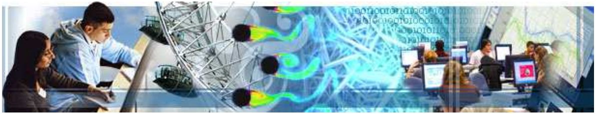
Dark Energy from Corrections to the Wheeler-DeWit Equation.

William Nelson

King's College London

25th Oct. 2008



Based on the paper:

Phys. Lett. B, Vol. 661 (2008) 37, arXiv:0709.1625 [gr-qc]

W. N., M. Sakellariadou[1].

[1] King's College London.

Motivation

We have seen that without Lattice Refinement, the equations of LQC exhibit an instability.

This instability comes from the fact that solutions to the WdW equation oscillate very fast on large scales i.e. the WdW equation requires very fine resolution at large scales.

Any theory of quantum gravity that produces the WdW in the classical limit and has a small scale at which corrections to General Relativity become important will have the problem.

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MOTIVATION

Typically we say that this problem needs to be avoided, because we don't see deviations from General Relativity at large scales.

Or do we?
Dark energy could be seen as a deviation from General Relativity that occurs only at the large scales.

Can (small scale) corrections to the WdW equation lead to Dark Energy like behaviour?

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Wheeler-DeWit

Wheeler-DeWit Equation

$$\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[a\psi\left(a\right)\right]\right]-\frac{9k}{16\pi^2 l_{\mathrm{Pl}}^2}a\psi\left(a\right)+\frac{3}{2\pi l_{\mathrm{Pl}}^2}\mathcal{H}_\phi\psi\left(a\right)=0\,,$$

where a is the scale factor, l_{Pl} is the Planck length, $k=\pm1,\ 0$ is the curvature and $\mathcal{H}_\phi$ is the matter Hamiltonian.

It must be remembered that the WdW equation can only be formally written down due to the symmetry reduction, but in cosmology this is what we always do!

Here a particular factor ordering is chosen, but the principle is the same for any factor ordering taken.

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Wheeler-DeWit

To solve the Wheeler-DeWit equation we need something for $\mathcal{H}_\phi$.

We parameterise the matter Hamiltonian as:

$$\frac{3}{(2\pi l_{\rm Pl}^2)}\mathcal{H}_\phi = \epsilon(\phi)\, a^\delta \ ,$$

We're going to be looking at the large scale, classical effects of this equation, where we expect the Hamiltonian to follow this form e.g.

$$\begin{aligned} \mathcal{H}_{\rm Dust} &\sim a^0 \qquad &&(\text{because } \rho_{\rm dust} \sim a^{-3}), \\ \mathcal{H}_{\rm Rad} &\sim a^{-1} \qquad &&(\text{because } \rho_{\rm Rad} \sim a^{-4}), \\ \mathcal{H}_{\rm Vac} &\sim a^3 \qquad &&(\text{because } \rho_{\rm Vac} \sim a^0). \end{aligned}$$

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WHEELER-DEWIT

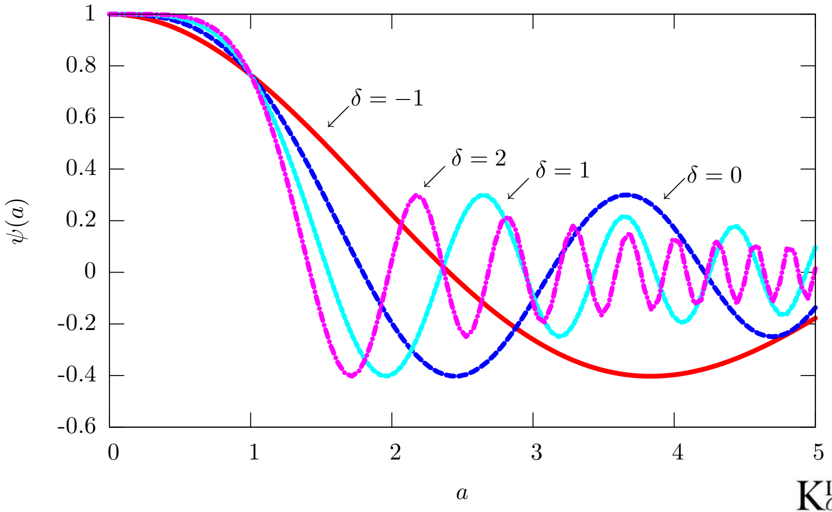
With this parameterisation of $\mathcal{H}_\phi$ we can solve the Wheeler-DeWit equation for $k = 0$ (which we will take from now on).

SOLUTION:

$$\begin{aligned}\psi(a) &= C_1 \, J_{\frac{\sqrt{2}}{3+\delta}} \left[\frac{2\sqrt{\epsilon}}{3+\delta} a^{(3+\delta)/2}\right] \\ &+ C_2 \, Y_{\frac{\sqrt{2}}{3+\delta}} \left[\frac{2\sqrt{\epsilon}}{3+\delta} a^{(3+\delta)/2}\right] ,\end{aligned}$$

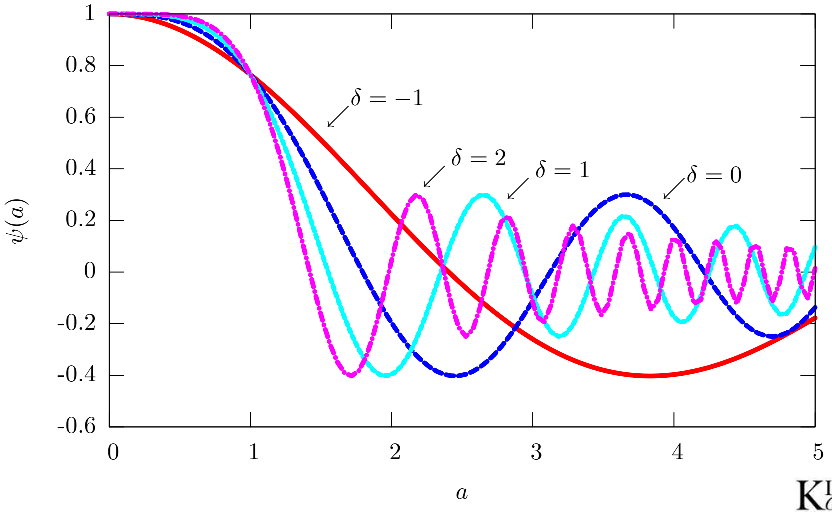
where J and Y are Bessel functions of the fi rst and second kind.

WHEELER-DEWIT



Notice that the larger δ the sooner the problem.

Wheeler-DeWit



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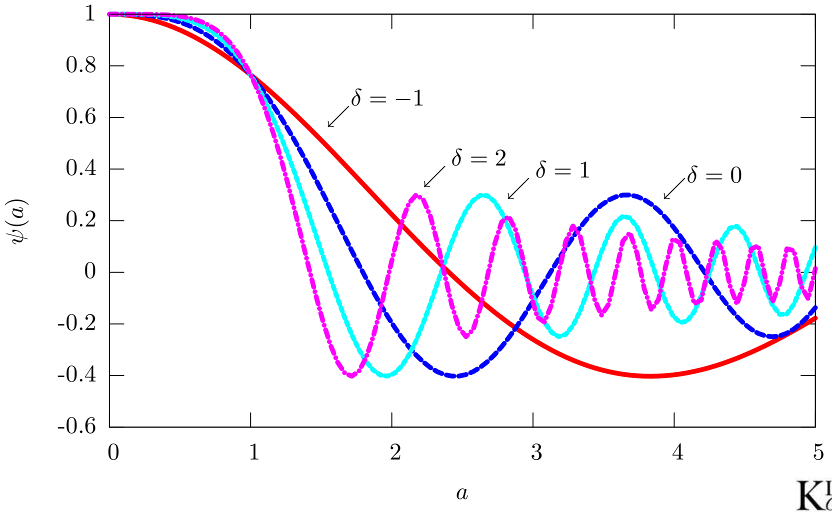

In particular this is a problem for inflation.

General Corrections

Standard Wheeler-DeWit equation

$$\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[a\psi\left(a\right)\right]\right]+\epsilon\left(\phi\right)a^{\delta}\psi\left(a\right)=0$$

where a_0 is the scale at which the corrections begin to become important ($a_\star$ in LQC) and f is some function.

In the semi-classical limit the corrections will be of this form.

Formally any (analytic) correction can be written as a Taylor expansion using these terms.

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General Corrections

Quantum Gravity Corrections

$$\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[a\psi\left(a\right)\right]\right]+\epsilon\left(\phi\right)a^{\delta}\psi\left(a\right)$$
$$+a_0 f\left(a,\psi\left(a\right),\partial_a\psi,\partial_a^2\psi\cdots\right)=0\,,$$

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Formally any (analytic) correction can be written as a Taylor expansion using these terms.

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General Corrections

We can no longer solve this exactly, but if we assume that the corrections are small, we can find their form.

We will then be able to calculate how the corrections behave initially i.e. before they begin to dominate.

Take the limit $a_0 \ll \epsilon(\phi)$ whilst $f\left(\psi(a), \partial_a\psi, \partial_a^2\psi \cdots\right)$ is small.

With this approximation, the previous solutions are approximately correct and we can use this to evaluate the *small* correction terms.

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General Corrections

In particular the derivatives are approximately:

$$\begin{aligned}
\partial_a \psi(a) &\approx \left(1 - \sqrt{\epsilon} Z_1(a) a^{(3+\delta)/2}\right) a^{-1} \psi(a) \ , \\
\partial_a^2 \psi(a) &\approx \left(Z_1(a) a^{(3+\delta)/2} - \sqrt{\epsilon} a^{3+\delta}\right) \sqrt{\epsilon} a^{-2} \psi(a) \ , \\
\partial_a^3 \psi(a) &\approx \left(\epsilon Z_1 a^{(9+3\delta)/2} - 3 Z_1 a^{(3+\delta)/2}\right) \sqrt{\epsilon} a^{-3} \psi(a) \\
&\quad - \sqrt{\epsilon}(1+\delta) a^{3+\delta} \sqrt{\epsilon} a^{-3} \psi(a) \ , \\
&\text{etc.}
\end{aligned}$$

where

$$\begin{aligned}
Z_1(a) \equiv C_1\, J_{\frac{\sqrt{2}}{3+\delta}+1} \left[\frac{2\sqrt{\epsilon}}{3+\delta} a^{(3+\delta)/2}\right] \left[\psi(a)\right]^{-1} \\
+ C_2\, Y_{\frac{\sqrt{2}}{3+\delta}+1} \left[\frac{2\sqrt{\epsilon}}{3+\delta} a^{(3+\delta)/2}\right] \left[\psi(a)\right]^{-1} .
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Notice: Highest power of a in all terms contains the highest power of $\epsilon(\phi)$ and one Z_1.

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General Corrections

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Recall,

$$\psi\,(a) = C_1\, J_{\frac{\sqrt{2}}{3+\delta}}\left[\frac{2\sqrt{\epsilon}}{3+\delta}a^{(3+\delta)/2}\right] + C_2\, Y_{\frac{\sqrt{2}}{3+\delta}}\left[\frac{2\sqrt{\epsilon}}{3+\delta}a^{(3+\delta)/2}\right] ,$$

So Z_1 basically increases the order of the Bessel functions in the wavefunction, by one.

General Corrections

We are interested in what these corrections look like at large scales, so we can expand the Bessel functions in the large a limit. Then

$$Z_1 \approx \left[\frac{\psi^\star(a)}{\psi(a)} \right] ,$$

where $\psi^\star$ is just ψ with the constants $C_1 \to -C_2$ and $C_2 \to -C_1$.

Now we can put this into the expressions for the derivatives.

And since we have already taken the large a limit, we keep only the terms with the highest power of a.

$$\begin{aligned}
\partial_a \psi(a) &\approx -\sqrt{\epsilon} \lim_{a\to\infty} \left[\frac{\psi^*(a)}{\psi(a)} \right] a^{(1+\delta)/2} \psi(a) , \\
\partial_a^2 \psi(a) &\approx -\epsilon a^{1+\delta} \psi(a) , \\
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General Corrections

In general the β^{th} derivative is proportional to,

$$\partial^{\beta}\psi(a) \propto a^{\beta(\delta+1)/2}\psi(a) \ ,$$

Whilst they are small the derivatives are proportional to the wavefunction.

Of course, we can generalise this to,

$$a^{\alpha}\partial^{\beta}\psi(a) \propto a^{\alpha+\beta(\delta+1)/2}\psi(a) \ .$$

So we can write any correction term (that is small) as proportional to the wavefunction.

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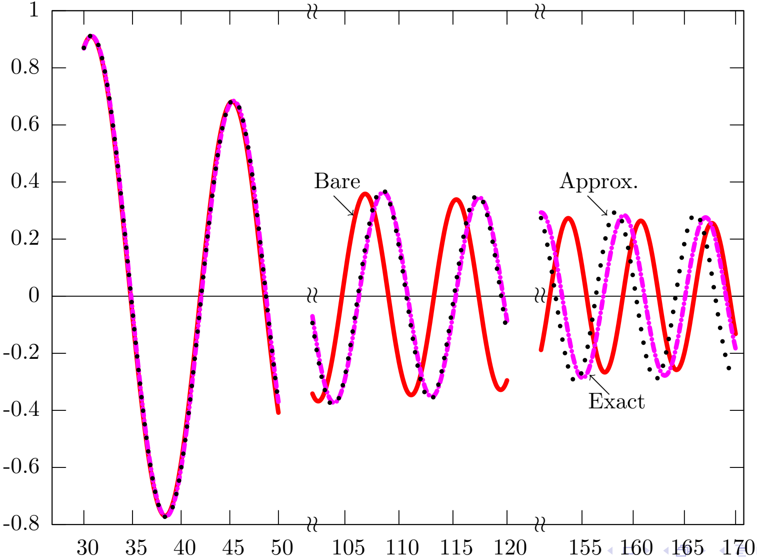
General Corrections

e.g. $\delta = 0$ (Dust), $\epsilon(\phi) \sim 5 \times 10^{-3}$, $a_0 = 9 \times 10^{-6}$.



Dark Energy

The correction terms (initially) look like additional terms in $\mathcal{H}_\phi$.

If we want these additional terms to act like (e.g.) Dark Energy, then we need

$$a^\alpha \partial^\beta \psi(a) \propto a^{\alpha+\beta(\delta+1)/2}\psi(a) \propto a^3 \psi(a) \ .$$

i.e. we need $\alpha + \beta(\delta+1)/2 = 3$.
notice that for a given type of QG correction (given α and β), we only get dark energy-like behaviour for a specific matter component (δ).

So here the 'Dark Energy' would be related to the background matter components.

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i.e. we need $\alpha + \beta(\delta + 1)/2 = 3$.
Also note that for a universe dominated by radiation, $\delta = -1$, the only QG correction that can produce Dark Energy behaviour is a^3, i.e. an explicit cosmological constant term.

Dark Energy

For example, lets consider a matter (dust) dominated universe, $\Rightarrow \delta = 0$.

For the corrections to act like a dark energy component, we need $\alpha + \beta/2 = 3$, i.e.

$$a^3, \quad a^2 \frac{\mathrm{d}^2\psi}{\mathrm{d}a^2}, \quad a \frac{\mathrm{d}^4\psi}{\mathrm{d}a^4}, \quad \frac{\mathrm{d}^6\psi}{\mathrm{d}a^6}, \quad a^{-1} \frac{\mathrm{d}^8\psi}{\mathrm{d}a^8}, \ldots$$

Other possibilities include: QG corrections to a Λ dominated universe giving Dark Matter like behaviour.

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Other possibilities include: QG corrections to a Λ dominated universe giving Dark Matter like behaviour.

Loop Quantum Cosmology

In the continuum limit the (fixed lattice) LQC constraint equation becomes,

$$
\begin{aligned}
\mathcal{H}_{\rm g}\psi(a) &= \frac{2\pi l_{\rm Pl}^2}{3}\left(\frac{1}{a}\frac{\rm d}{{\rm d}a}\left[\frac{1}{a}\frac{\rm d}{{\rm d}a}\left[a\psi(a)\right]\right] + \frac{\rm d}{{\rm d}a}\left[\frac{1}{a}\frac{{\rm d}\psi(a)}{{\rm d}a}\right]\right)\\
&+\frac{2\pi l_{\rm Pl}^2 a_0}{9}\left(\frac{1}{a^3}\frac{{\rm d}^4\psi(a)}{{\rm d}a^4} - \frac{4}{a^4}\frac{{\rm d}^3\psi(a)}{{\rm d}a^3} + \frac{47}{8a^5}\frac{{\rm d}^2\psi(a)}{{\rm d}a^2}\right.\\
&\left.+\frac{1}{2a^6}\frac{{\rm d}\psi(a)}{{\rm d}a} - \frac{135}{16a^7}\psi(a)\right),
\end{aligned}
$$

to first order in $\frac{a_0}{a} = \left(\frac{\mu_0}{\mu}\right)^2$.

So we can apply what we did before to see what these corrections look like as be approach the instability.

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$$+\frac{2\pi l_{\mathrm{Pl}}^2 a_0}{9}\left(\frac{1}{a^3}\frac{\mathrm{d}^4\psi(a)}{\mathrm{d}a^4} - \frac{4}{a^4}\frac{\mathrm{d}^3\psi(a)}{\mathrm{d}a^3} + \frac{47}{8a^5}\frac{\mathrm{d}^2\psi(a)}{\mathrm{d}a^2}\right.$$
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Keeping only the largest term, as before we find that the 'corrected' Wheeler-DeWit equation is,

$$\frac{1}{2}\left(\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[a\psi\left(a\right)\right]\right]+\frac{\mathrm{d}}{\mathrm{d}a}\left[\frac{1}{a}\frac{\mathrm{d}\psi\left(a\right)}{\mathrm{d}a}\right]\right) \\ +\frac{a_0}{3}\epsilon\left(\phi\right)^2 a^{2\delta-1}\psi\left(a\right)+\epsilon\left(\phi\right)a^{\delta}\psi\left(a\right)\approx 0\,,$$

where $\epsilon\left(\phi\right)a^{\delta}\psi\left(a\right)$ is the dominant matter Hamiltonian

and $\frac{a_0}{3}\epsilon\left(\phi\right)^2 a^{2\delta-1}\psi\left(a\right)$ is how the Loop Quantum corrections initially behave.

Loop Quantum Cosmology

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$$\frac{1}{2}\left(\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}a}\left[a\psi(a)\right]\right]+\frac{\mathrm{d}}{\mathrm{d}a}\left[\frac{1}{a}\frac{\mathrm{d}\psi(a)}{\mathrm{d}a}\right]\right)$$
$$+\frac{a_0}{3}\epsilon(\phi)^2 a^{2\delta-1}\psi(a)+\epsilon(\phi)a^{\delta}\psi(a)\approx 0\,,$$

where $\epsilon(\phi)a^{\delta}\psi(a)$ is the dominant matter Hamiltonian

and $\frac{a_0}{3}\epsilon(\phi)^2 a^{2\delta-1}\psi(a)$ is how the Loop Quantum corrections initially behave.

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where $\epsilon\left(\phi\right)a^{\delta}\psi\left(a\right)$ is the dominant matter Hamiltonian

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LOOP QUANTUM COSMOLOGY

The Loop Quantum corrections initially behave as $\frac{a_0}{3}\epsilon\,(\phi)^2\,a^{2\delta-1}\psi\,(a)$.

e.g. For $\delta = 0$, matter dominated: LQC correction looks like radiation.

e.g. For $\delta = 2$, (stiff matter ?): LQC corrections look like Cosmological Constant...

but it has the wrong sign!

For $\delta = 3$, Λ dominated e.g. inflation: LQC corrections scale like a^5. So this will eventually dominate over the background matter source, which is precisely the instability that motivates lattice refinement.

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Conclusions

- Quantum gravity corrections to the Wheeler-DeWit equation can resemble additional matter components, such as Dark Energy.
- For LQC, they have the wrong sign and would require a dominant matter field that scales like a^2 to get Dark Energy.
- Different LQC corrections might play a role
  e.g. correlations/ fluctuations.
- Given a QG correction we can immediately calculate how it would behave at large scales.

Small scale Quantum Gravity corrections can produce large scale corrections to General Relativity.

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Thanks for Listening

ANY QUESTIONS?