th/0112054]

- Supertubes are tubular D-brane configurations.

- There are static E- and B- fields on their worldvolumes that produce angular momentum, which stabilizes them against collapse.

D6 brane Supertube (Bena-Kraus construction)

- The supertube is comprised of k coincident D6-branes, wrapped on $T^4 \times S^1_z = T^5$, where k is an even integer.

- By construction, the supertube carries net D0, D4, and F1 charge $\{\mathfrak{q}_{D0}, \mathfrak{q}_{D4}, \mathfrak{q}_{F1}\}$, but no D2 charge.

- $(10 - 5)$ dim. $\rightarrow$ effectively a five-dimensional scenario.

Rotating Black Holes in 4+1 Dimensions

- In D=4+1 there are [(D-1)/2]=2 independent planes of rotation $\rightarrow$ two independent angular momenta J_1 and J_2.

- If $J_1 = J_2$ the spacetime can still carry net angular momentum even if the black hole event horizon is nonrotating!

Lift of BMPV Black Hole to IIA Supergravity

$$\begin{aligned} ds^2 &= -H_{D0}^{-1/2} H_{D4}^{-1/2} H_{F1}^{-1} (dt + \gamma_1(\theta) d\phi_1 + \gamma_2(\theta) d\phi_2)^2 \\ &\quad + H_{D0}^{1/2} H_{D4}^{1/2} H_{F1}^{-1} dz^2 \\ &\quad + H_{D0}^{1/2} H_{D4}^{1/2} (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi_1^2 + r^2 \cos^2\theta d\phi_2^2) \\ &\quad + H_{D0}^{1/2} H_{D4}^{-1/2} ds^2_{T^4} \end{aligned}$$

$$\begin{aligned} \gamma_1 &= \frac{4G_5}{\pi}\frac{J}{r^2}\sin^2\theta, \quad \gamma_2 = \frac{4G_5}{\pi}\frac{J}{r^2}\cos^2\theta, \\ H_{D0} &= 1 + \frac{Q_{D0}}{r^2}, \quad H_{D4} = 1 + \frac{Q_{D4}}{r^2}, \quad H_{F1} = 1 + \frac{Q_{F1}}{r^2} \end{aligned}$$

- The angular momenta $J_1 = J_2 \equiv J$ of the BH satisfy

$$J^2 \leq N_{D0} N_{D4} N_{F1}$$

where the N_i's represent the integer-valued charges of the BH.

Dirac-Born-Infeld (DBI) Action

- Low energy effective action for a D-brane

- It assumes a fixed background, which is the most straightforward method to describe moving supertubes.

$$
\begin{aligned}
S &= \int L\,dt = \int \mathcal{L}\,d^7x = \int (\mathcal{L}_{DBI} + \mathcal{L}_{WZ})\,d^7x \\
&= -\tau_{D6} \int d^7x\, e^{-\Phi} \sqrt{-\det(g_{ab} + b_{ab} + F_{ab})} \\
&\quad + \tau_{D6}\, g_s \int \sum_{7-forms} c^{(m)} \wedge e^{(F+b)^{(2)}}.
\end{aligned}
$$

- $g_{ab} = g_{\mu\nu} \frac{\partial x^\mu}{\partial y^a} \frac{\partial x^\nu}{\partial y^b}, \quad b_{ab} = b_{\mu\nu} \frac{\partial x^\mu}{\partial y^a} \frac{\partial x^\nu}{\partial y^b}, \quad c_a^{(1)} = C_\mu^{(1)} \frac{\partial x^\mu}{\partial y^a}$, *etc.*

Static Supertube Configurations

- In a supersymmetric configuration the supertube charges and black hole charges have the same signs, and the repulsion of the like charges exactly cancels the gravitational attraction.

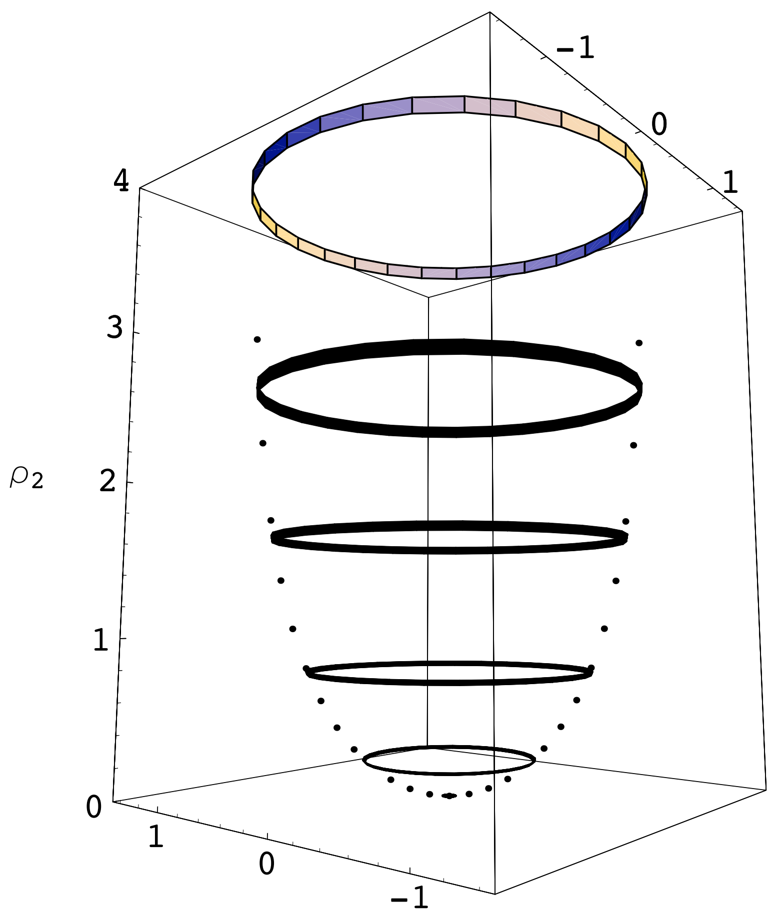
- This is a configuration of minimum energy, called a Bogolmol'nyi-Prasad-Sommerfield (BPS) configuration.

Critical Angular Momentum

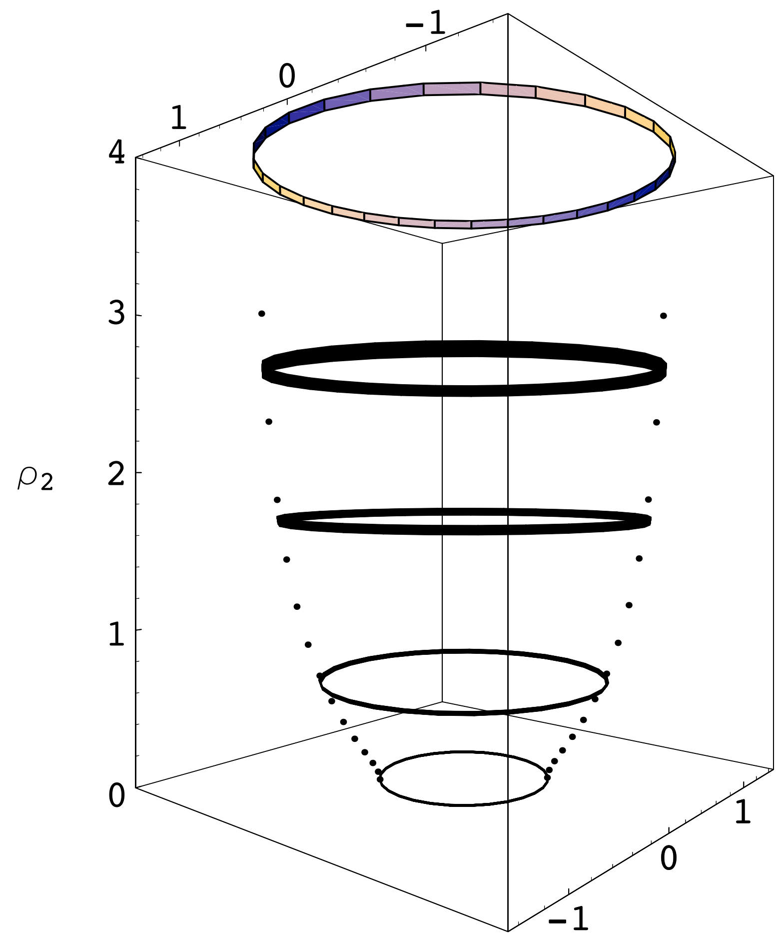
- The three-charge supertube, when in the vicinity of the BMPV black hole, has a critical value of the angular momentum:

$$j_{crit} = N_{D0} + \frac{\mathfrak{q}_{D0}}{\mathfrak{q}_{D4}} N_{D4}.$$

Static Configuration Locations



$j_1 \leq j_{crit}$



$j_1 > j_{crit}$

Moving Supertubes

- When a supertube moves with respect to the black hole, the delicate cancellation of forces is spoiled. There is now an excess of energy ΔE over the minimum E_{BPS},

$$\begin{aligned} E &= E_{BPS} + \Delta E \\ &= 2\pi R_z \, \tau_{F1} \, \mathfrak{q}_{F1} + \tau_{D0} \, \mathfrak{q}_{D0} + V_{T^4} \, \tau_{D4} \, \mathfrak{q}_{D4} + \Delta E. \end{aligned}$$

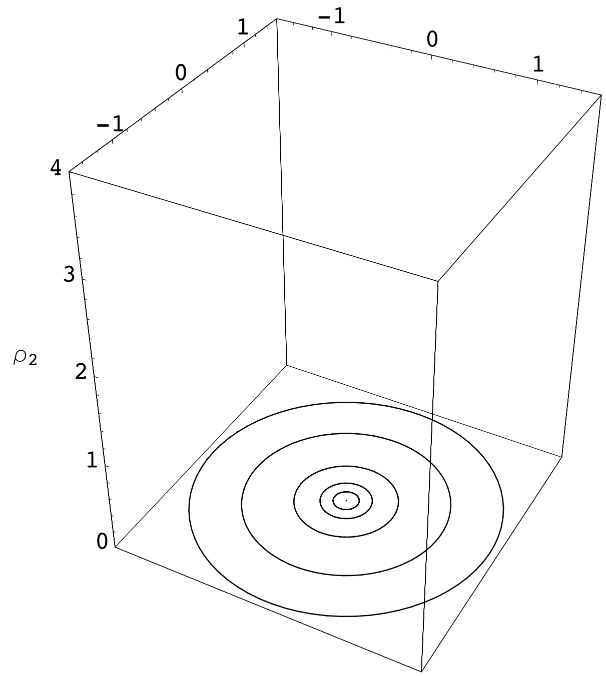
- This can be used to obtain an effective potential $V(r, \theta)$.

Effective Potential

- We use the coordinates $r, \theta, \phi_1, \phi_2$. Eliminating $\dot{r}$ and $\dot{\theta}$ for conserved quantities gives an effective potential $\mathsf{V}(r,\theta)$ for $0 < \theta < \frac{\pi}{2}$.

$$\begin{aligned}
\mathsf{V}(r,\theta) &= \Delta E|_{\dot{r}=0,\dot{\theta}=0} \\
&= \frac{\tau_{D6} V_6\, k\, F_{z\sigma} r^2}{2r^2(H_{D0} + B_0^2 H_{D4})} \\
&\times \frac{1}{(F_{z\sigma}^2 H_{F1} r^2 + H_{D0} H_{D4}\, r^4 \sin^2\theta - 2\omega F_{z\sigma} \sin^2\theta)} \\
&\times \left(\frac{[j_1/(\tau_{D6} V_6) - (H_{D0} + B_0^2 H_{D4})\, r^2 \sin^2\theta]^2}{\sin^2\theta} \right. \\
&\left. + \frac{[j_2/(\tau_{D6} V_6) - (Q_{D0} + B_0^2 Q_{D4}) \sin^2\theta]^2}{\cos^2\theta} \right).
\end{aligned}$$

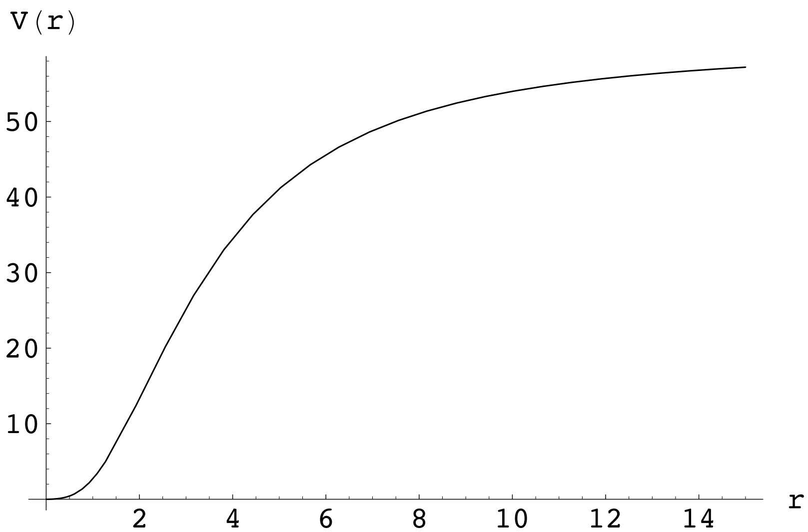
Motion in the plane $\theta = \frac{\pi}{2}$

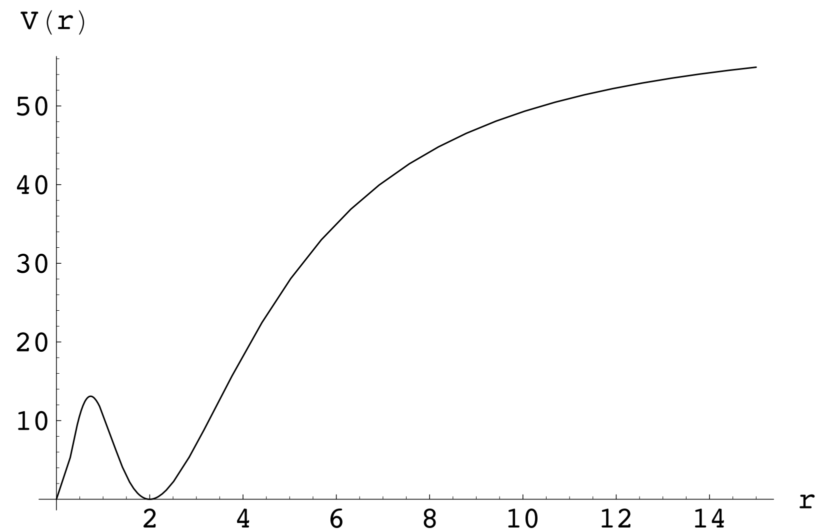


- There is a special set of purely radial trajectories along $\theta = \frac{\pi}{2}$. In this case the center of mass of the supertube stays motionless (centered on the black hole) while the radius changes. The potential V(r) is

$$
\begin{aligned}
\mathrm{V}(r) &= \Delta E|_{\dot{r}=0,\ \dot{\theta}=0,\ \theta=\frac{\pi}{2}} \\
&= \tau_{D6} V_6\, k \frac{F_{z\sigma}\, r^2 \left[j_1/(\tau_{D6}V_6) - (H_{D0} + B_0^2 H_{D4})\, r^2\right]^2}{2r^2(H_{D0} + B_0^2 H_{D4})\,(F_{z\sigma}^2 H_{F1} r^2 + H_{D0} H_{D4}\, r^4 - 2\omega F_{z\sigma})}\,.
\end{aligned}
$$

Motion in the plane $\theta = \frac{\pi}{2}$





- When $j_1 \leq j_{crit}$ there is no potential barrier.

- When $j_1 > j_{crit}$ there is a potential barrier and local potential minimum.

BMPV Angular Momentum Bound

- The angular momenta $J_1 = J_2 \equiv J$ of the BH satisfy

$$J^2 \leq N_{D0} N_{D4} N_{F1}.$$

- Now, supertubes also have charges and angular momenta. Thus we can attempt to violate this bound by dropping supertubes of appropriate charge and orientation into the BH.

Attempting to Violate the Angular Momentum Bound

- If the BH angular momentum is slightly below the maximum value, *i.e.*

$$J \approx \sqrt{N_{D0} N_{D4} N_{F1}}$$

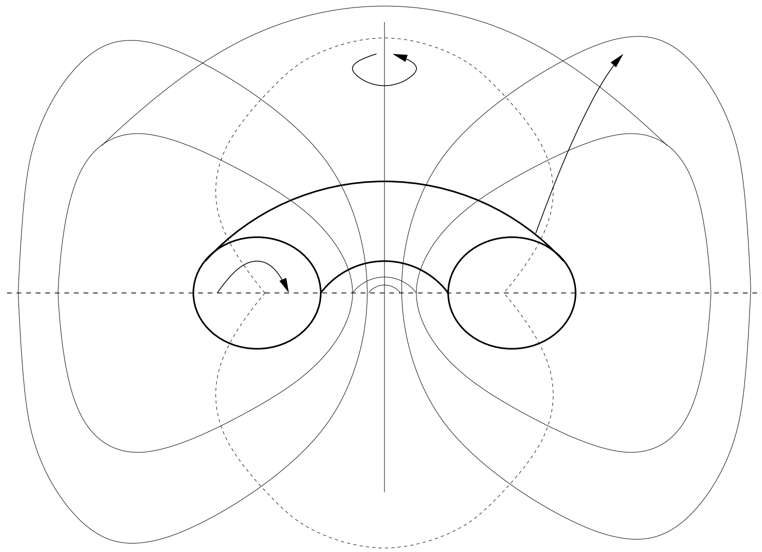
we can add two identical supertubes to the BH (one along each axis so that $J_1' = J_2'$) and see if doing so violates the angular momentum bound.

Attempting to Violate the Angular Momentum Bound

- If there is no potential barrier present, no overspin occurs.

- If there is a potential barrier present, overspin can formally occur if $j_1 > 4 j_{crit}$
  ...but the resulting object is no longer a BMPV black hole!

Post-Merger State: Fragmentation of the Black Hole?



[Elvang, Emparan, Mateos and Reall, hep-th/0408120]

- The true nature of the final bound state of BH + supertubes has yet to be determined.

- Marolf and Virmani (2005) have argued that it is a fragmentation into several black objects; the most natural candidate seems to be a concentric black hole/black ring configuration.

Entropy Considerations

$$S_{BMPV} = 2\pi\sqrt{N_{D0}N_{D4}N_{F1} - J^2}$$

- If, post-merger, we have

$$N'_{D0}N'_{D4}N'_{F1} - J'^{\,2} < N_{D0}N_{D4}N_{F1} - J^2,$$

then the GSL implies that the result cannot be a black hole.

Entropy Considerations

- Since the condition

$$N'_{D0}N'_{D4}N'_{F1} - J'^2 < N_{D0}N_{D4}N_{F1} - J^2$$

is necessary but not sufficient for an overspin, we see that fragmentation can occur even *without* an overspin, *i.e.* under circumstances more general than realized previously.

- A similar argument applies to the two-charge supertube, thus generalizing that analysis as well.

Possible Future Directions

- A configuration of a black ring surrounding a black hole, called 'black Saturn' has recently gained attention. It appears to be one of the most stable black objects for a given mass and angular momentum, as phase diagrams for five dimensional black holes indicate (Elvang *et. al.* 2007).

- If charged versions of black Saturn are found, perhaps some of them will be supersymmetric. If so they would be natural candidates for endpoints of supertube/BMPV mergers.

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Grateful acknowledgements to:

- The Eastern Gravity Meeting Committee
- The Pennsylvania State University
- Howard University Dept. of Physics and Astronomy
- Howard University Graduate School
- United States Department of Energy
- Kavli Institute of Theoretical Physics at UCSB
- UCSB Graduate Division and Prof. John Mohr
- Prof. Don Marolf, Prof. Iosif Bena, Prof. Per Kraus, Prof. Joe Polchinski and Amitabh Virmani.